Implementation of Practical Unforgeable Quantum Money

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LABORATOIRE Aimé Cotton

Experimental Investigation of Practical Unforgeable Quantum Money
npj Quantum Information 4, 5 (2018)

Semi Device-Independent Practical Quantum Money
to appear on arXiv

Background : Quantum Communications

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And many more...

Quantum Banknotes

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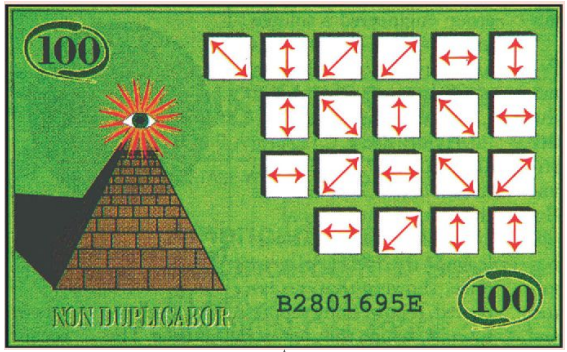
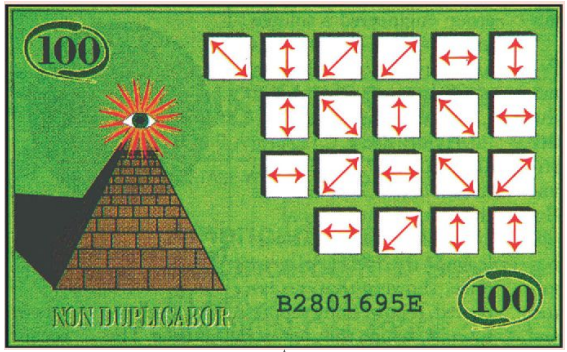


MINT
public serial number **s,**
secret classical key **k(s)**

Quantum Banknotes

CLIENT
stores banknote in a quantum memory



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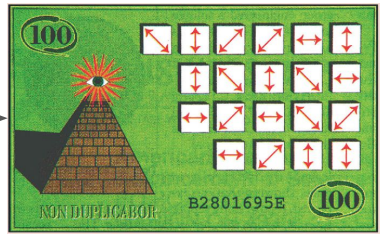

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measurements for
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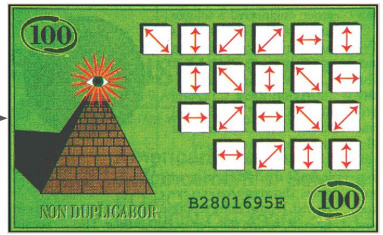

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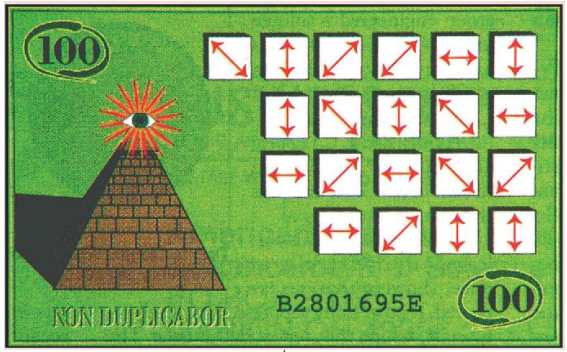

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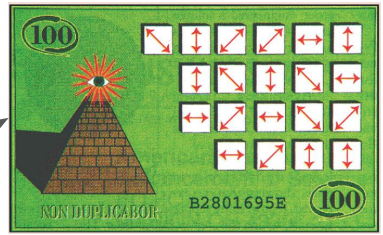

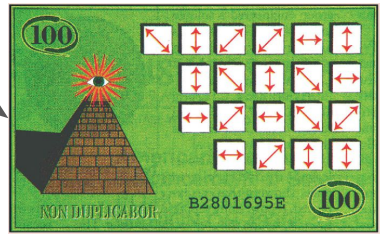

NO-CLONING THEOREM

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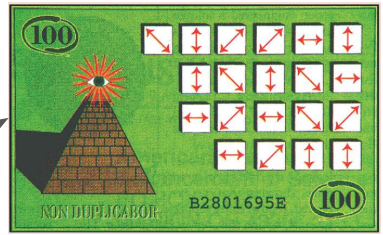

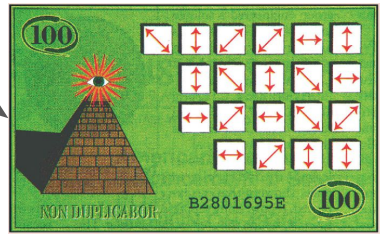

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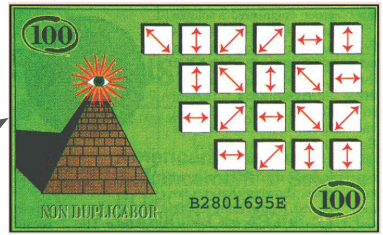

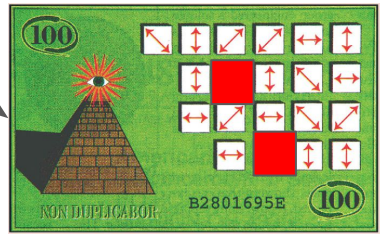

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Quantum Credit Cards

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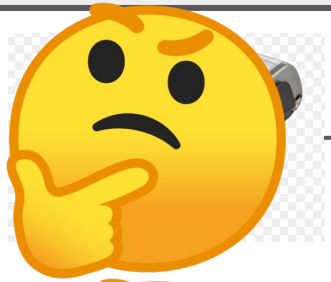

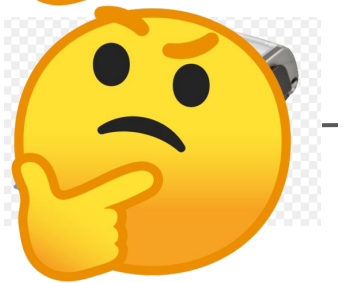

SHOULD WE TRUST THE TERMINAL ?

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Trusted Terminal : Protocol

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Allow classical verification by choosing **qubit pairs** :

$$S_{pair} = \{|0,+\rangle, |0,-\rangle, |1,+\rangle, |1,-\rangle, |+,0\rangle, |+,1\rangle, |-,0\rangle, |-,1\rangle\}$$

Secret classical key : 3 bits $\{b, c_0, c_1\}$, $b =$ basis of the first qubit,
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Correctness challenges (c=1, asked by the bank)
Q_{xx} : Guess the two bits c_0 and c_1 such that the guess corresponding to the qubit prepared in the σ_x basis is correct.
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Security challenge ($\epsilon = 3/4$ = cloning probability)
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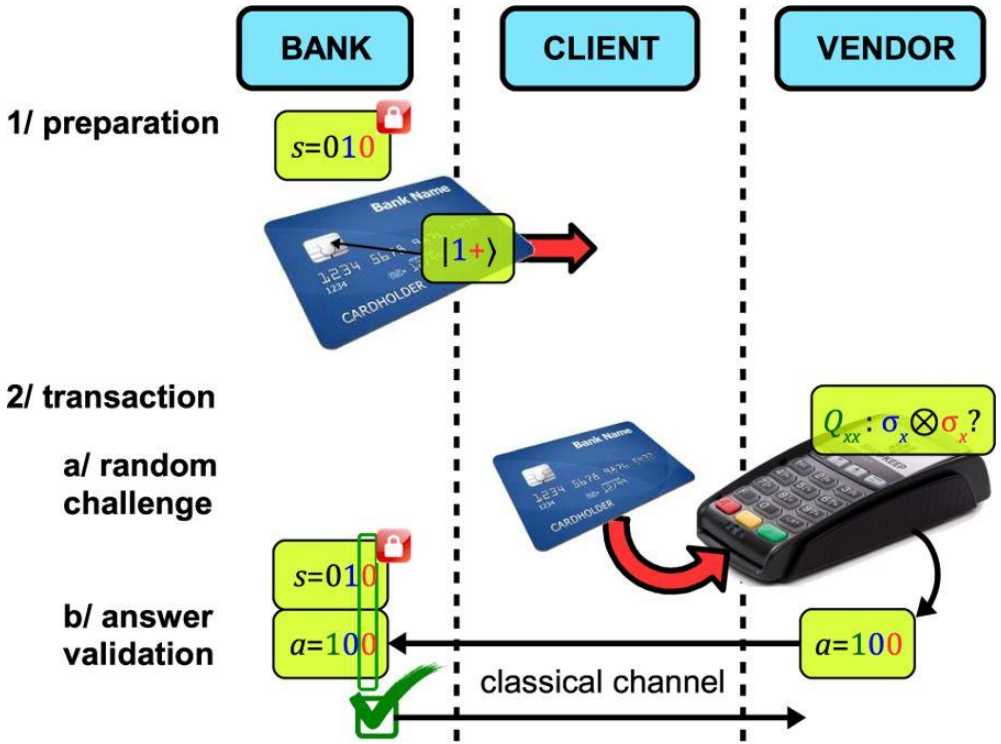

Mapping onto Coherent States

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Qubit states may be mapped onto 2-mode coherent states as:

$$|0\rangle \rightarrow |\alpha\rangle \otimes |vac\rangle \qquad |1\rangle \rightarrow |vac\rangle \otimes |-\alpha\rangle$$
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What are the security implications of such a mapping ?

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- **True single photon scenario**

  $\Rightarrow$ Single emitter quantum memories with coherent state input.

  $\Rightarrow$ Atomic ensemble quantum memories with single photon input.

  $\Rightarrow$ Upper bound on cheating probability $\epsilon = \frac{3}{4}$.

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- **Weak coherent state scenario (μ)**

  $\Rightarrow$ Atomic ensemble quantum memories with coherent state input.
  $\Rightarrow$ New attacks such as Unambiguous State Discrimination.
  $\Rightarrow$ Memory introduces attenuation, therefore :

$$c > f(\mu, \epsilon)$$

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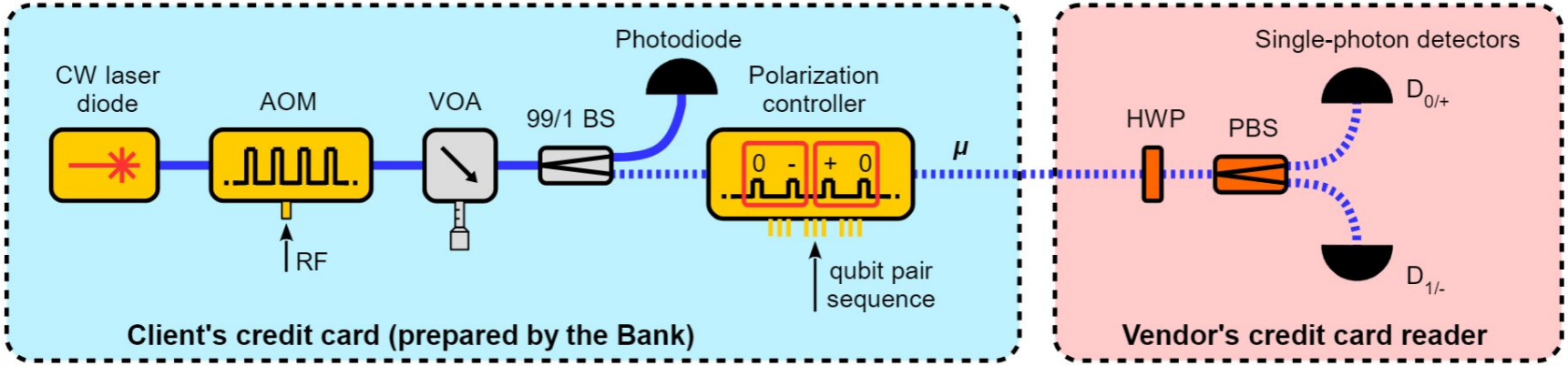

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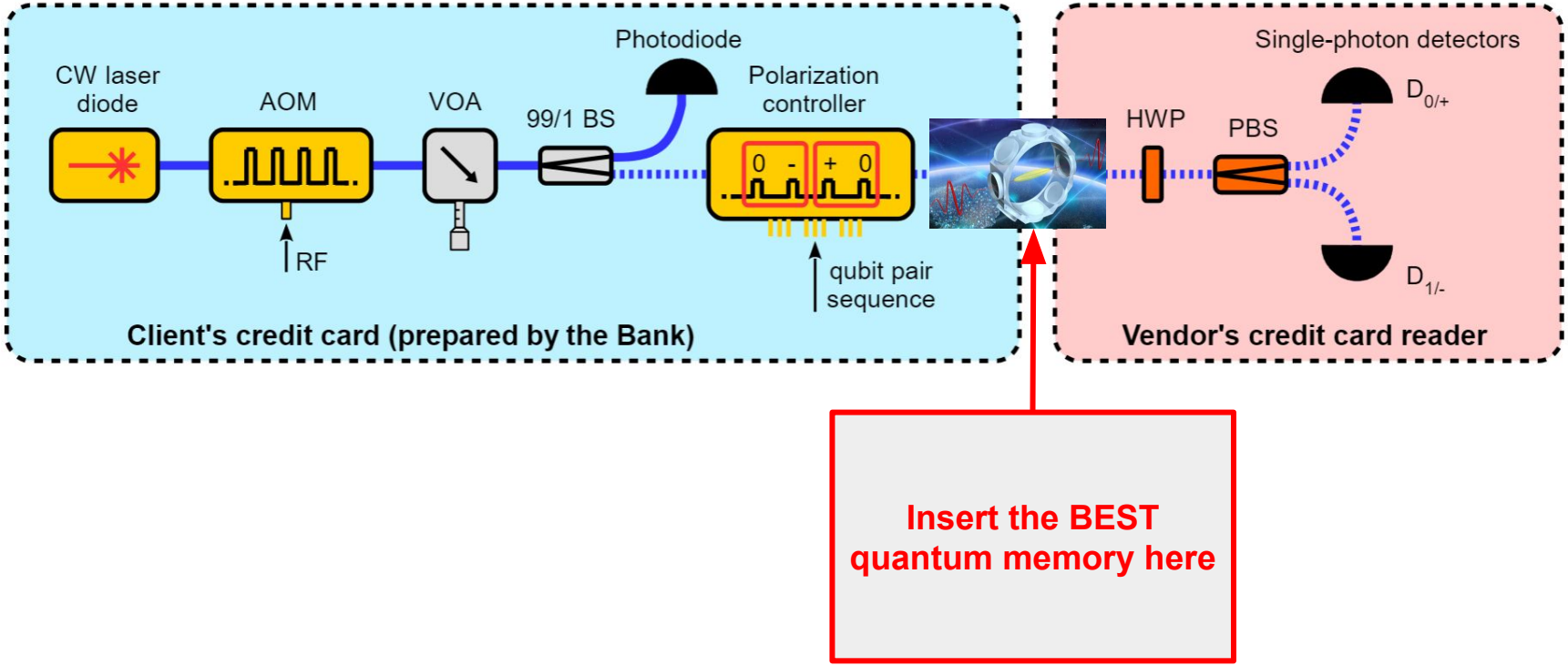

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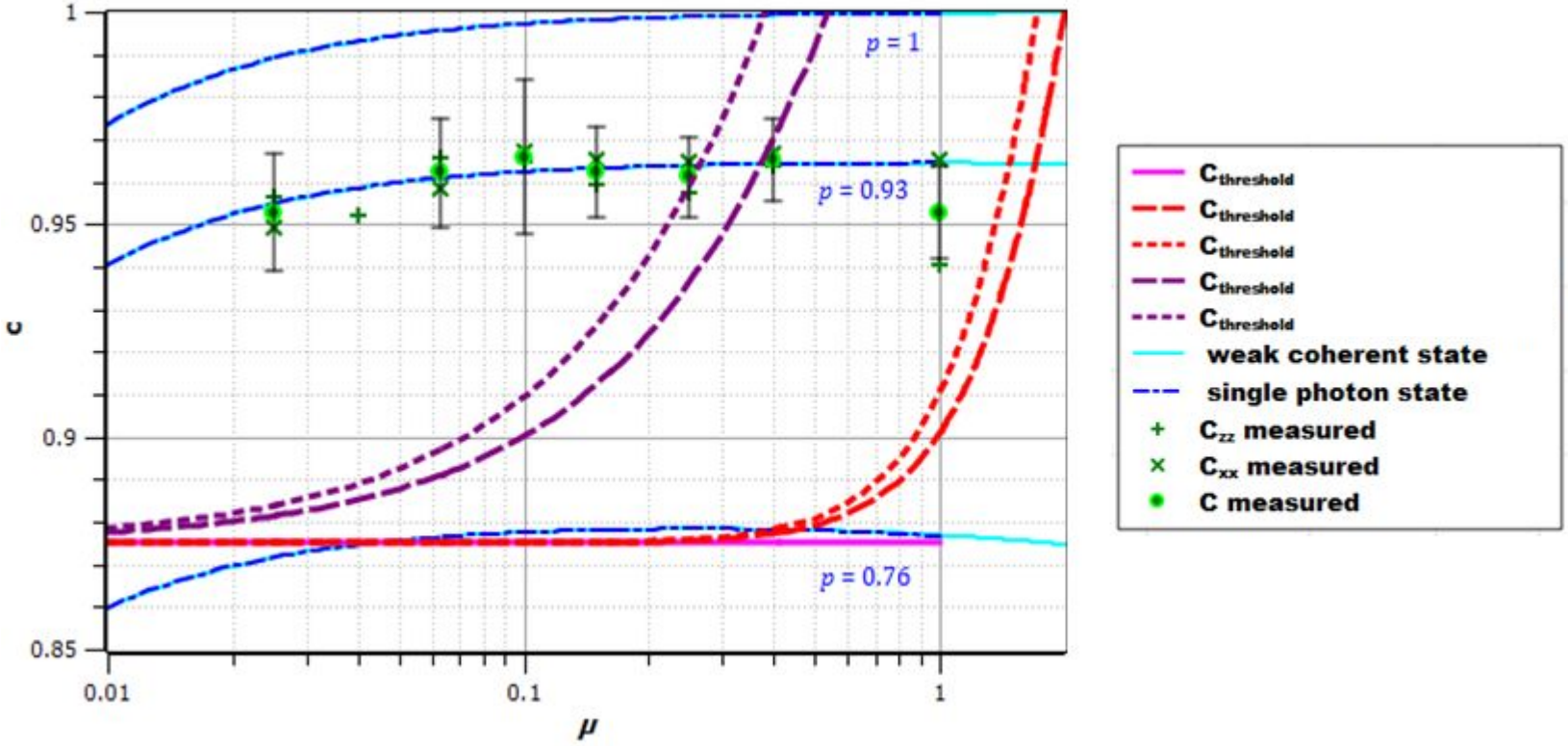

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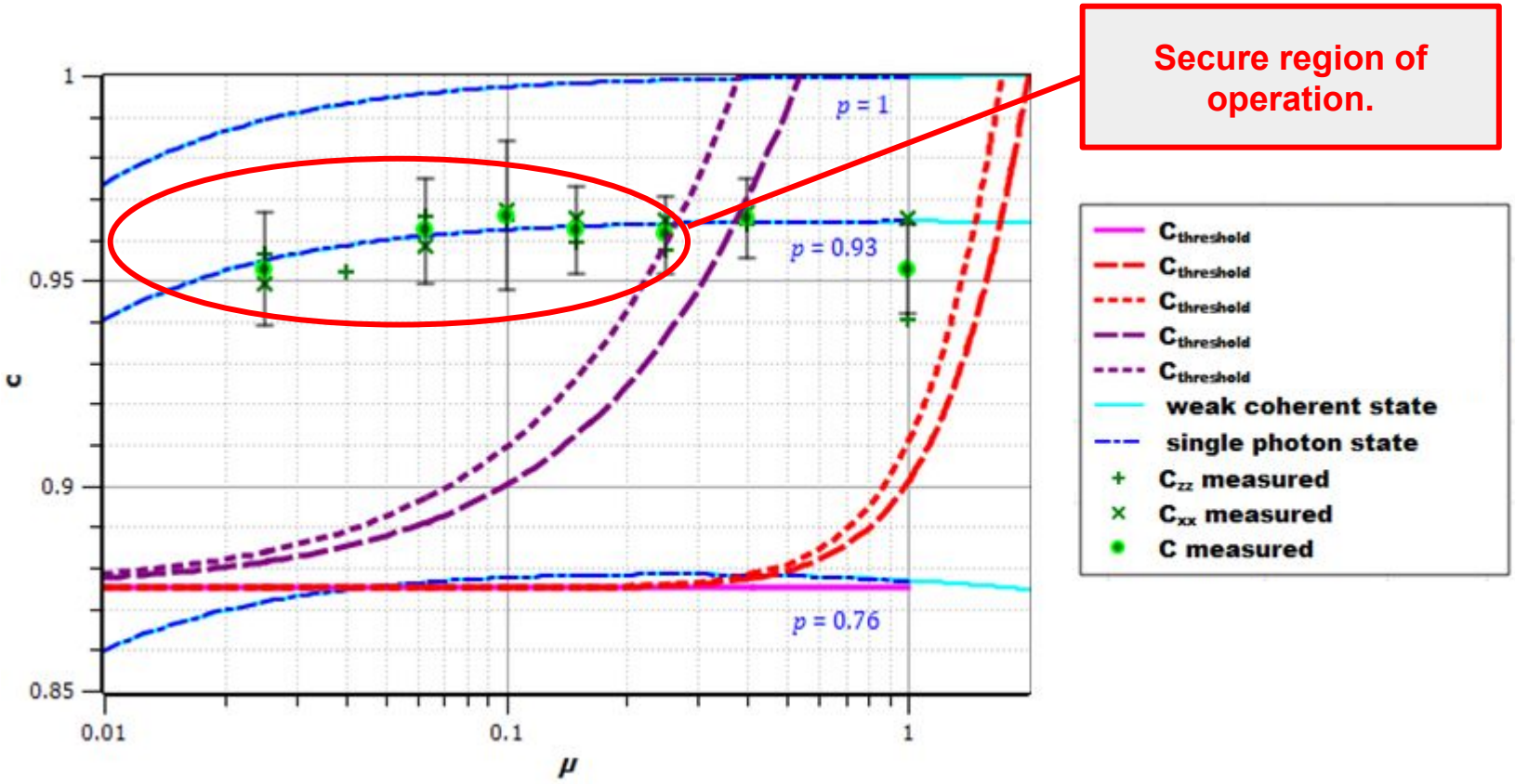

General Security Framework

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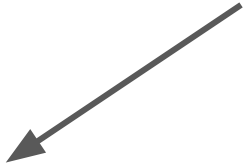

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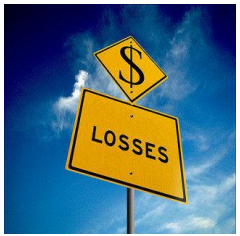

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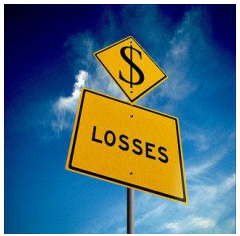

Minimize losses AND error using semidefinite programming (SDP)

Theoretical Challenges

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Unified Optimization Framework :

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Figures of Merit :

- Trusted terminal : quantum measurements
- Untrusted terminal : acceptance of classical data

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Quantum Memory :

- Time-dependent security proof

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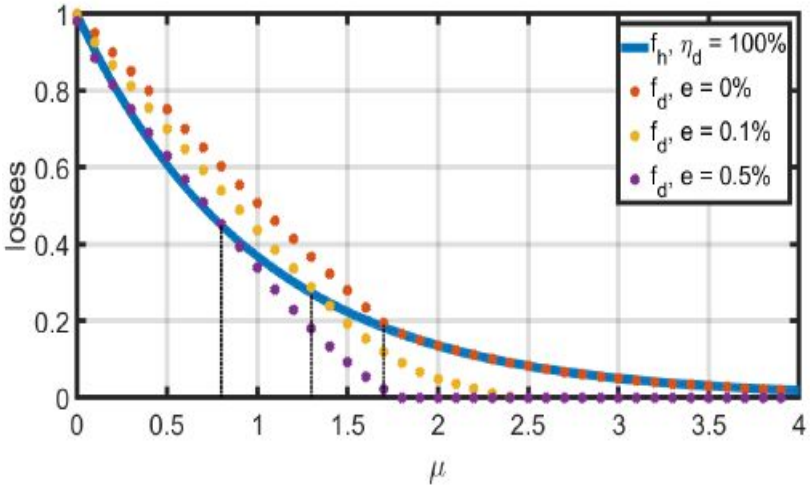

Trusted terminal

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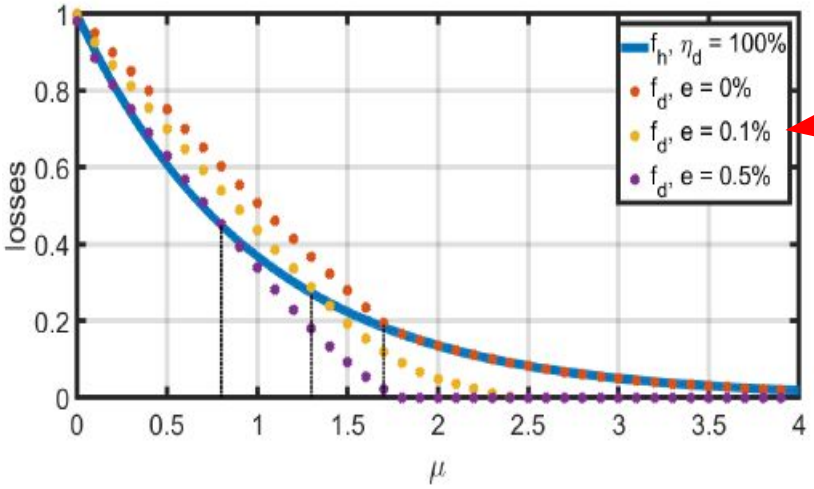

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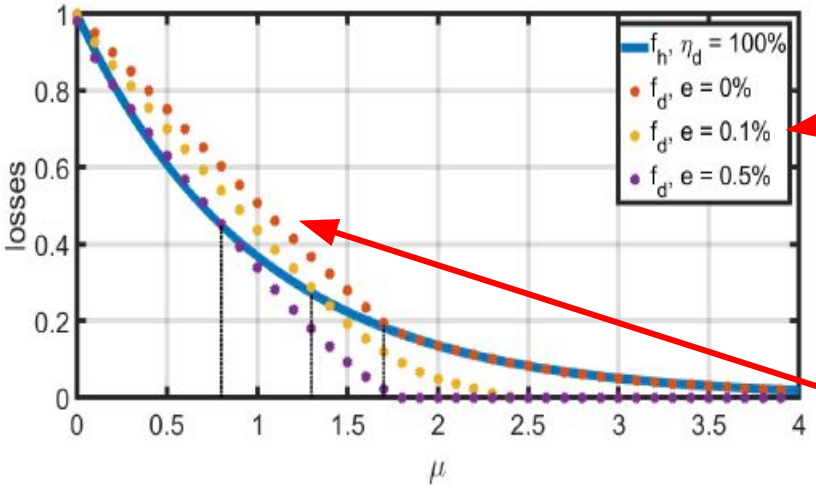

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The dishonest excess losses f_d must lie above the honest losses f_h for the bank to detect the attack.

Trusted terminal

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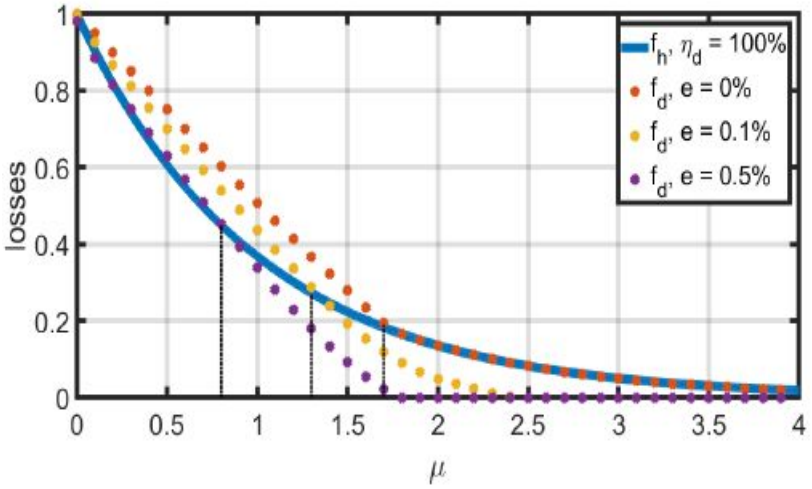

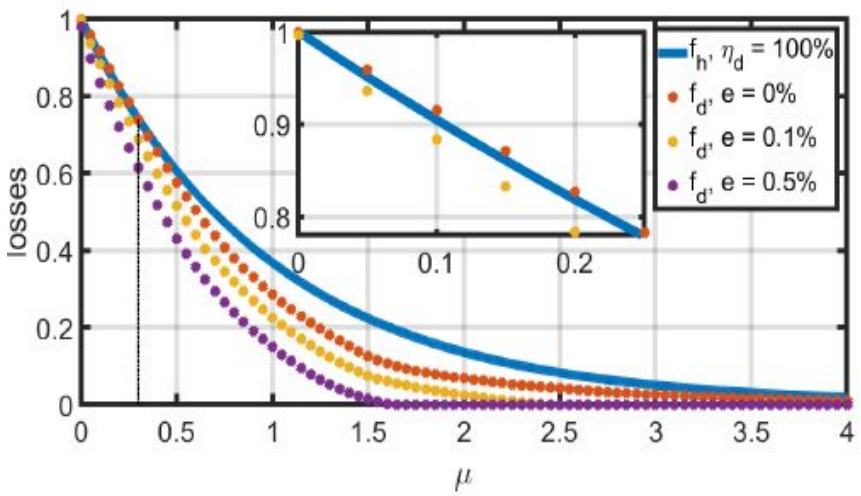

Trusted terminal

Untrusted terminal

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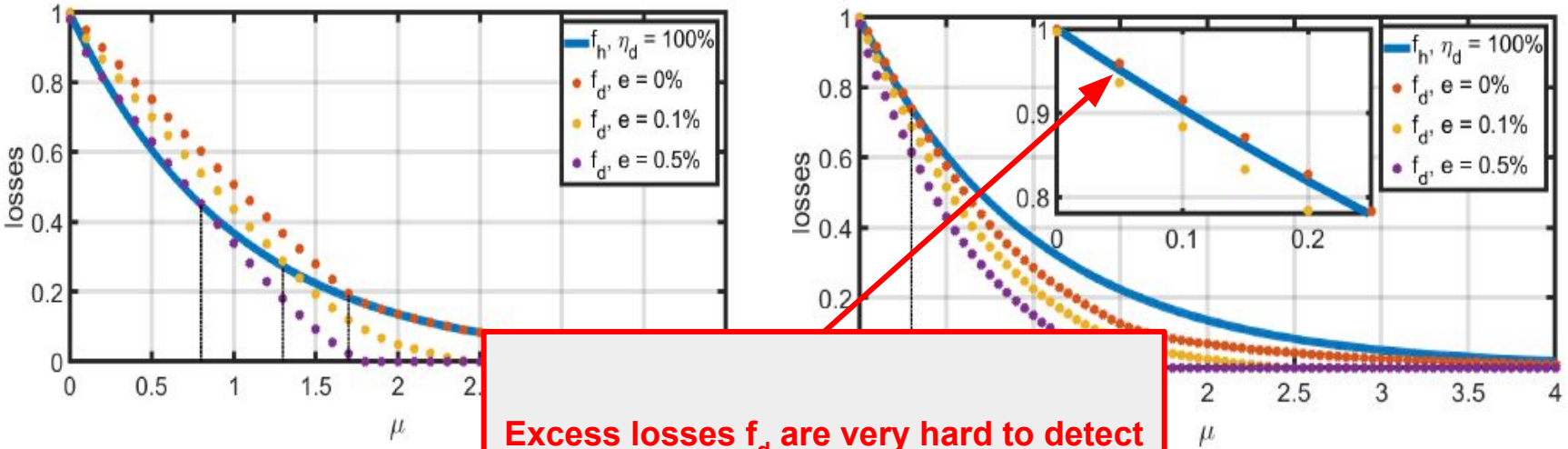

Trusted termi … usted terminal

Excess losses f_d are very hard to detect even for zero error.

Noise and loss tolerance can be improved with phase randomization.

Security and Quantum Memories

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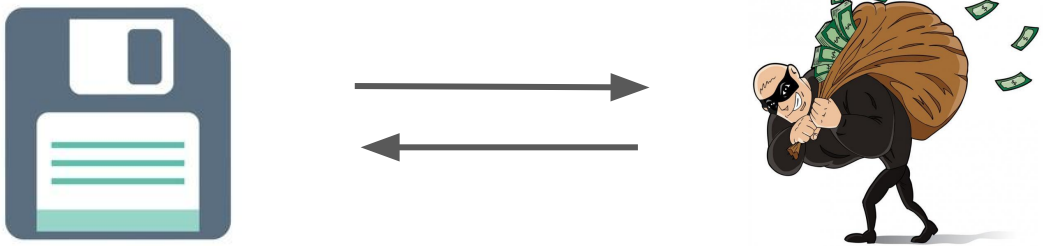

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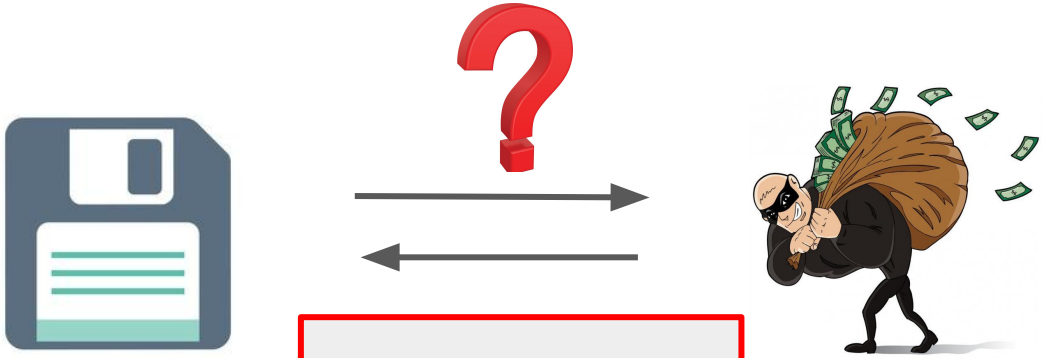
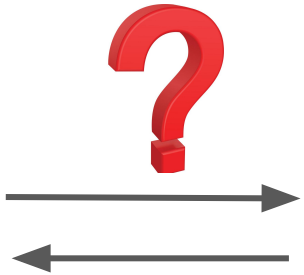


How can the adversary exploit the quantum memory to get more information?

Security and Quantum Memories



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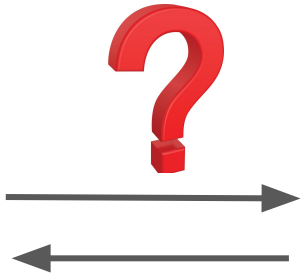
Retrieval efficiency decreases with time

Security and Quantum Memories



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Some physical processes preserve information

Summary

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NEXT : Implement the protocol with a quantum memory !

S.Wiesner, ACM Sigact News 15, 78 (1983).
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Thanks for listening !

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to appear on arXiv