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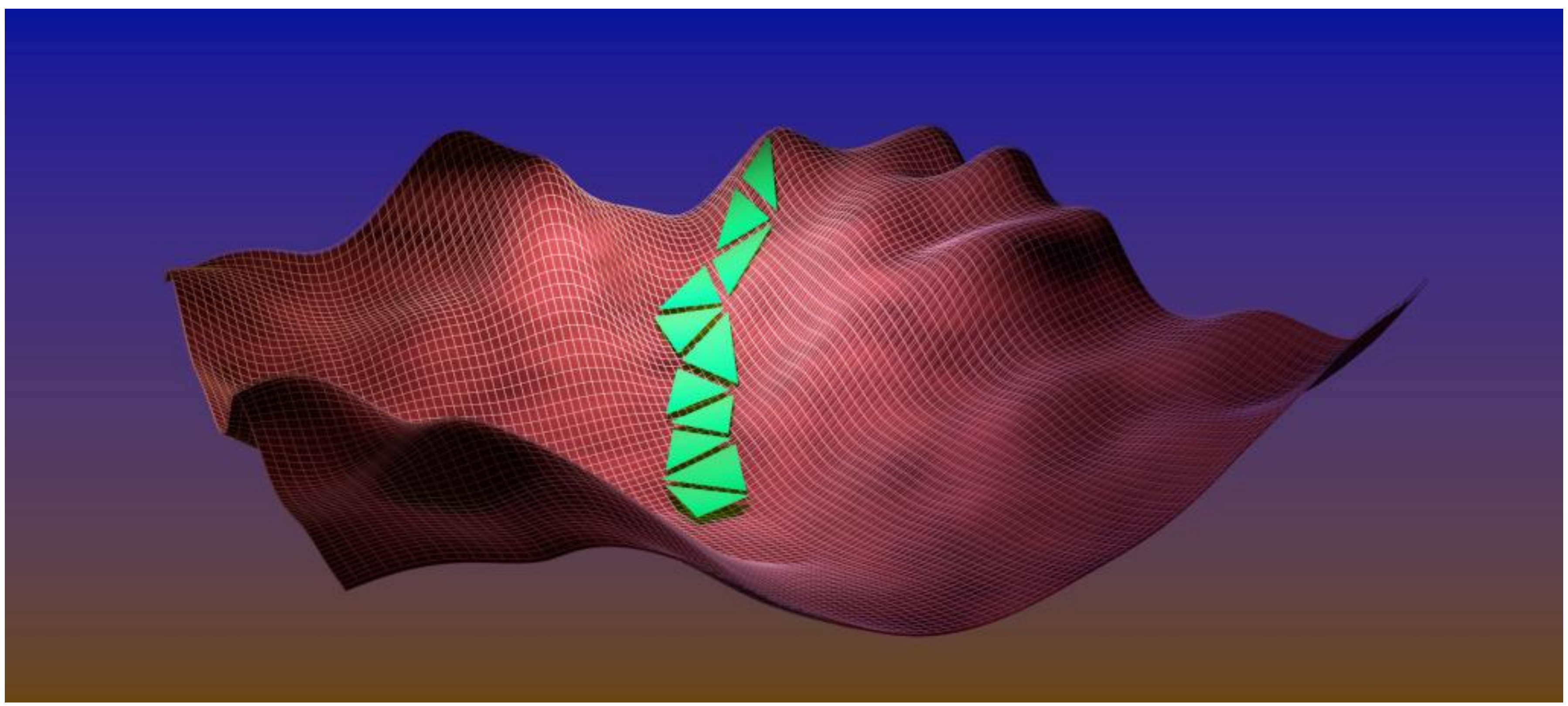
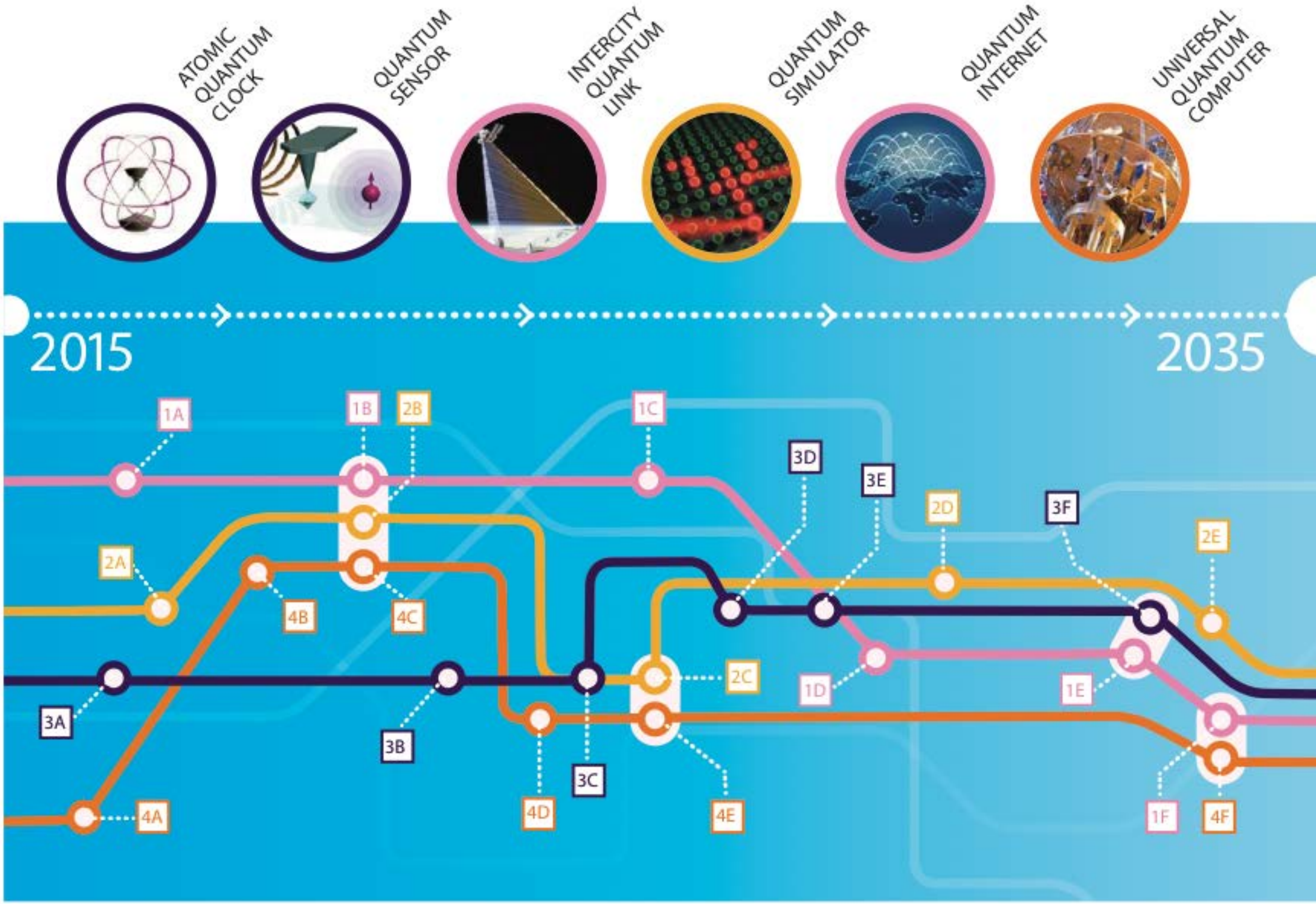
IQ ST INTEGRATED QUANTUM SCIENCE AND TECHNOLOGY

Quantum optimal control for quantum technologies

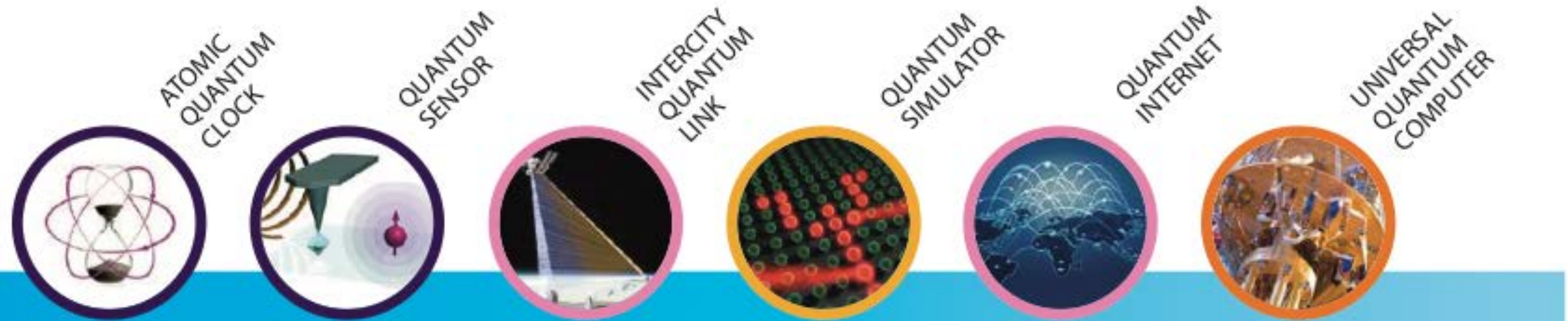


Tommaso Calarco

Quantum Manifesto



Quantum Manifesto



	1. Communication	2. Simulators	3. Sensors	4. Computers
0 – 5 years	A Core technology of quantum repeaters B Secure point-to-point quantum links	A Simulator of motion of electrons in materials B New algorithms for quantum simulators and networks	A Quantum sensors for niche applications (incl. gravity and magnetic sensors for health care, geosurvey and security) B More precise atomic clocks for time stamping of high-frequency financial transactions	A Operation of a logical qubit protected by error correction or topologically B New algorithms for quantum computers C Small quantum processor executing technologically relevant algorithms
5 – 10 years	C Quantum networks between distant cities D Quantum credit cards	C Development and design of new complex materials D Versatile simulator of quantum magnetism and electricity	C Quantum sensors for larger volume applications including automotive, construction D Handheld quantum navigation devices	D Solving chemistry and materials science problems with special purpose quantum computer > 100 physical qubit
> 10 years	E Quantum repeaters with cryptography and eavesdropping detection F Secure Europe-wide internet merging quantum and classical communication	E Simulators of quantum dynamics and chemical reaction mechanisms to support drug design	E Gravity imaging devices based on gravity sensors F Integrate quantum sensors with consumer applications including mobile devices	E Integration of quantum circuit and cryogenic classical control hardware F General purpose quantum computers exceed computational power of classical computers

Outline

1. **Controlling quantum dynamics**
    - quantum optimal control
    - gradient methods
2. **Pushing the limits**
    - time: the Quantum Speed Limit
    - size: the CRAB algorithm
3. **Applying CRAB to experiments**
    - open-loop vs closed-loop
    - few-body vs many-body
4. **Understanding complexity**
    - control complexity vs bandwidth
    - integrable vs non-integrable systems

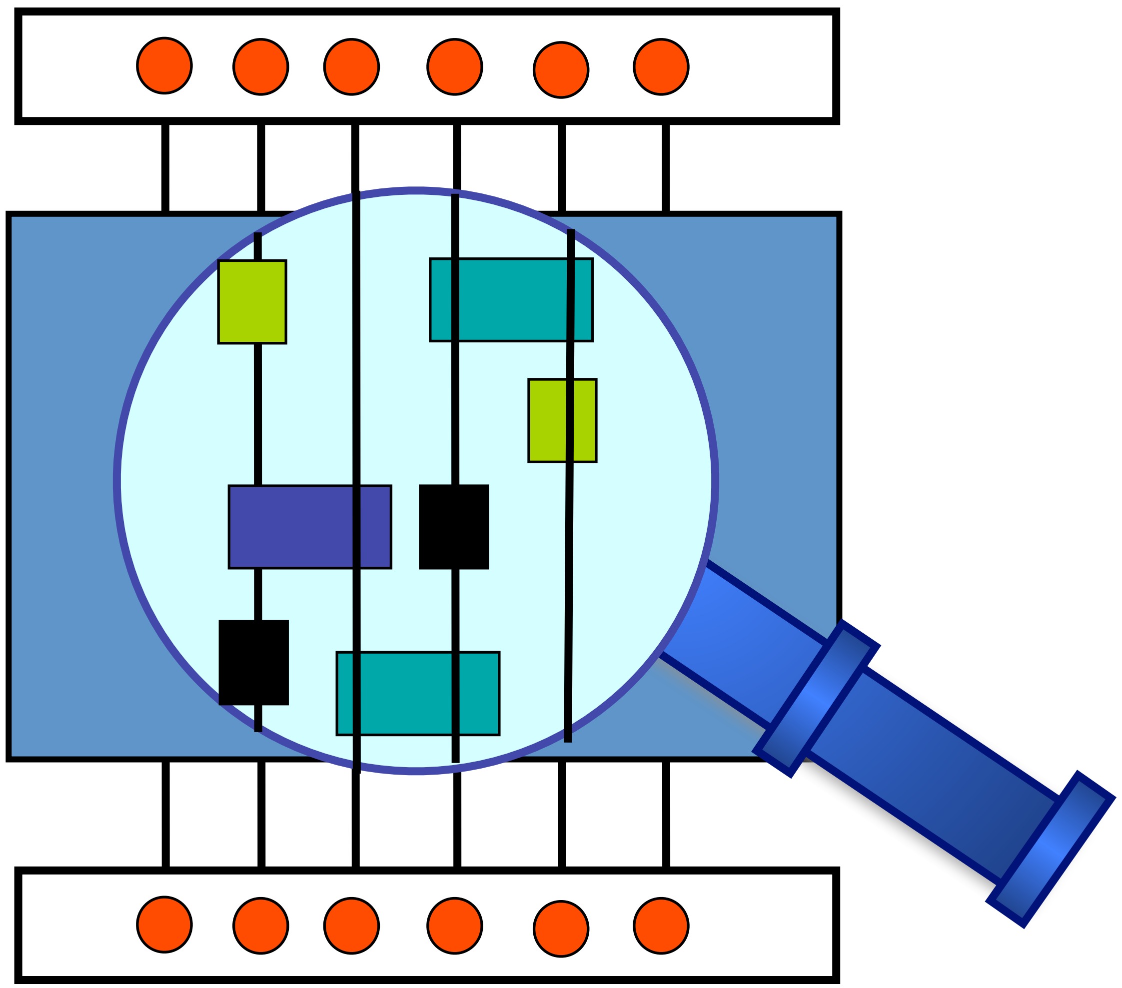
Scalability

What do we need?

[TC, Grangier, Walraff, Zoller, Nature Phys. '08]

What do we need for quantum computing?

1. scalable system of well-characterized qubits
2. initialize qubits
3. long decoherence times
4. universal set of quantum gates
5. qubit readout



Goal: fault-tolerance error threshold

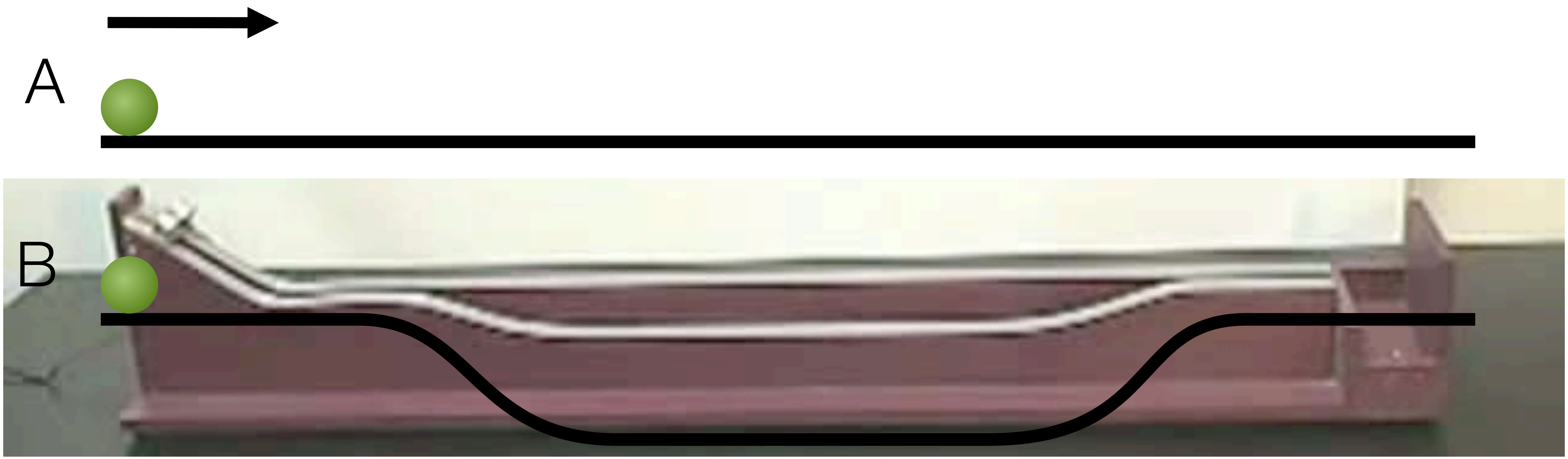
What do we need for scalability?

- Memory:
  - Quantum register with many qubits
  - Low decoherence rates
- Gates:
  - Fast operation
  - High fidelity
- …implementation with ultracold systems:
  - Good isolation from environment
  - Individual control
  - Periodic potentials

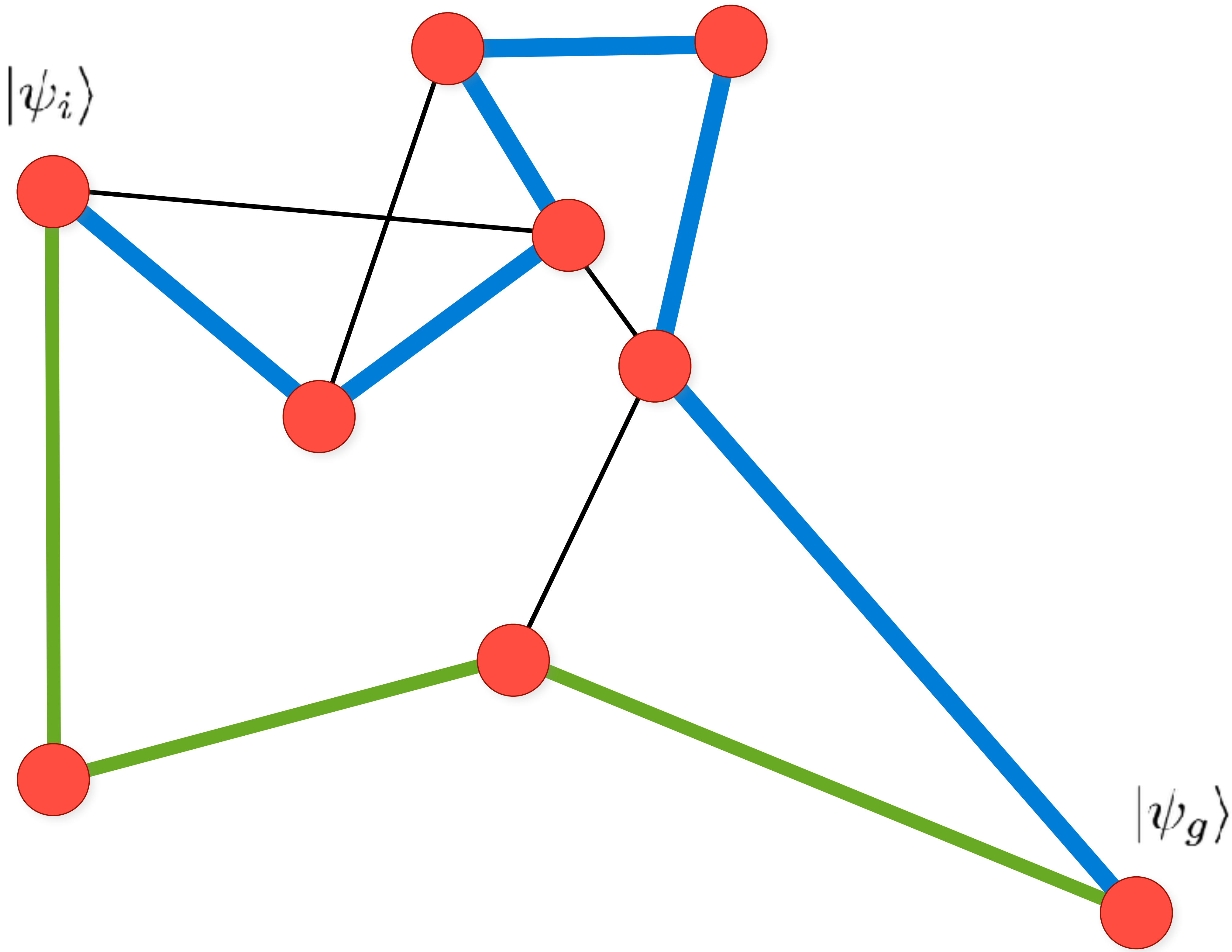
Controlling quantum dynamics

What can we do?

Time optimality: a classical ‘wiggle’



1. Ball A wins.
2. Ball B wins.
3. It's a draw.



Quantum controllability: an intuitive picture

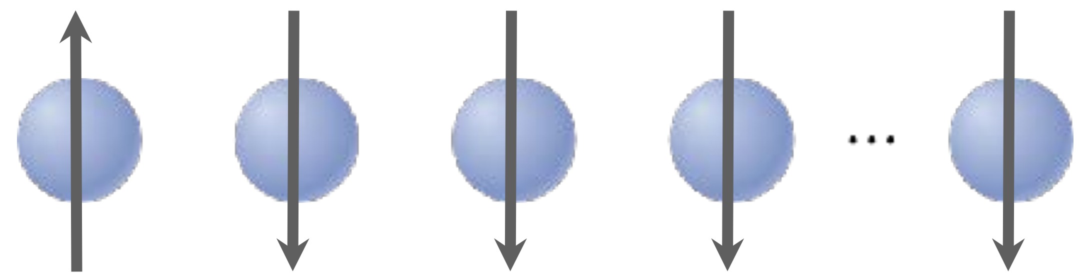
Which path is ‘optimal’?

How can we steer quantum processes?

- You can't get it right the first time
- Retrying helps improve
- You've got to know your stuff
- Once you know how to do it, you may do remarkable things
- Don't forget to tilt the tray!
- ... then you can make it more interesting

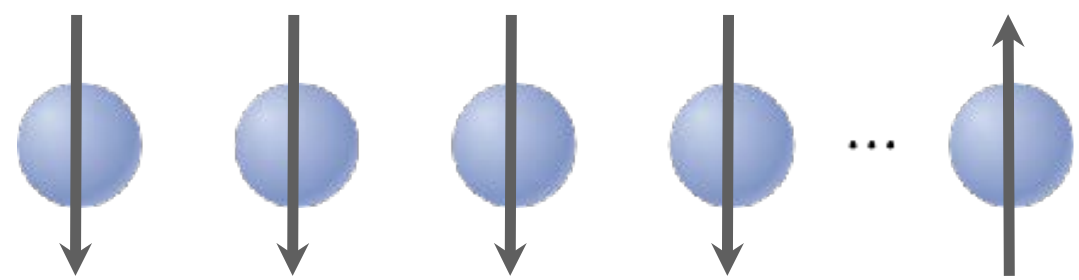
A simple “many-body” system: the spin chain

Initial state

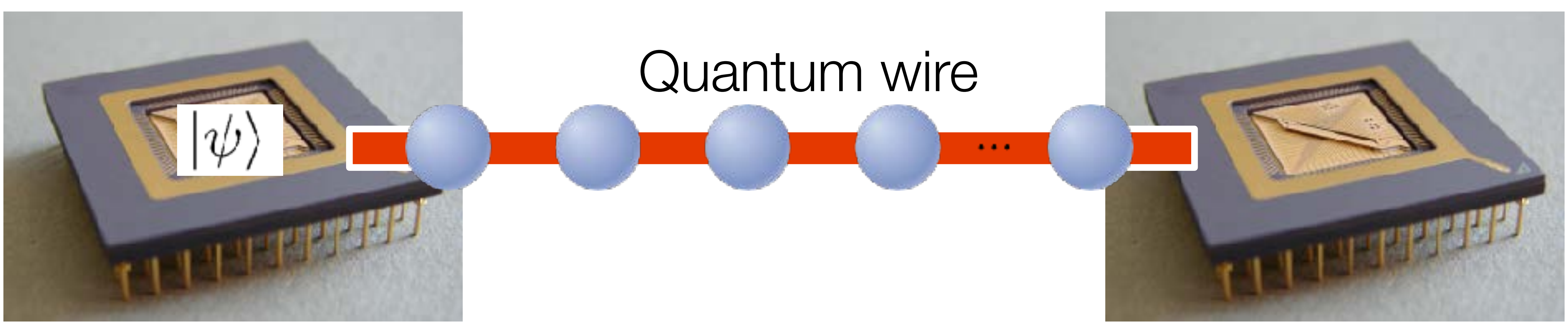


$$|\psi(0)\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |00\ldots\rangle$$

Final state



$$|\psi(T)\rangle = |00\ldots\rangle \otimes (\alpha|0\rangle + \beta 1\rangle)$$



S. Bose, PRL 91 207901 (2003)

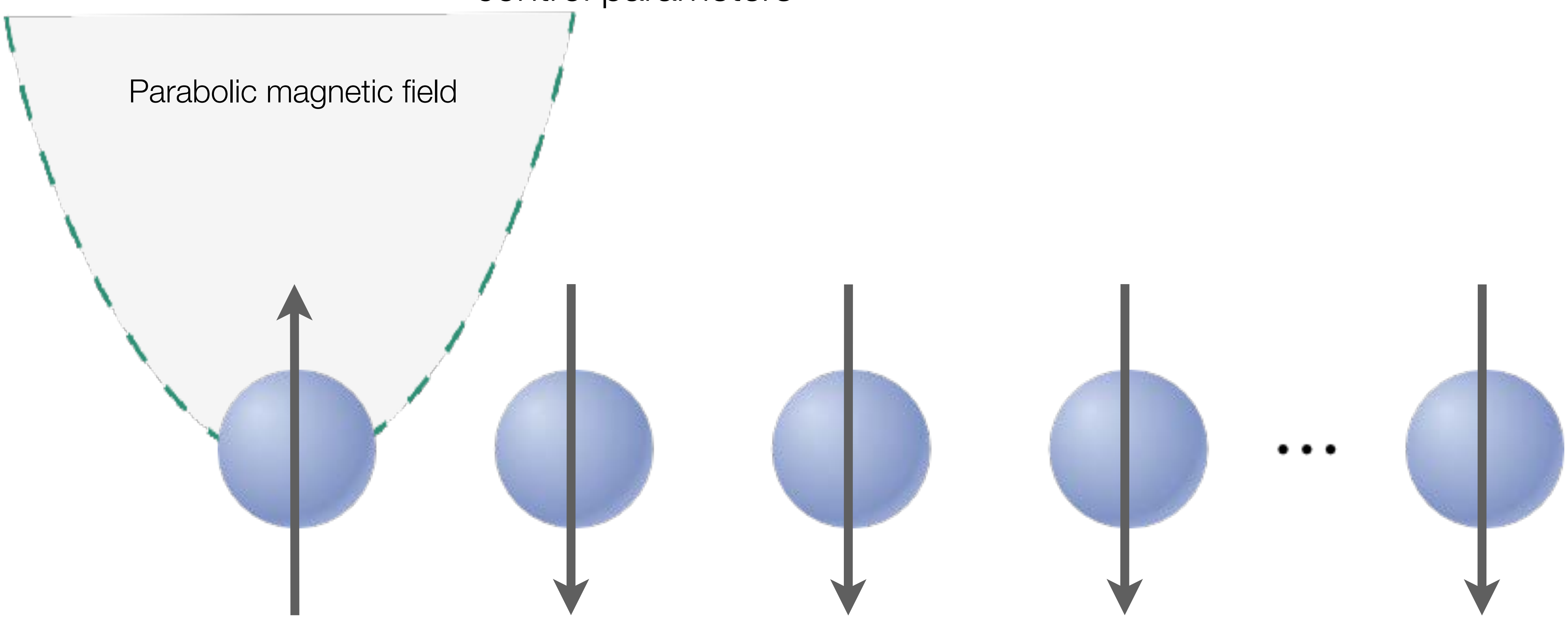
Transporting a state through the chain

$$H = -\frac{J}{2}\sum_{n=0}^{N-2} \vec{\sigma}_n \cdot \vec{\sigma}_{n+1}$$

Balachandran and Gong, Phys. Rev. A 77, 012303 (2008)

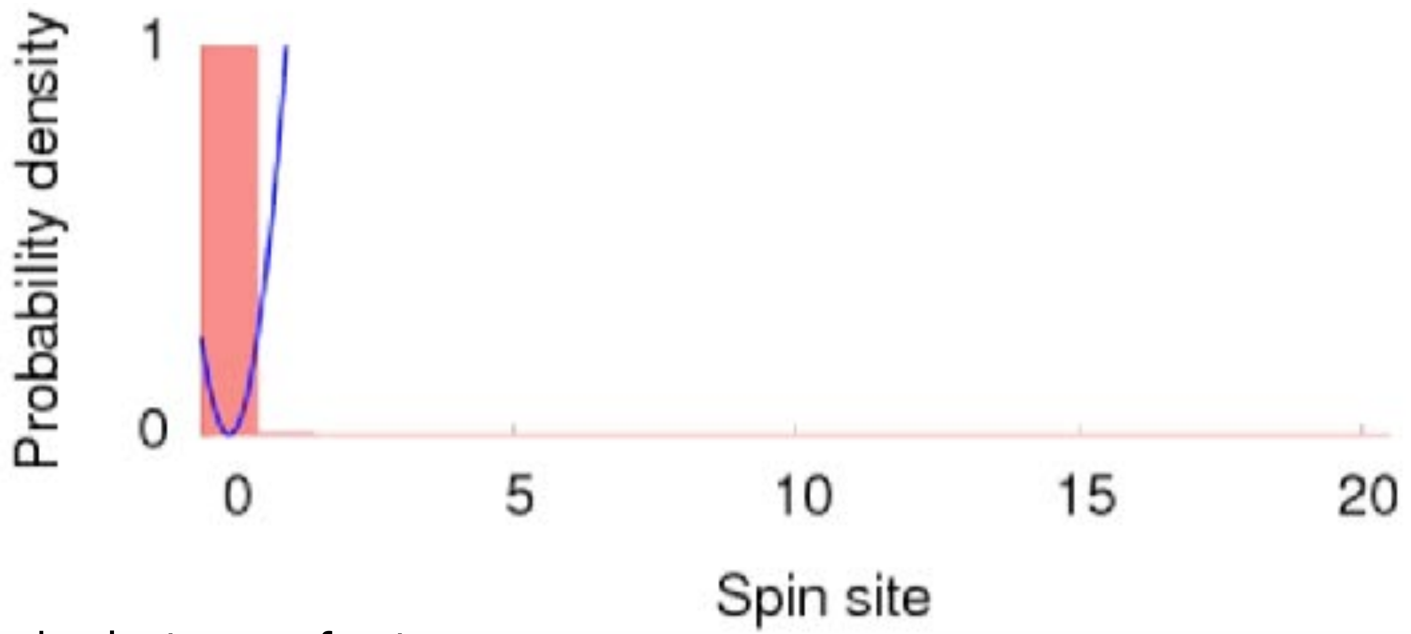
spin – spin coupling (nearest neighbour)

control parameters



Speeding up state transfer

No problem adiabatically,



But if we try naively to go faster...

Tilting the tray: the Krotov algorithm

Initial state $|\psi_0\rangle$

Iterate

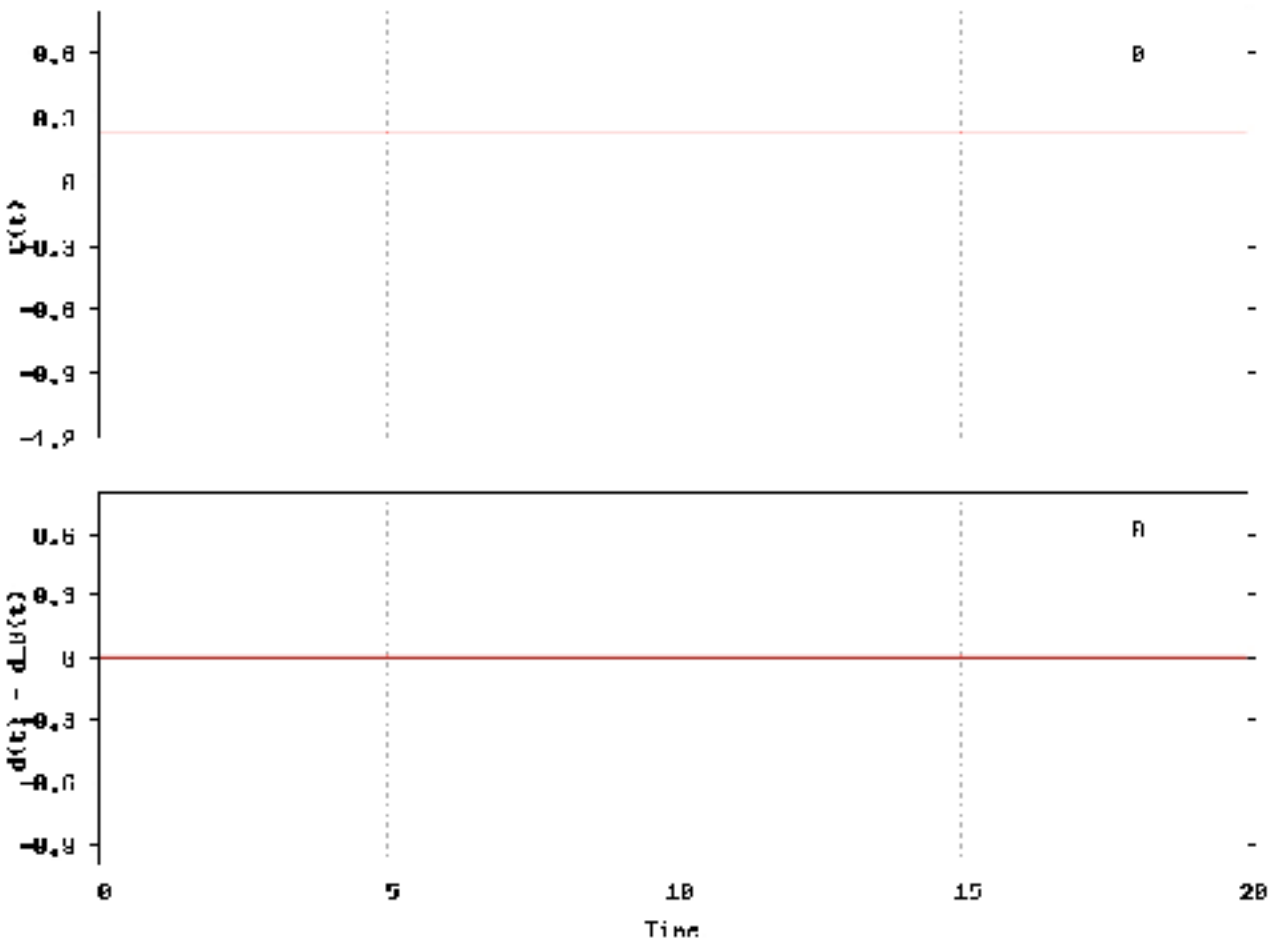
Evolve with control *d* $$|\psi_0\rangle \longrightarrow |\psi_d(T)\rangle \equiv \mathcal{U}(d,T)|\psi_0\rangle$$

Project onto goal state and evolve back $$|\chi_d(T)\rangle = |\psi_{\rm goal}\rangle\langle\psi_{\rm goal}|\psi_d(T)\rangle$$

Update the control pulse $$d \longrightarrow d + \mathcal{F}[\mathcal{H}, |\psi_d\rangle, |\chi_d\rangle]$$

$$d \longrightarrow d + \frac{2}{\lambda}\Im\langle\chi_d|\frac{\partial\mathcal{H}}{\partial d}|\psi_d\rangle$$

Juggling the system until destination



Running the optimization...

99%

After optimization:

es faster

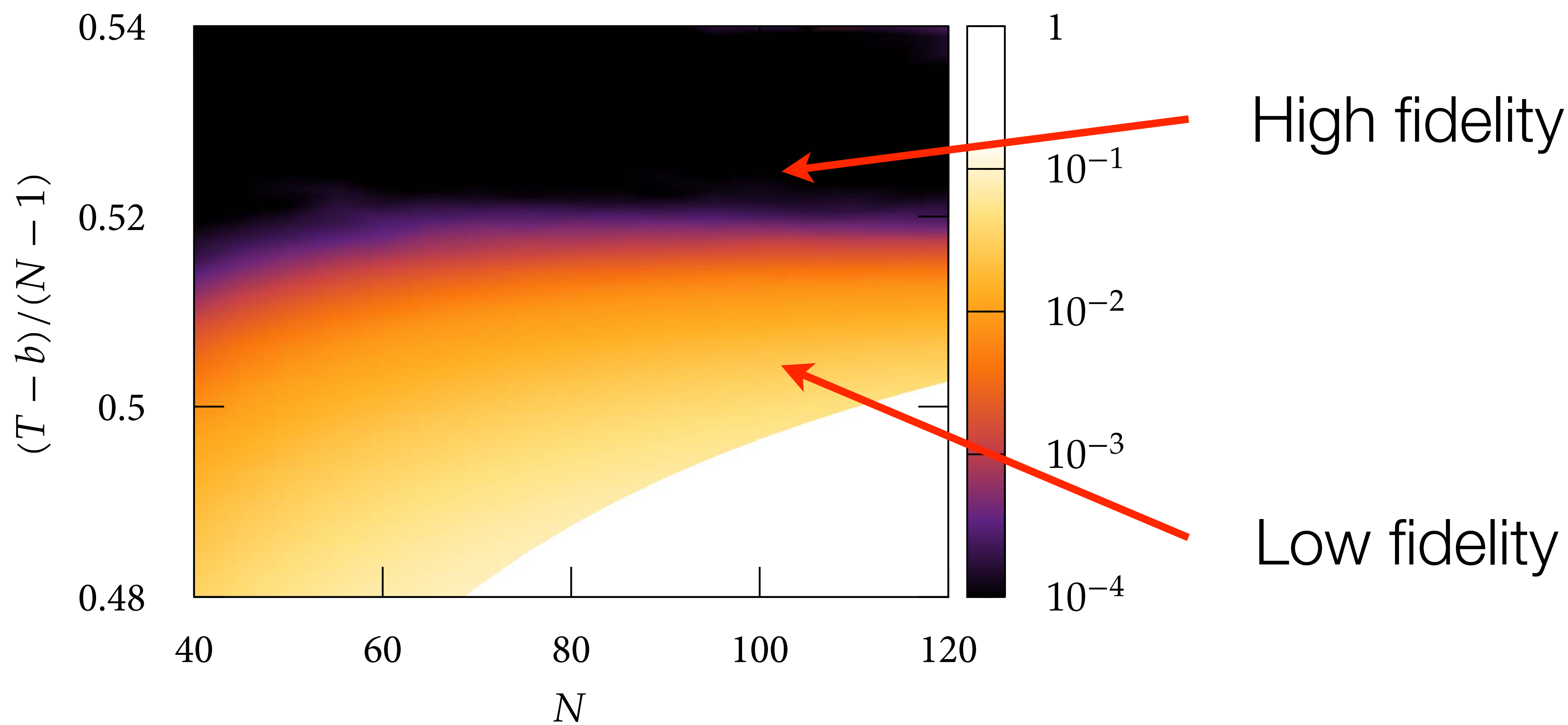
Pushing the limits

in time: the Quantum Speed Limit

How fast can we go?

Hitting the Quantum Speed Limit

T. Caneva, M. Murphy, TC, R. Fazio, S. Montangero, V. Giovannetti, and G. E. Santoro, PRL 2010

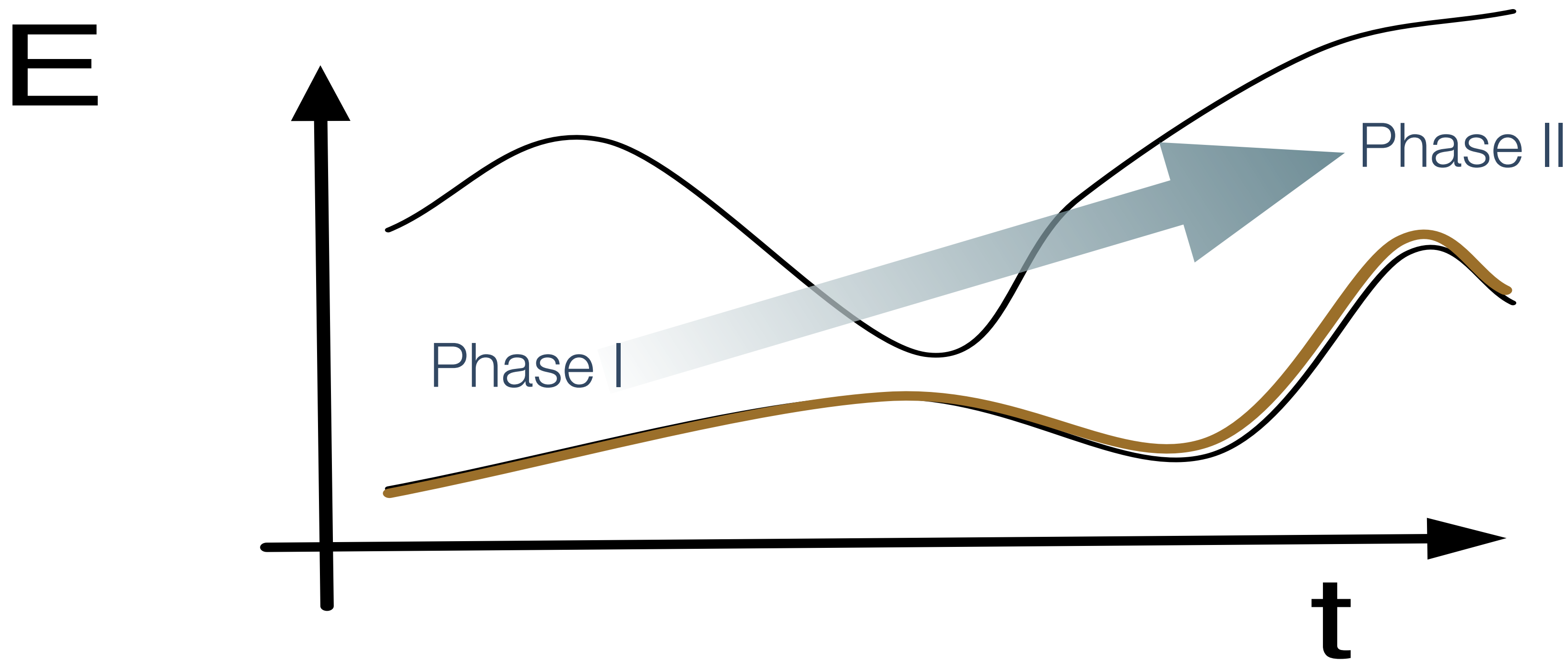


Theory: max speed is given by N repeated swaps

Optimal Control reaches the Quantum Speed Limit

An intuitive explanation

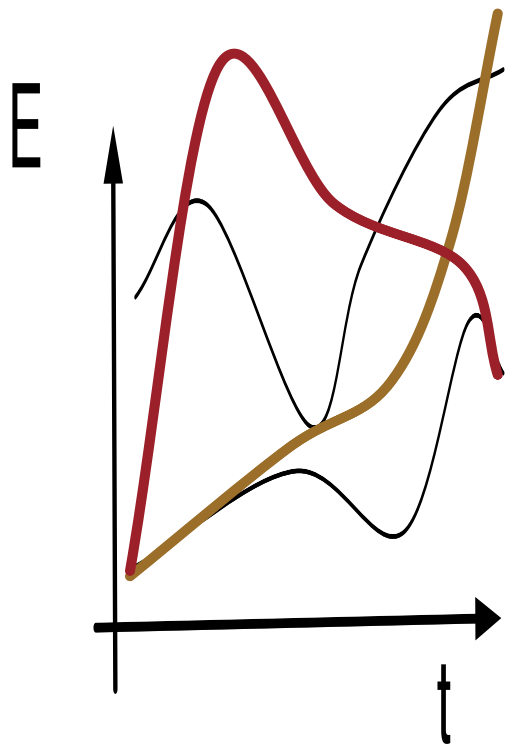
Slow



Adiabatic strategy

An intuitive explanation

Fast

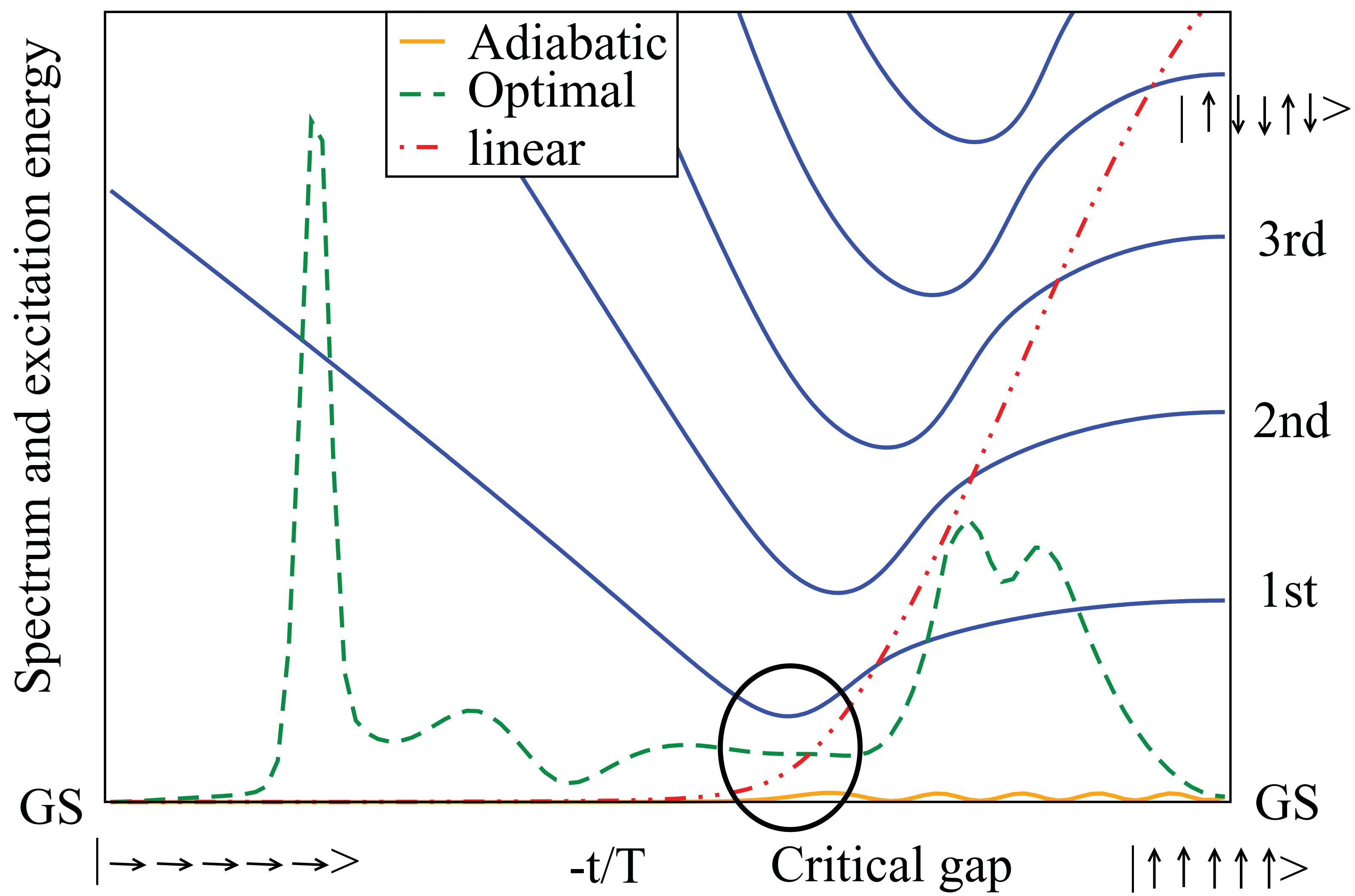


Optimal control

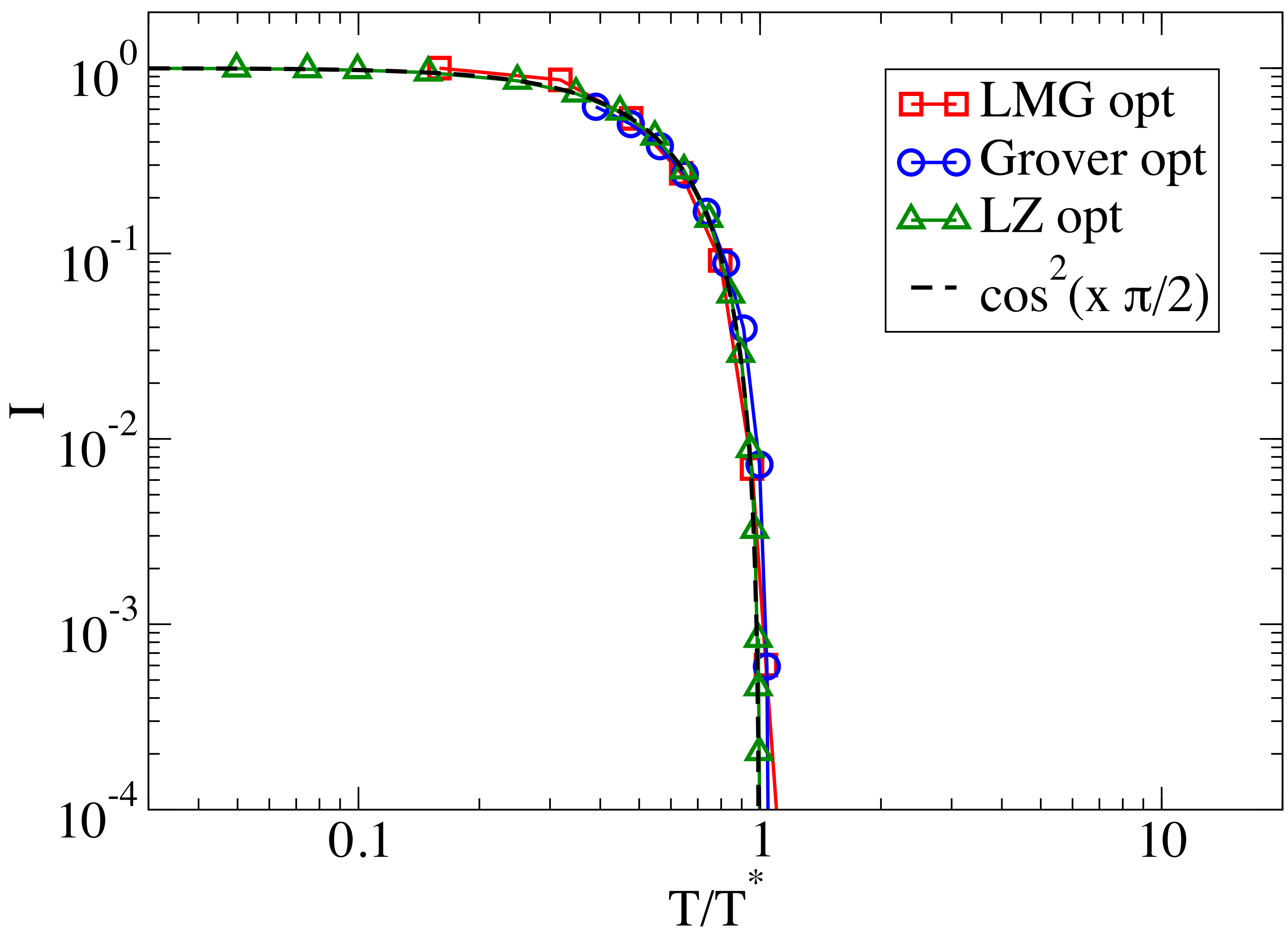
An actual many-body example

Lipkin-Meshkov-Glick model

$$H^{\mathrm{LMG}} = -\sum_{i<j}^{N} J_{ij}\sigma_i^x\sigma_j^x - \Gamma(t)\sum_{i}^{N}\sigma_i^z$$



Universal error scaling at the Quantum Speed Limit



T. Caneva, TC, R. Fazio, G. E. Santoro, and S. Montangero, Phys. Rev. A 84, 012312 (2011)

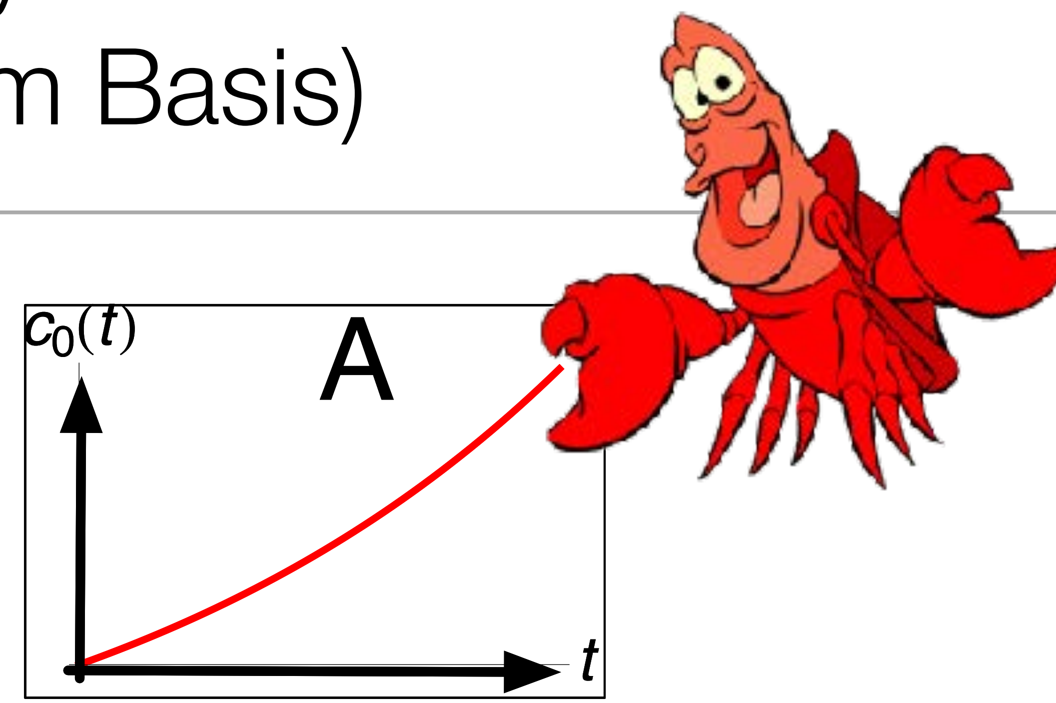
Pushing the limits

in size: the CRAB algorithm

How BIG can we go? $\Bigg|\psi_g\Bigg\rangle$ $|\psi_g\rangle$

Controlling many-body systems: CRAB (Chopped RAndom Basis)

Initial guess: $c_0(t)$



Correction:

$$g(t) = \sum_{k=1}^{n} a_k \tilde{f}_k(t)$$

$\tilde{f}_k(t)$ "randomized" basis functions

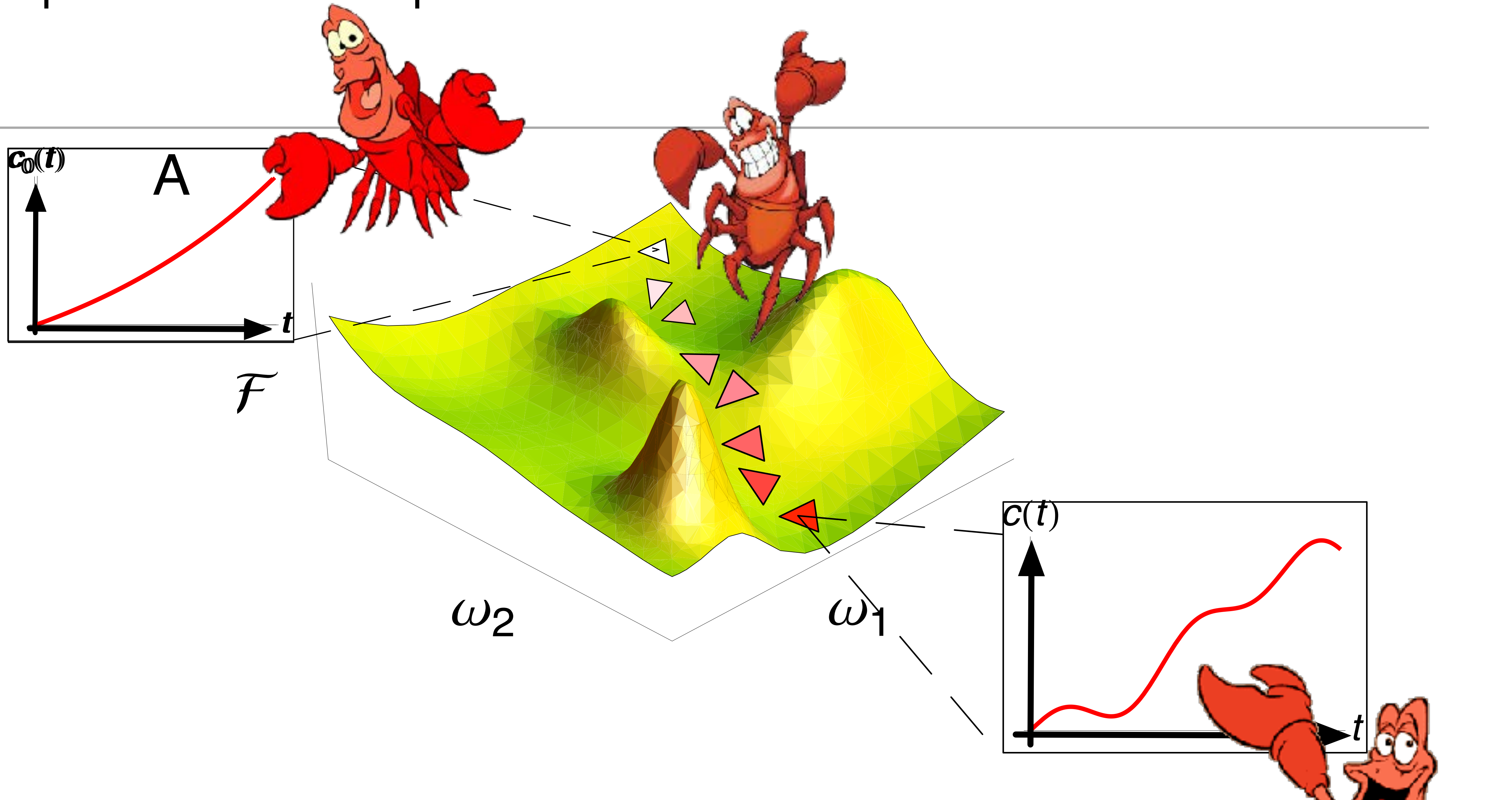
Examples: $f_k(t) = sin(\omega_k t),\ x_k^{\alpha},\ H_k(x), ...$

Trial pulse: $c(t) = c_0(t) g(t)$

Optimize only n=O(10) parameters

Doria, Calarco, Montangero, PRL 2011

Simplex search optimization



- No need for gradient (Nelder-Mead, simplex, etc.)
- No need for (semi-)analytical solutions
- Figures of merit: energy, fidelity, purity, entanglement.

Applying CRAB to experiments

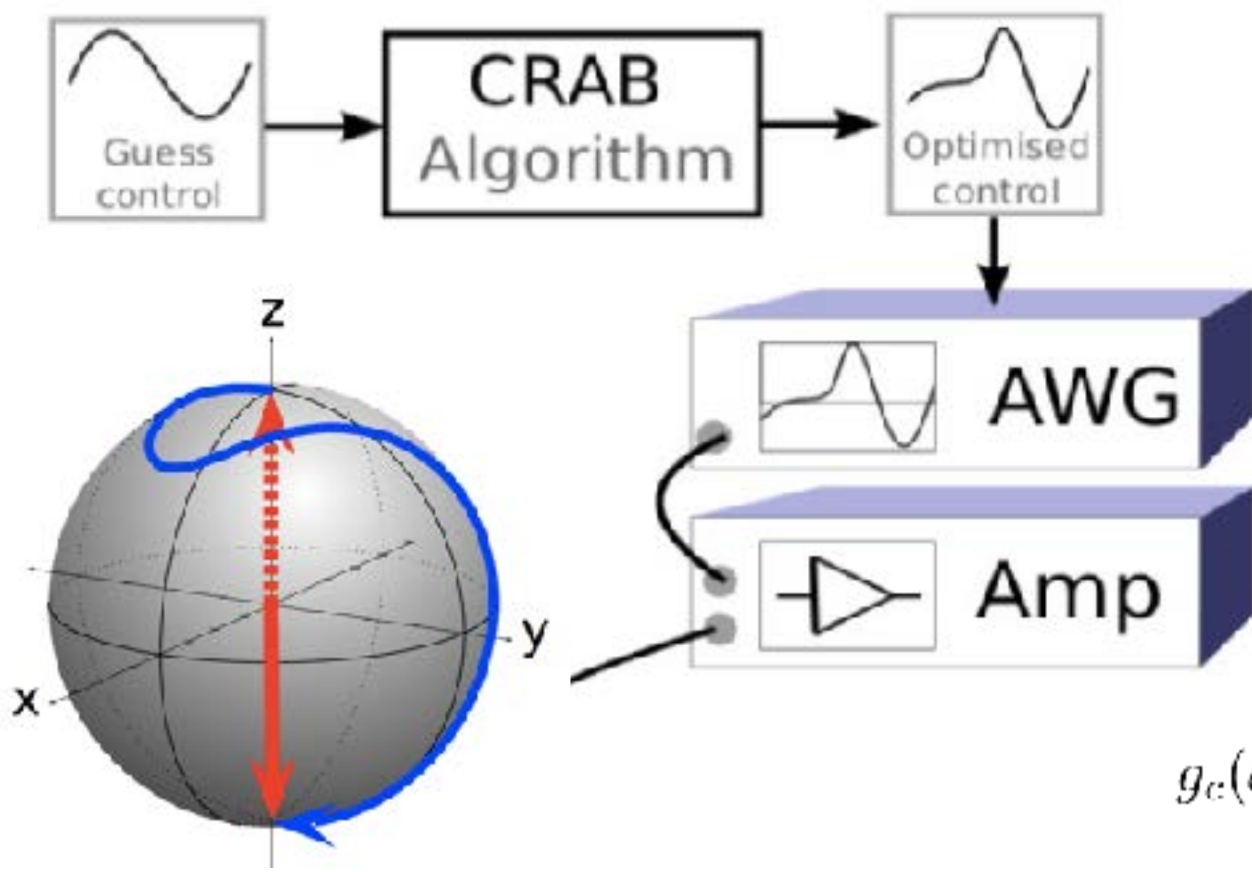
in NV centers: open- and closed-loop

CRAB control of NV centers

- Diamond NV Centers

$$\hat{\mathcal{H}} = 2\pi\Delta\hat{S}_z^2 + 2\pi g_e\mu_B^* B_z\hat{S}_z + 2\sqrt{2}\pi\Gamma(t)\hat{S}_x$$

Zero Field Zeeman Splitting Single control

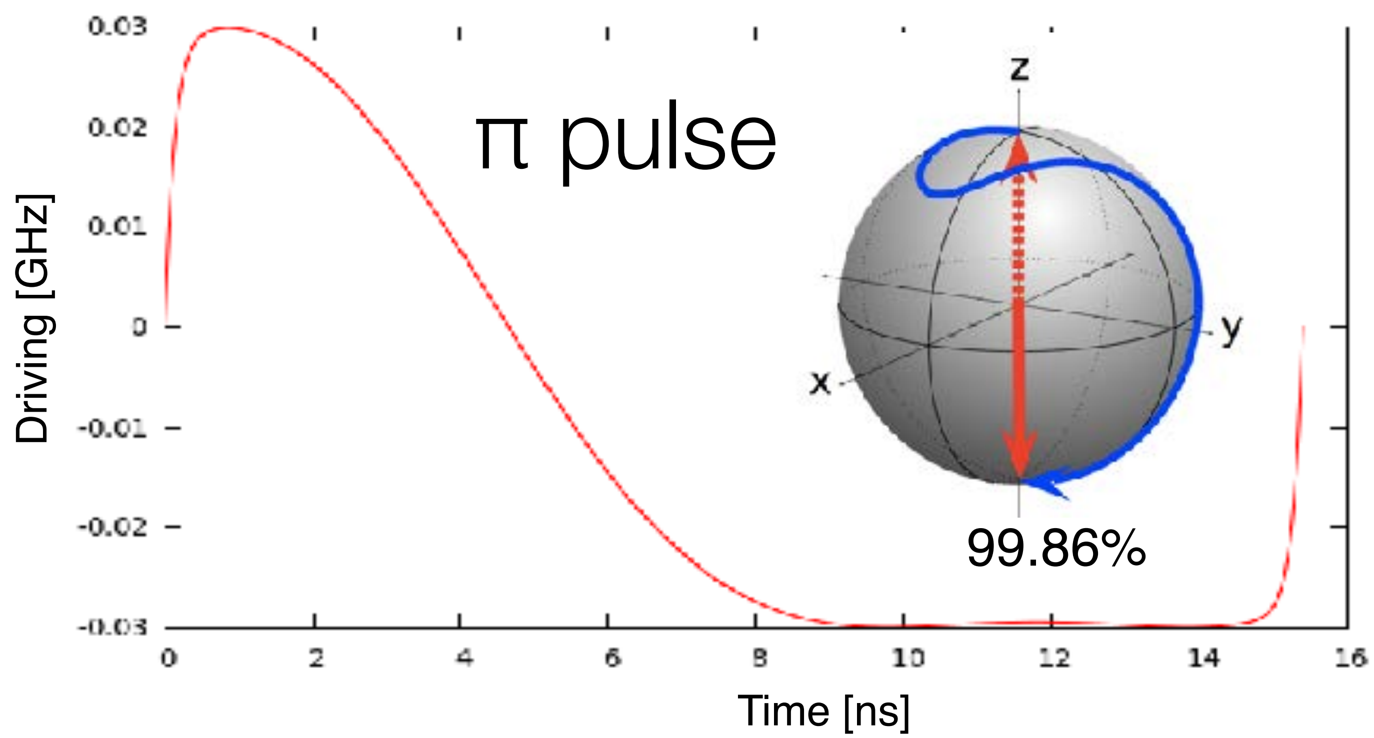


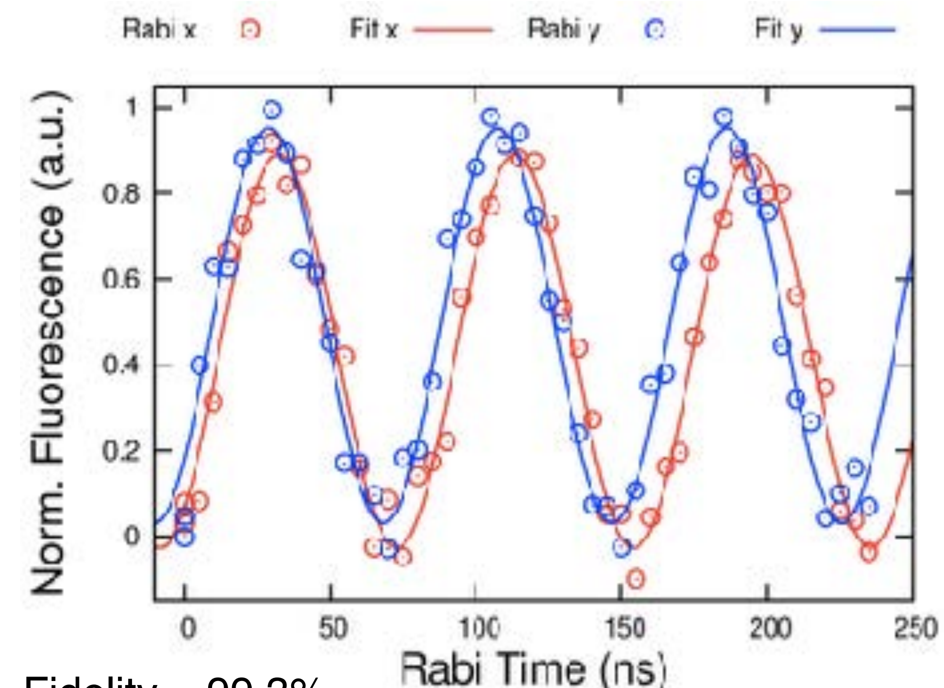
$$\Gamma(t) = \Gamma_0 \times g_c(\omega, t)$$

Guess Update

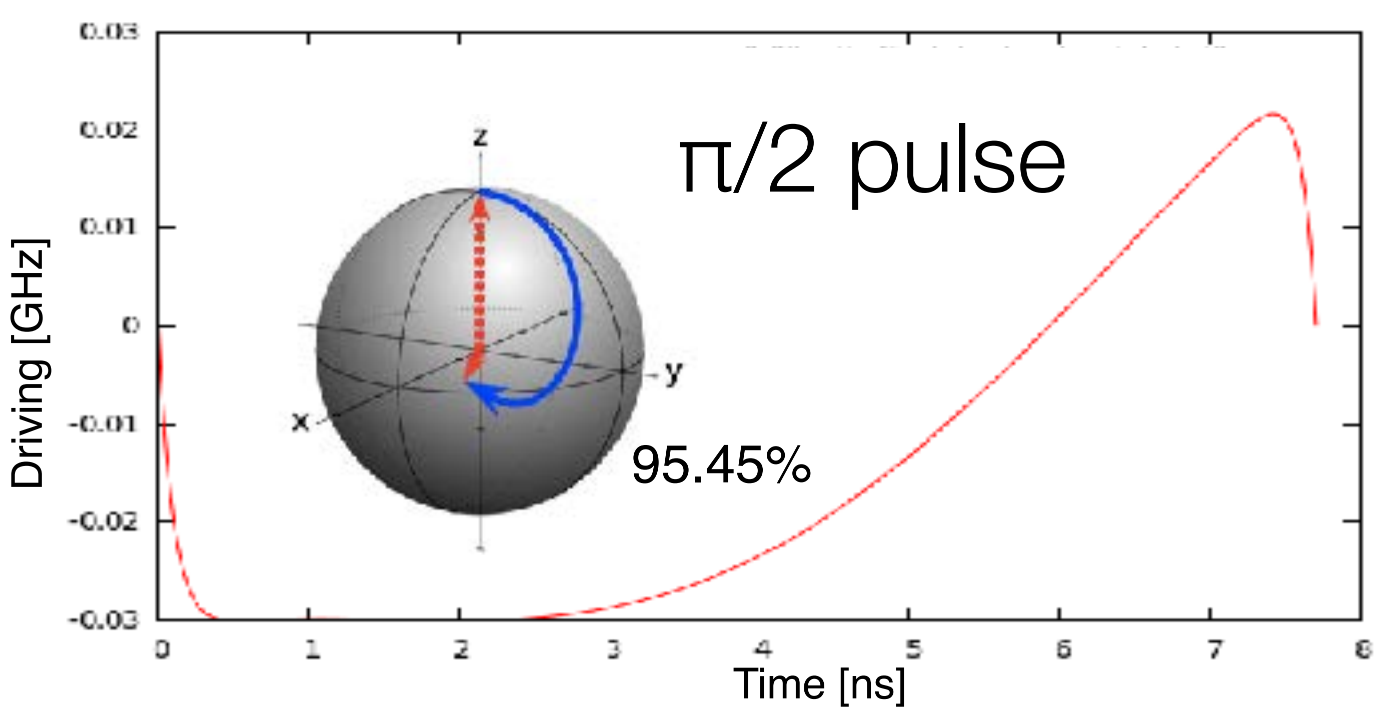
$$g_c(\omega, t) = \frac{1}{2N\lambda(t)}\sum_n^N a_n \sin(\omega_n t) + b_n \cos(\omega_n t)$$

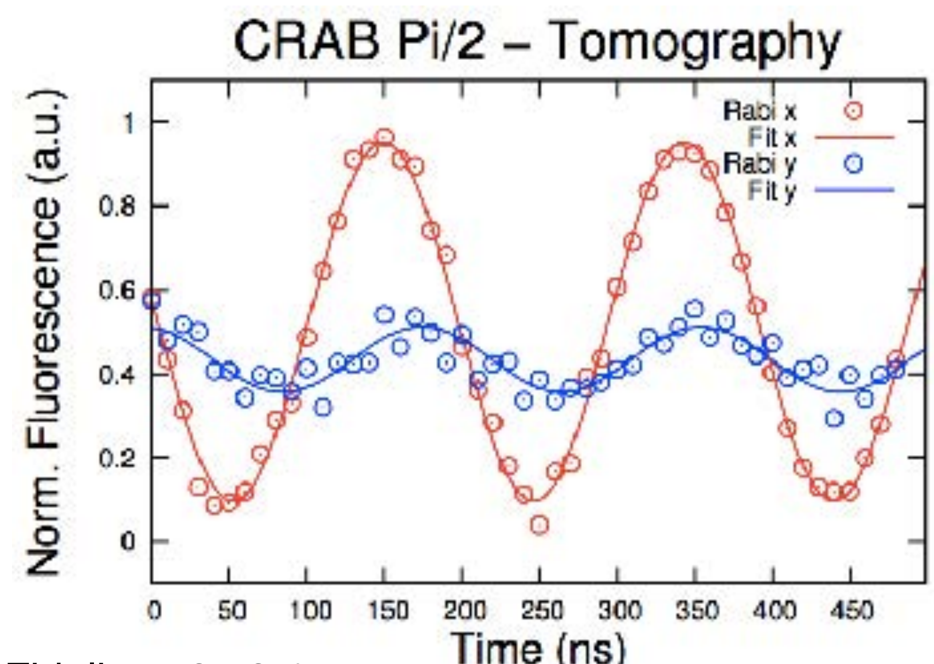
Open-loop CRAB control of NV centers





Fidelity = 99.3%



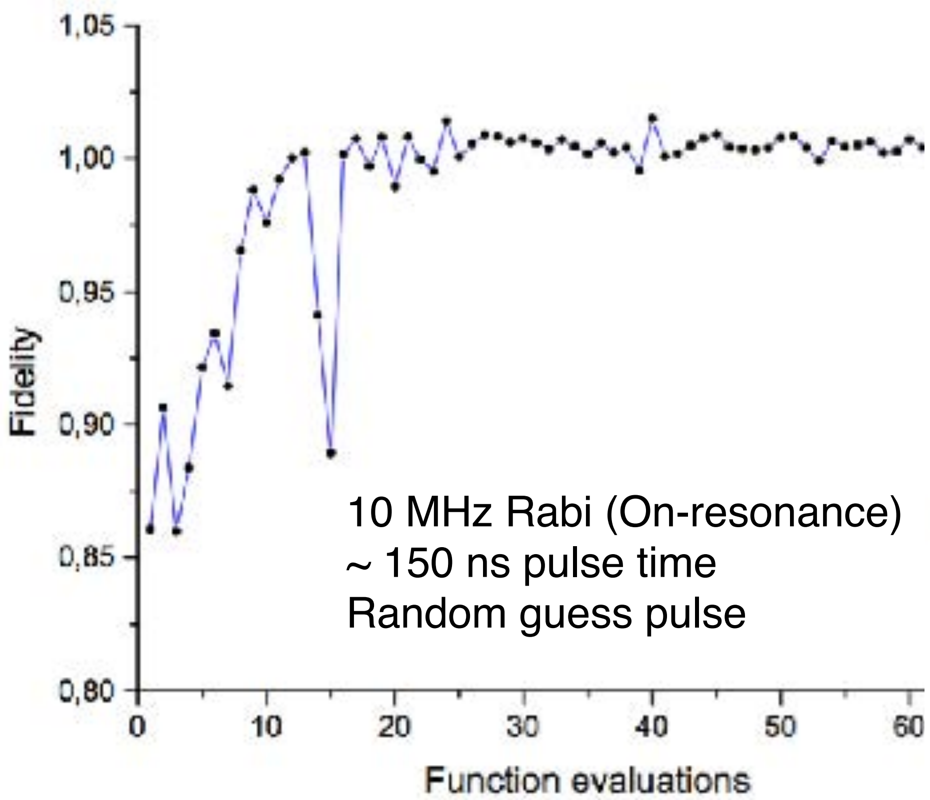


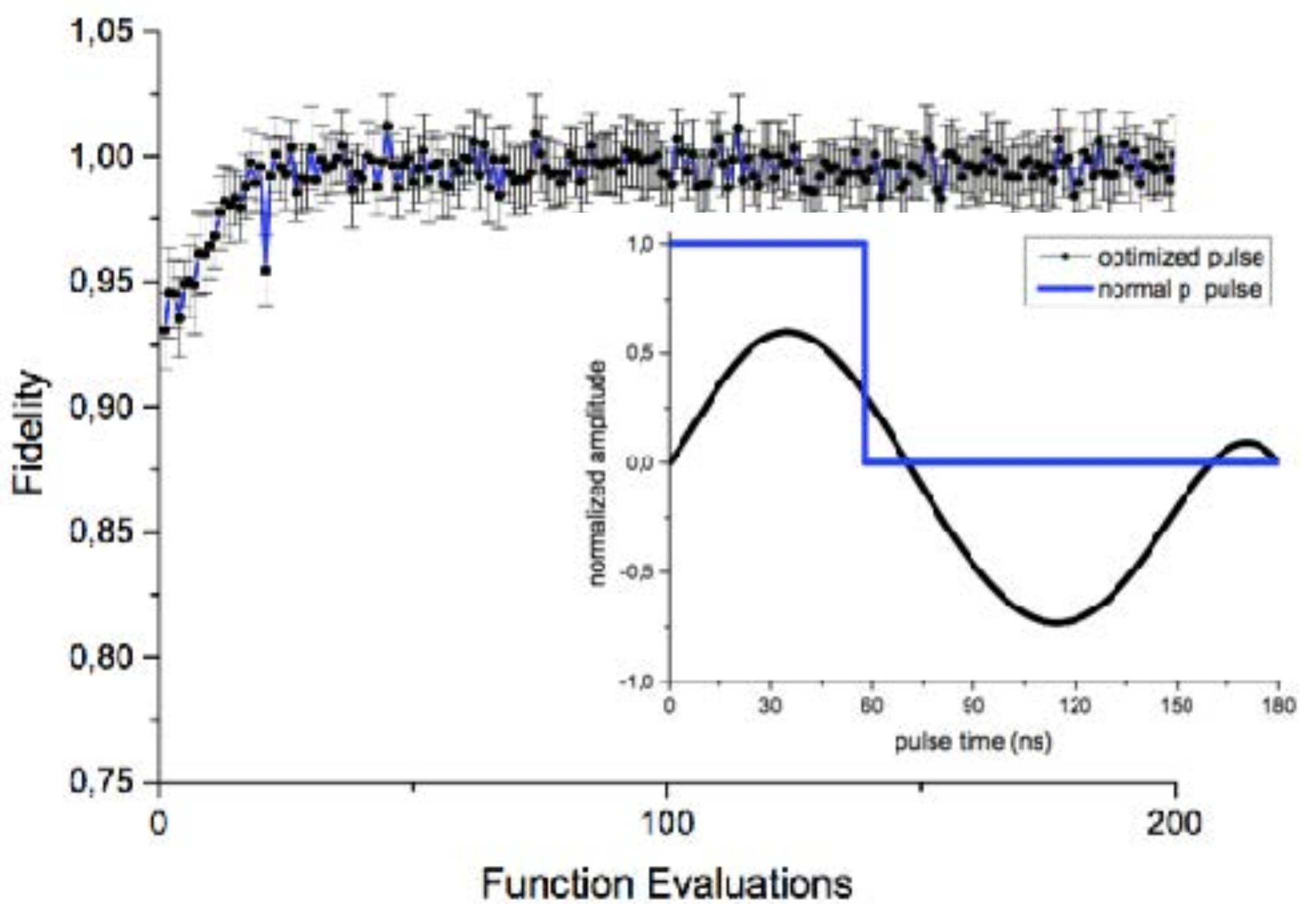
Fidelity = 95.9%

Scheuer et al. (F. Jelezko group), NJP 2014

Closed-loop CRAB control of NV centers

- Simple pi-rotation in RWA regime
- Random guess, fidelity to 99% in 25 FE ~50 mins





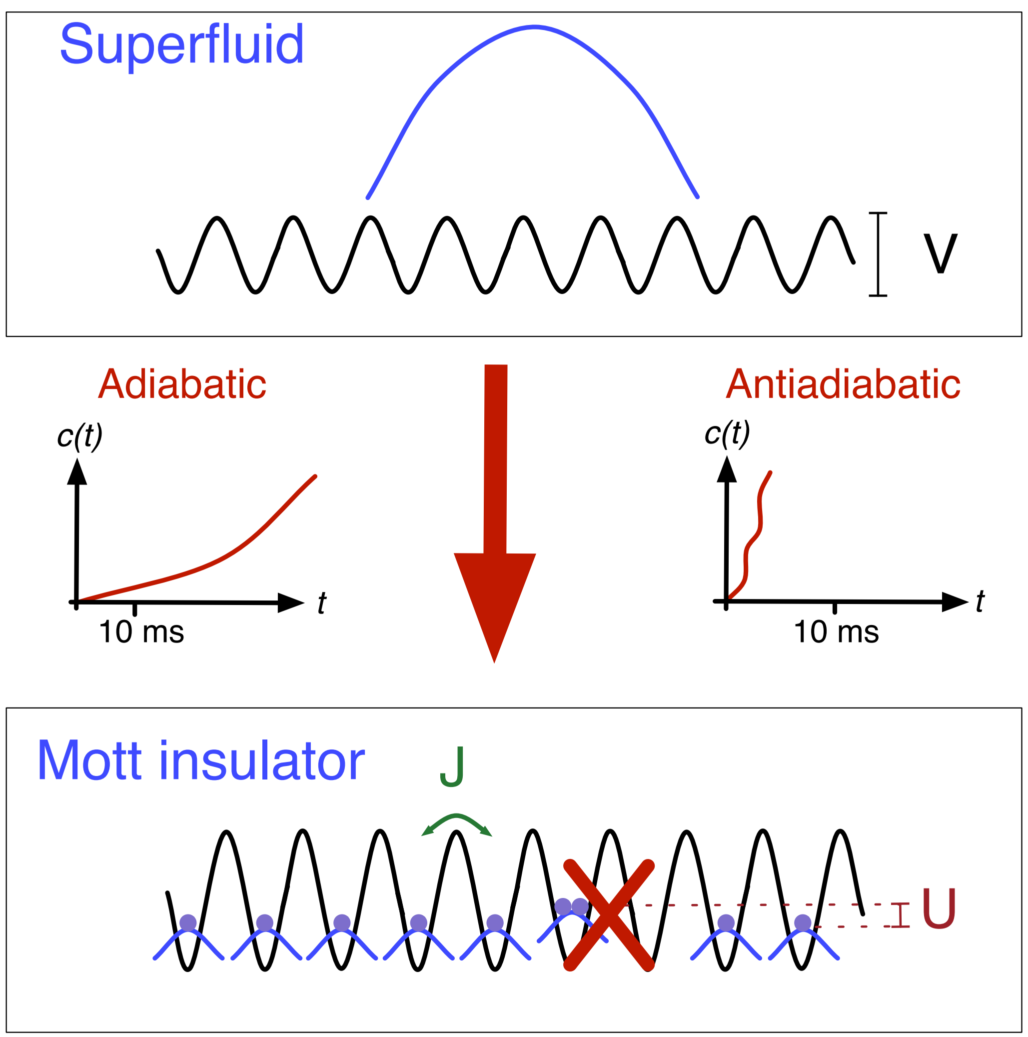
Applying CRAB to experiments

in atomic many-body systems

Antiadiabatic phase transition with cold atoms

Bose Hubbard model

$$\mathrm{H}=\sum_j\left[-J(b_j^\dagger b_{j+1}+\mathrm{h.c.})+\Omega\left(j-\frac{N}{2}\right)^2 n_j+\frac{U}{2}(n_j^2-n_j)\right]$$



J/U >> 0.1

J Hopping

U Onsite energy

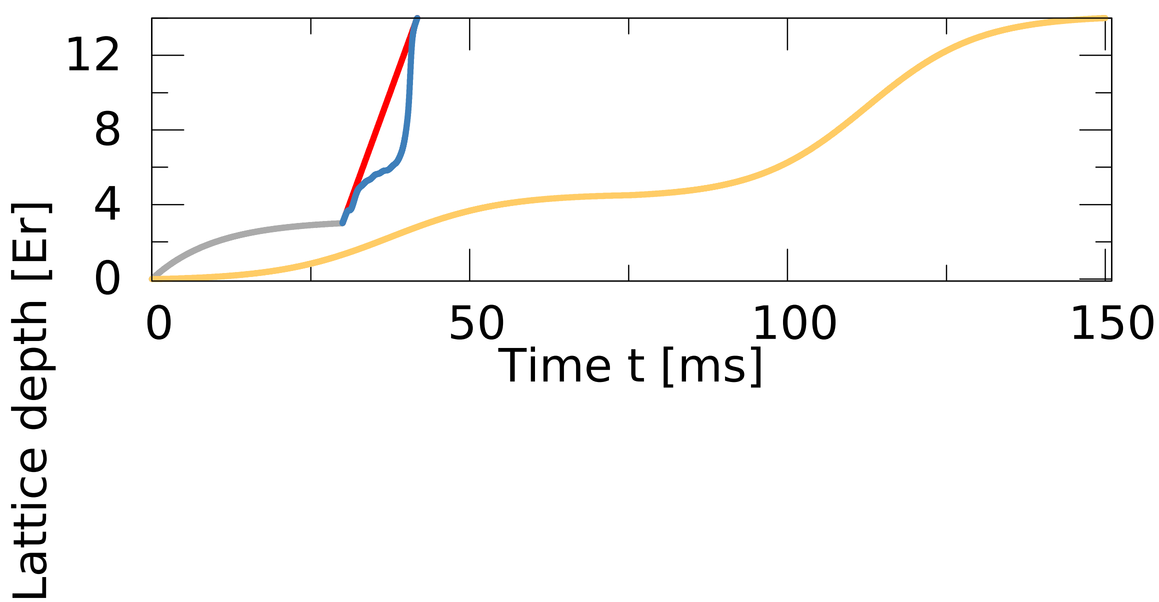
Ω Trapping

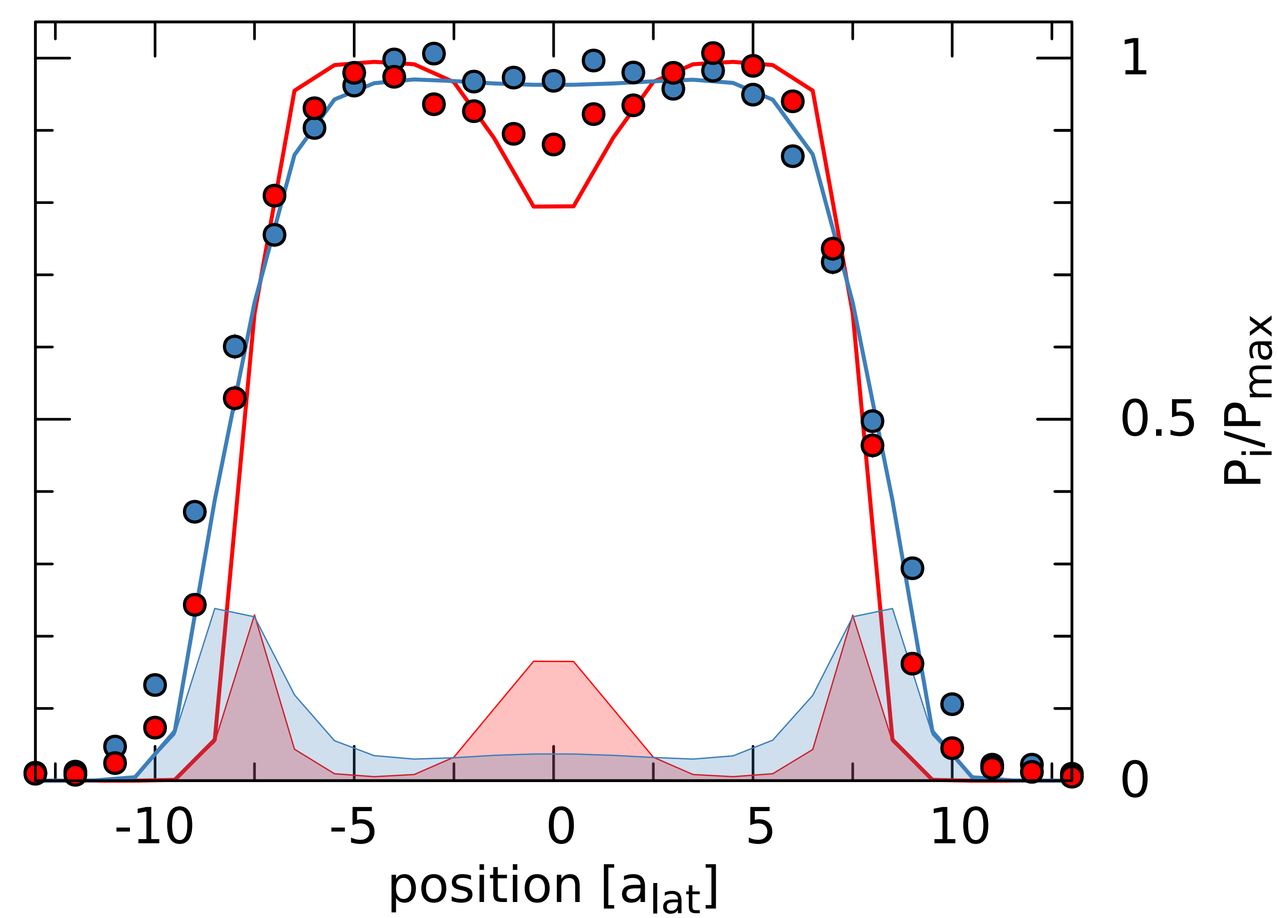
J/U << 0.1

M. Greiner, O. Mandel, T. Esslinger, T. W. Hansch and I. Bloch, Nature 415, 39 (2002).

Experimental results

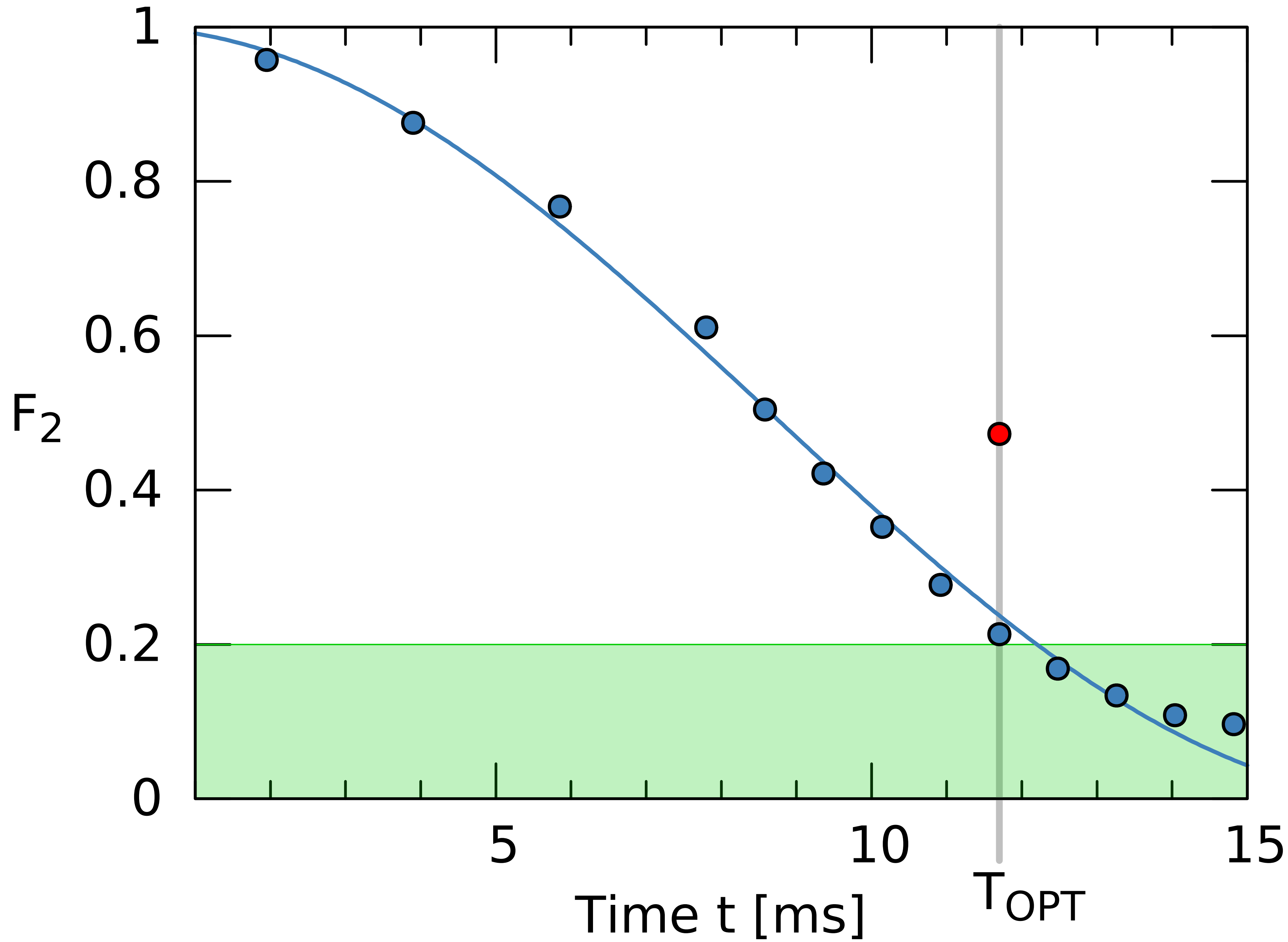
- Experiment by I. Bloch, MPQ Garching





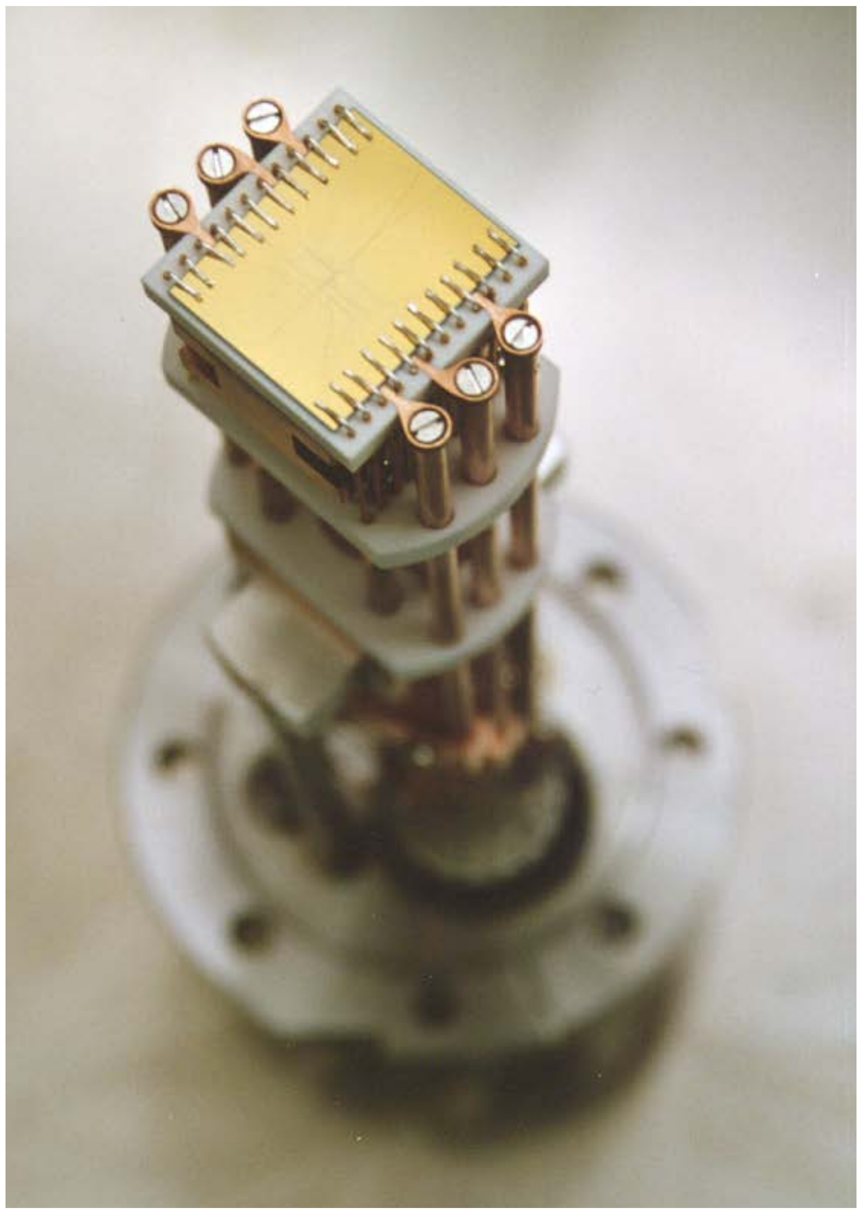
S. van Frank et al. (Bloch, Schmiedmayer and TC groups), arXiv:1511.02247

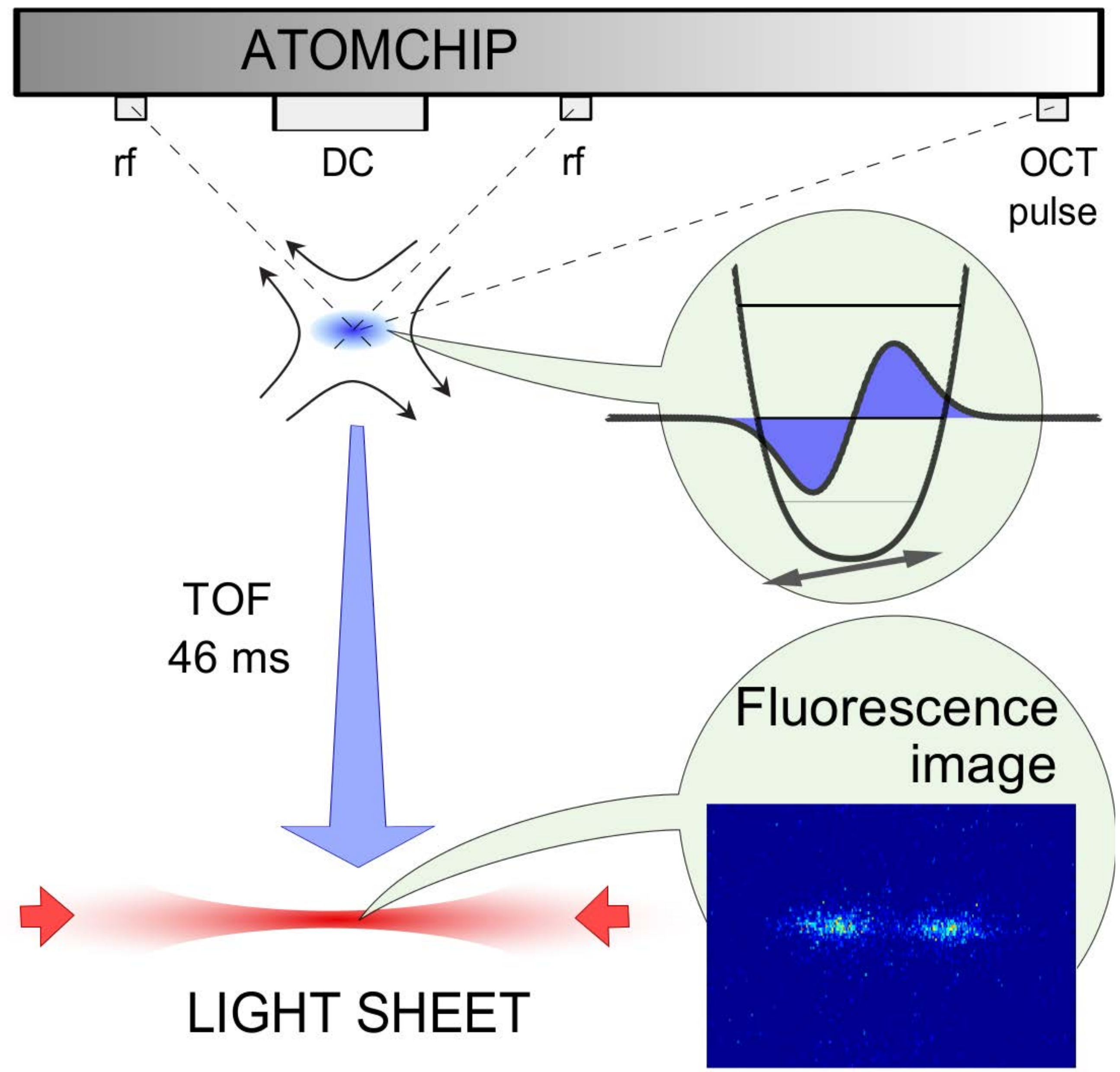
Quantum phase transition at the Speed Limit



S. van Frank et al. (Bloch, Schmiedmayer and TC groups), arXiv:1511.02247

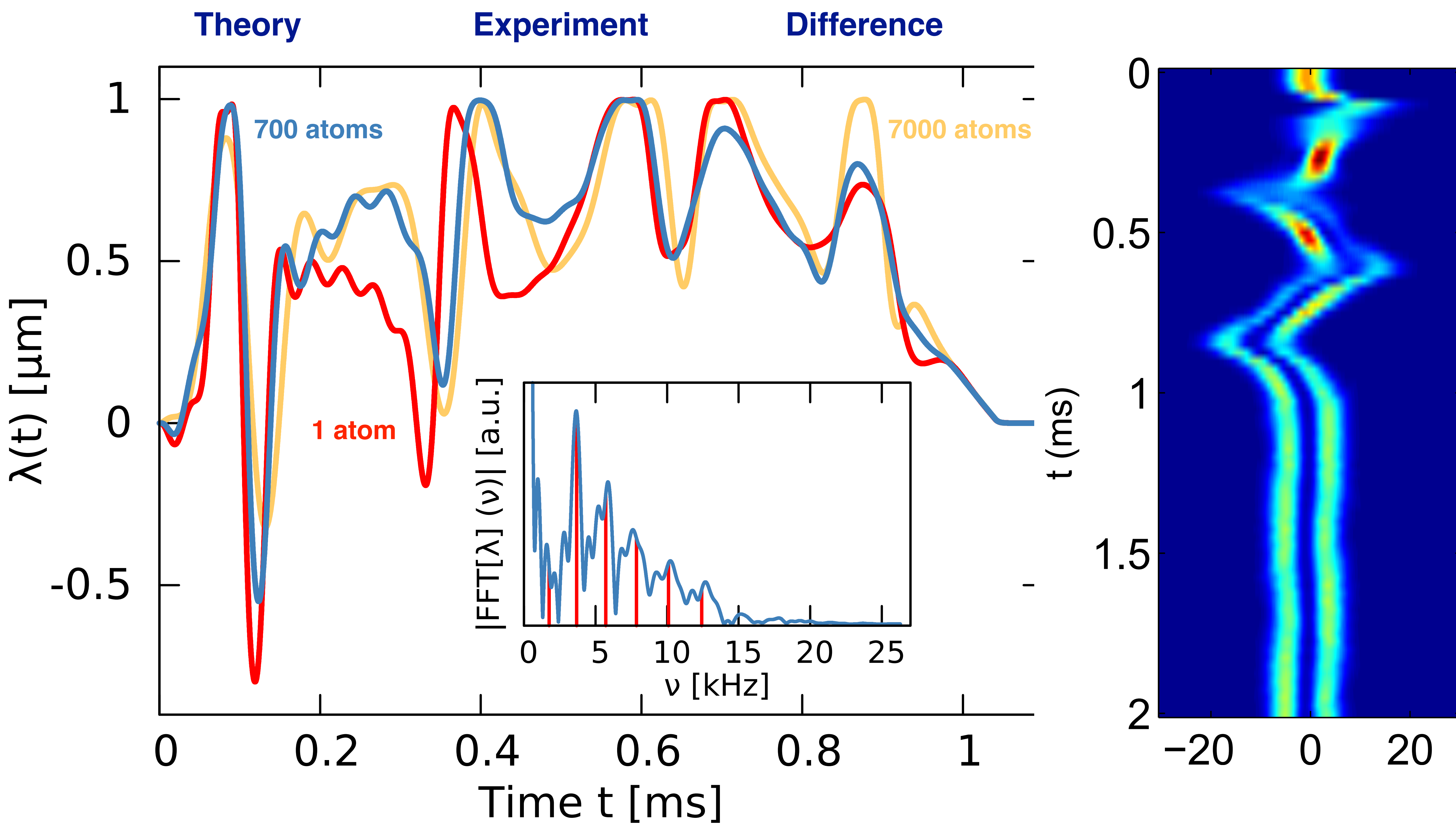
Atom chip experiments at Quantum Speed Limit





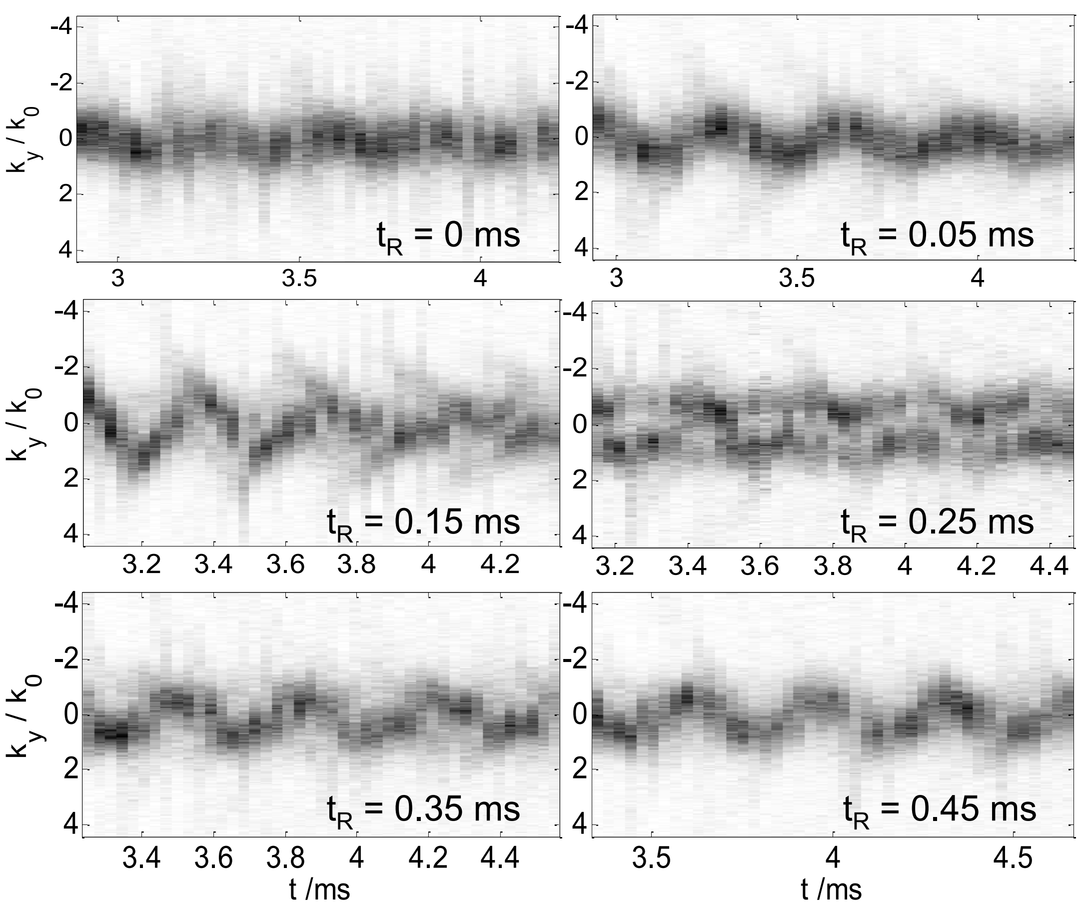
S. Van Frank et al. (Schmiedmayer group), Nature Communications 2014

Effect of nonlinearity in the pulse shape

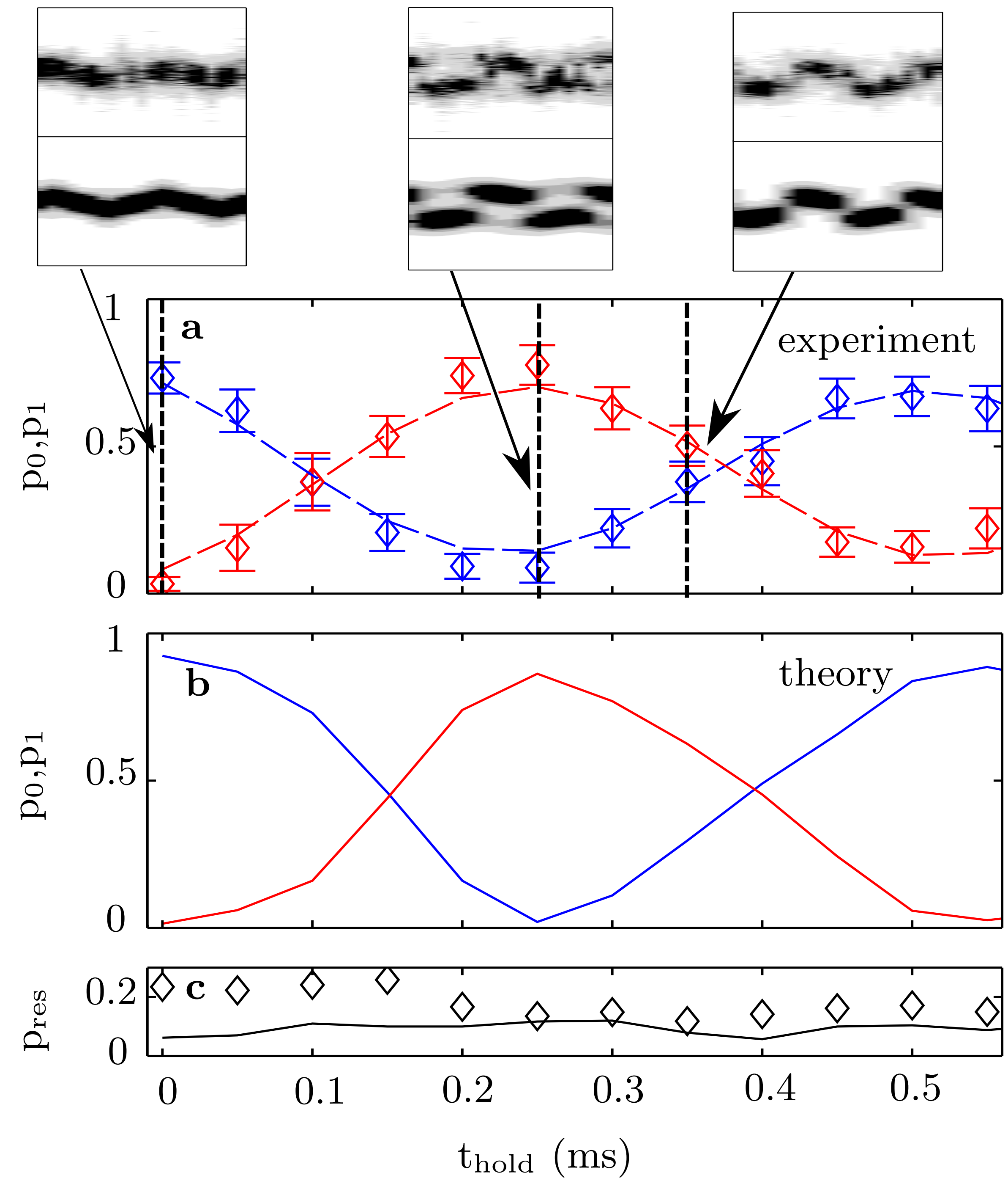


Experimental Ramsey interferometer with a BEC

Detailed analysis of the ‘carpets’ = the interference patterns recorded after time of flight allow to extract the populations in the ground, first and higher excited states.



Momentum distribution after 46 of time-of-flight for 12 points of the first oscillation period.

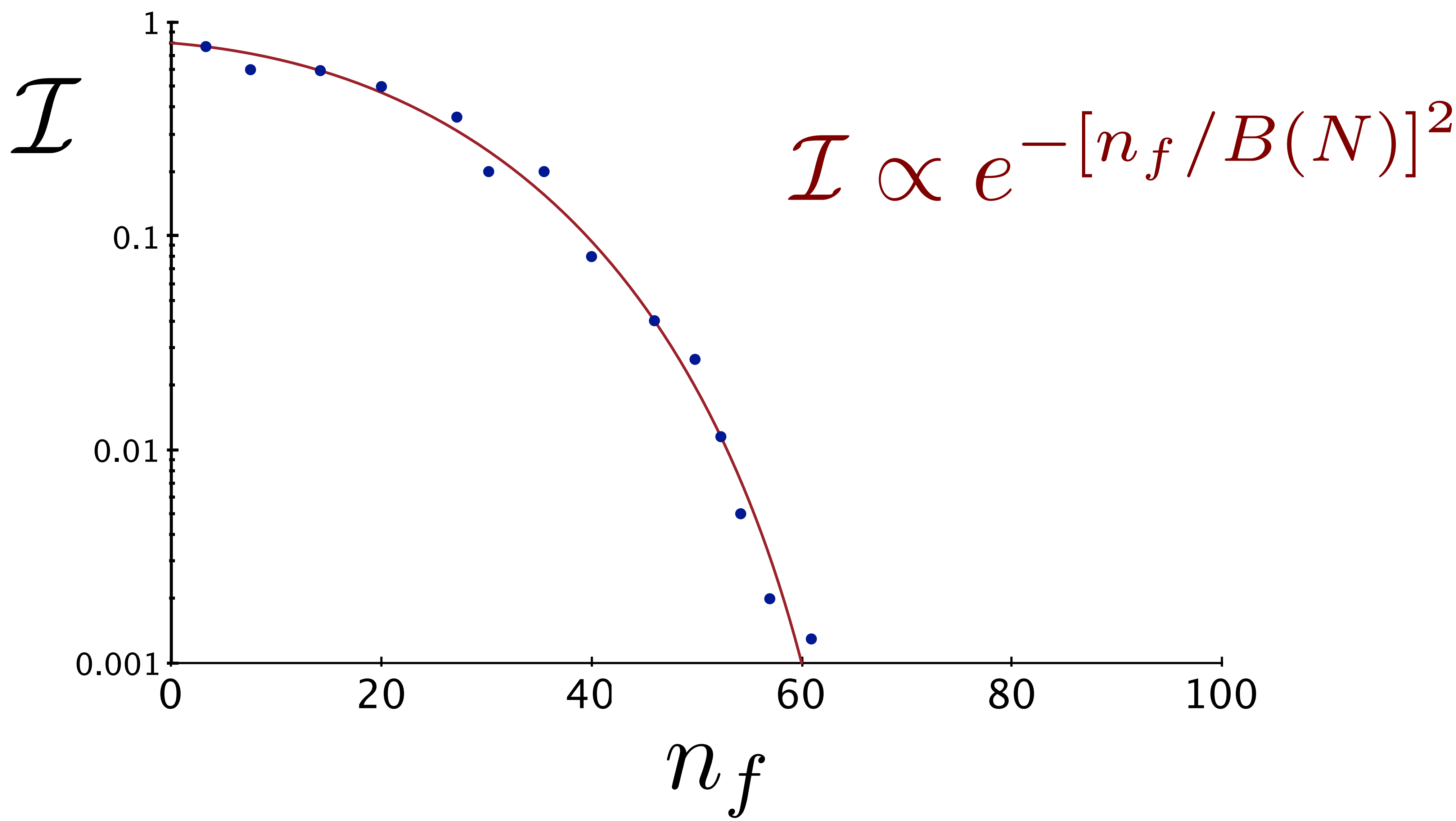


S. Van Frank et al. (Schmiedmayer group), Nature Communications 2014

Understanding complexity

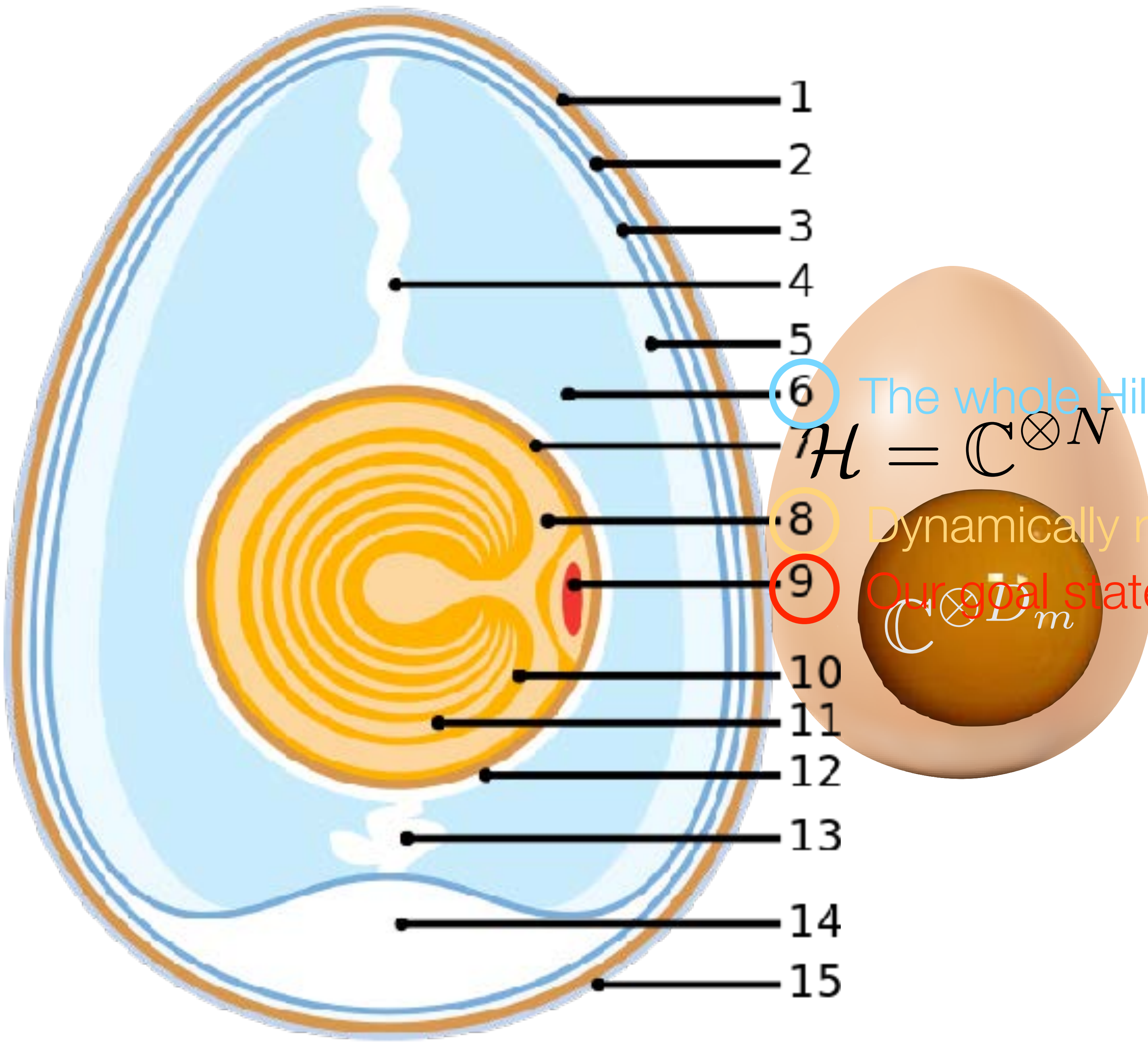
in terms of control bandwidth

4. Complexity



Scaling of the number of parameters with the system size: B(N)

A simple explanation - Columbus’s egg



The whole Hilbert space of N qubits

$\mathcal{H} = \mathbb{C}^{\otimes N}$

Dynamically relevant subspace of dimension D_m

$\mathbb{C}^{\otimes D_m}$

Our goal state, within accuracy ε

A simple explanation

Dynamically relevant subspace of dimension D_m

Our goal state, within accuracy ε

Bits needed to specify the ***goal state***

Bits needed to specify the ***control pulse***

$$b_\varepsilon = \log_2(1/\varepsilon^{D_m}) < b_s = T \cdot \Delta\Omega \cdot k_s = (n_f - 1)k_s$$

Duration   Bandwidth   Bit depth

Solve for the accuracy: $\varepsilon > 2^{-(n_f - 1)k_s / D_m}$

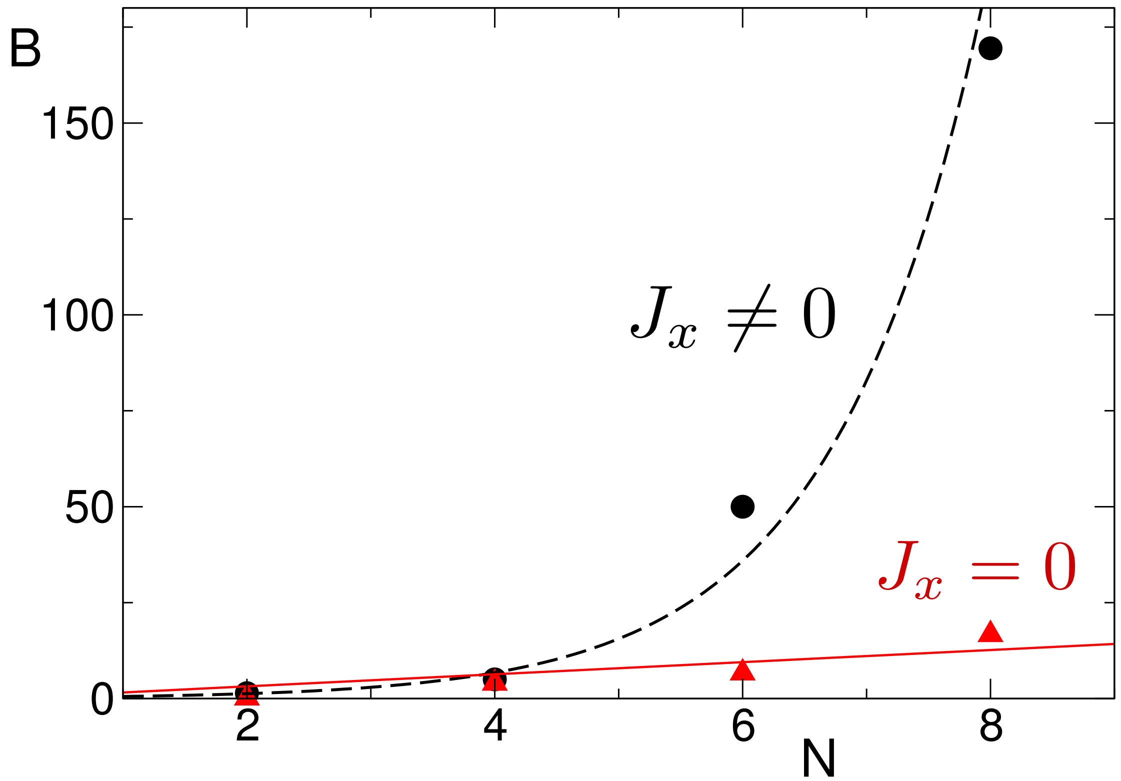
For fixed accuracy: $n_f \sim D_m(N)$

The control bandwidth scales like the complexity of the dynamics

Complexity and integrability

Does the complexity of the control task (control bandwidth) have to do with integrability?

$$H = -\sum_{i,j} J_{ij}\sigma_i^x\sigma_j^x - \Gamma(t)\sum_i^N \sigma_i^z - J_x\sum_i^N \sigma_i^x$$



$$B(N) \propto D_m(N)$$

T. Caneva, A. Silva, R. Fazio, S. Lloyd, T. Calarco, S. Montangero, PRA 2014

Summary

- **Optimal control of quantum systems**
  - possible with the CRAB method
  - from simple (few-body) to complex (many-body) systems
- **Theoretical predictions experimentally confirmed**
  - both in open- and in closed-loop scenarios
  - reaching the Quantum Speed Limit
- **Control complexity**
  - characterized by the degrees of freedom of the driving field
  - linked to Hilbert space dimension, depends on integrability

Thanks for your attention