Certifying quantum entanglement in many-body systems via inverse statistical methods

GDR IQFA
2020 December 2nd

Irénée Frérot

- ICFO – The Institute of Photonic Sciences (Castelldefels, Barcelona, Spain)
- Max Planck Institute of Quantum Optics (Garching, Germany)

IF & T. Roscilde, arxiv: 2004.07796 (2020)

Generalitat de Catalunya
Fundació Privada CELLEX
Fundació Privada MIR-PUIG
MINISTERIO DE ECONOMÍA, INDUSTRIA Y COMPETITIVIDAD
AGENCIA ESTATAL DE INVESTIGACIÓN



Guillem Müller, Albert Aloy, Maciek Lewenstein
(ICFO, Barcelona)

Flavio Baccari
(MPQ, Munich)

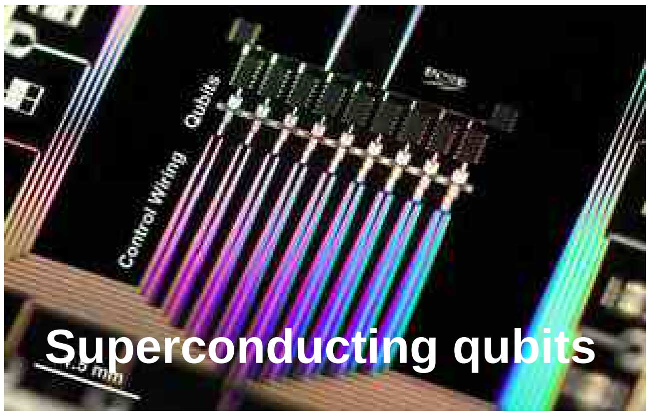
Tommaso Roscilde
(ENS Lyon)

Quantum info group – Antonio Acín
(ICFO, Barcelona)

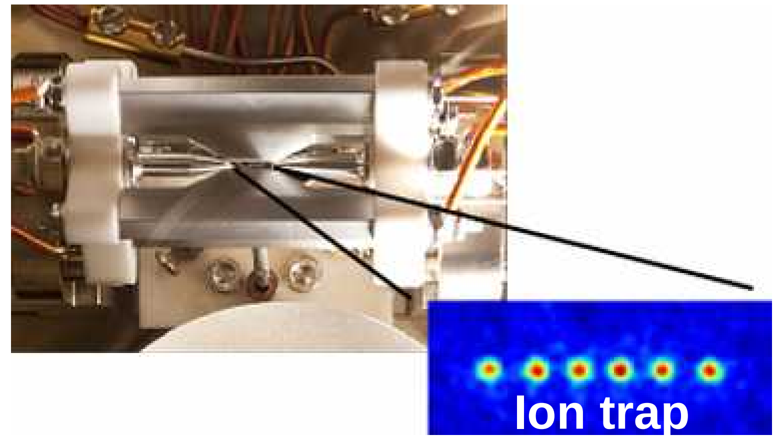
Noisy intermediate-scale quantum devices

(Preskill, Quantum 2018)

trapped ions, superconducting qubits, cold atoms, photons, quantum dots, vacancy centers in diamond, phosphorous embedded in silicon
50-70 qubits, error/gate 0.01%.



Google
IBM (in the cloud)...



Monroe, Wineland (NIST)
Blatt (Innsbruck, Austria)
Cirac & Zoller PRL 1995

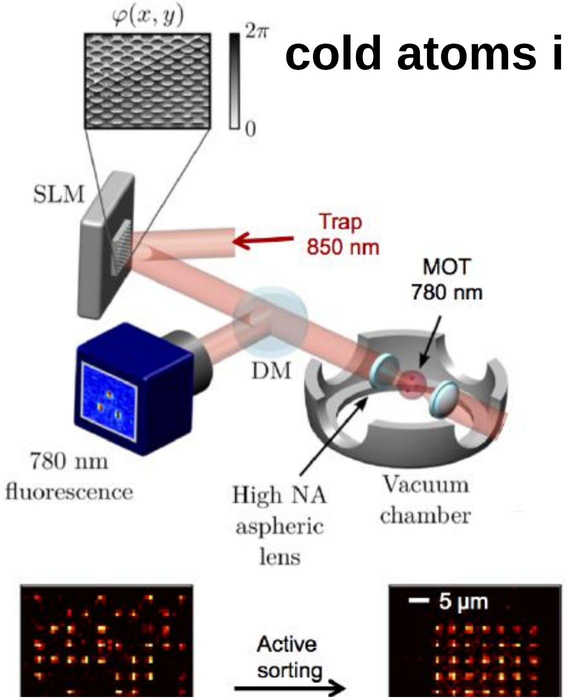
→ **Optimization problems**

e.g. compute ground state energies of quantum Hamiltonians
More generally: minimize some objective function
- lattice gauge theories (Kokail *et al*, Nature 2019)
- quantum chemistry (water molecule: Nam *et al*, NJP quantum info 2020)

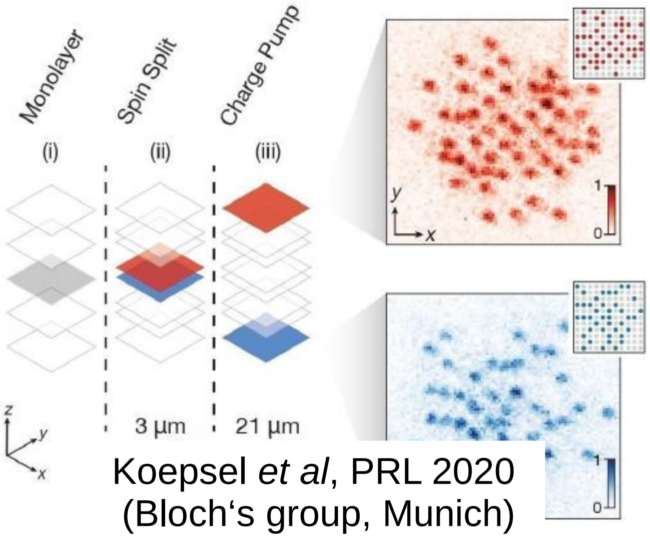
→ **Not useful yet, but reasonable to expect near-term applications of quantum subroutines, for very specific optimization problems**

Quantum simulators
for many-body physics

cold atoms in tweezers or lattices, trapped ions
100-1000 atoms



Browaeys & Lahaye, Nat. Phys. 2020
(Institut d'optique, Palaiseau)



Koepsel *et al*, PRL 2020
(Bloch's group, Munich)

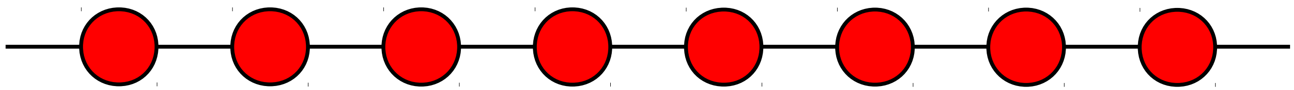
→ Simulation of condensed matter (Wecker *et al*, PRA 2015)
(Hubbard models, spin liquids, topological phases)
Magnified lattice structure, access to individual d.o.f

→ Simulation of quantum chemistry (Reiher *et al*, PNAS 2017)

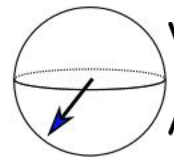
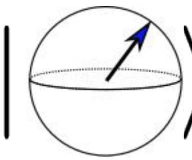
→ Simulation of lattice-gauge-theory dynamics
(Zohar, Cirac & Reznik, Rep. Prog. Phys. 2016; Mil *et al* Science 2020)

Quantum entanglement: A key computational resource

Quantum computers are useful insofar as they are hard to simulate on classical machines



N qubits

Product states: $|\mathcal{C}\rangle = |$  $\rangle \otimes \cdots \otimes |$  $\rangle$

$\mathcal{C} = \{\mathbf{n}^{(i)}\}$

"classical configuration"

State of 1 qubit (Bloch sphere)

$\cos\frac{\theta}{2}|\uparrow\rangle + e^{i\phi}\sin\frac{\theta}{2}|\downarrow\rangle$

easy states

Separable states = statistical mixture of product states (probab. ***p(C)***)

O(N) classical resources + random variable

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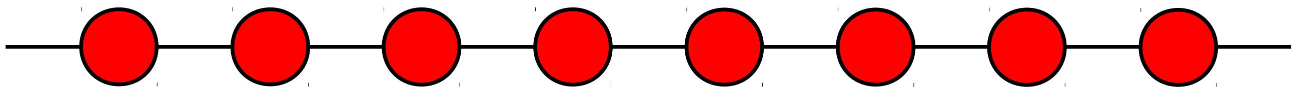

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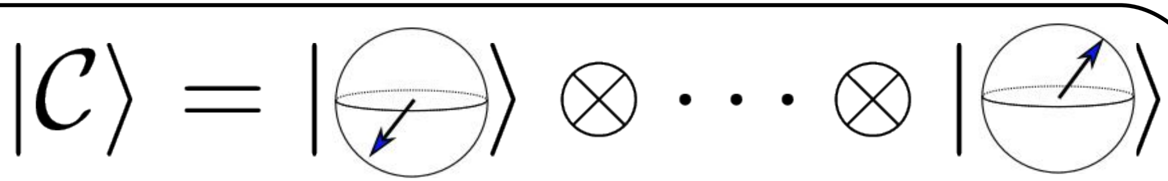

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Entangled states = non-separable

hard states → resource

$$|\Psi\rangle_{\text{entangled}} = \sum_{\mathcal{C}} \Psi(\mathcal{C})|\mathcal{C}\rangle = \sum_{\vec{\sigma}\in\{\pm 1\}^N} \Psi'(\vec{\sigma})|\vec{\sigma}\rangle$$

Complex amplitude

Exponentially-many terms

O(exp N) classical resources

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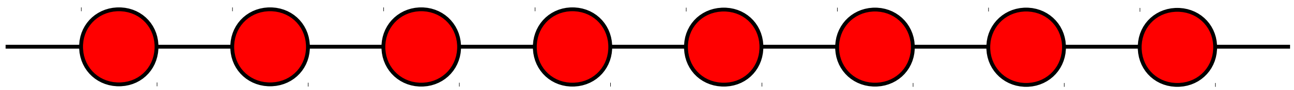

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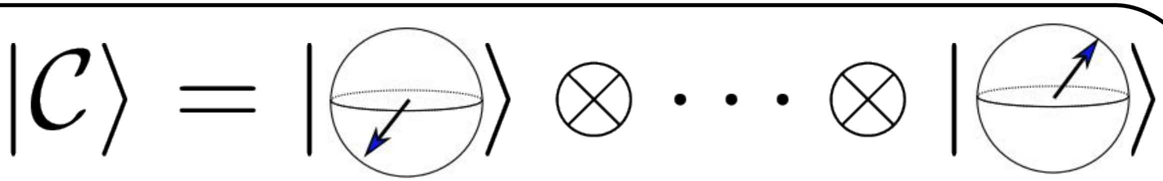

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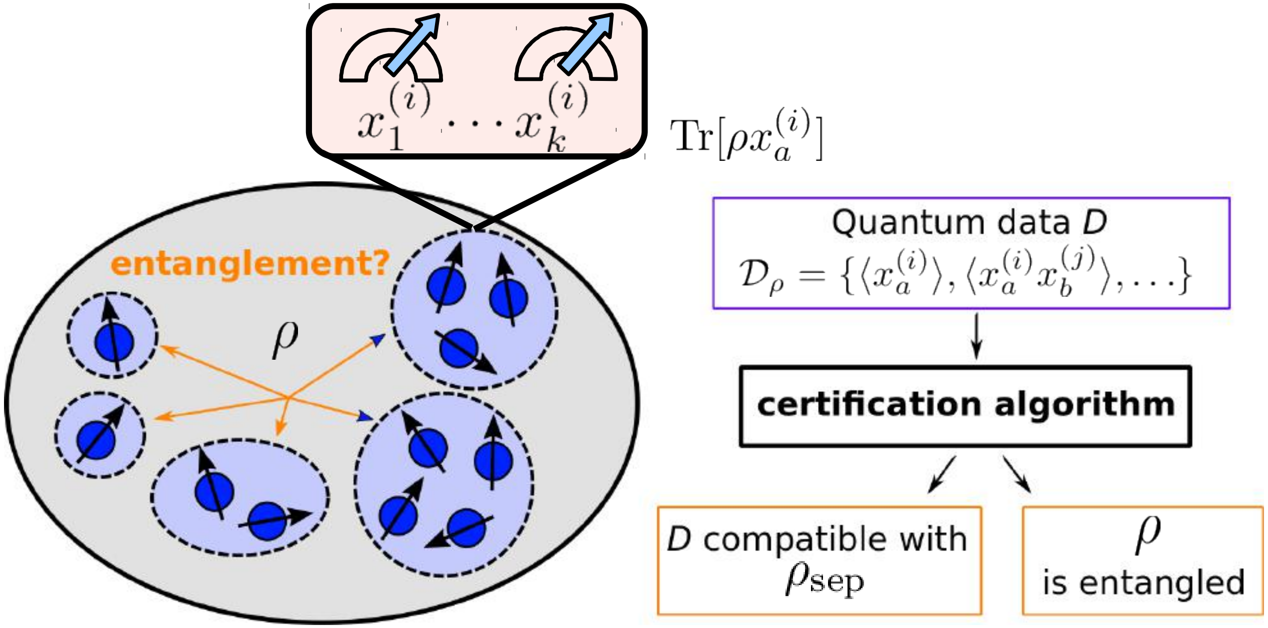
Entanglement problem: no scalable solution → witness

The problem: entanglement certification

1) Move the focus from the unaccessible state ρ (d^Nxd^N matrix) to the **available dataset *D***



2) Can we can explain the data **without entanglement**?

Is *D* compatible:
With a **separable state**?
With a **local-variable model** *à la* John Bell?
Answer 'no' → entanglement

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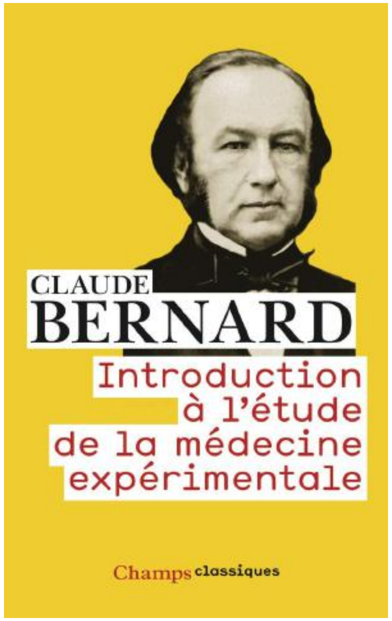

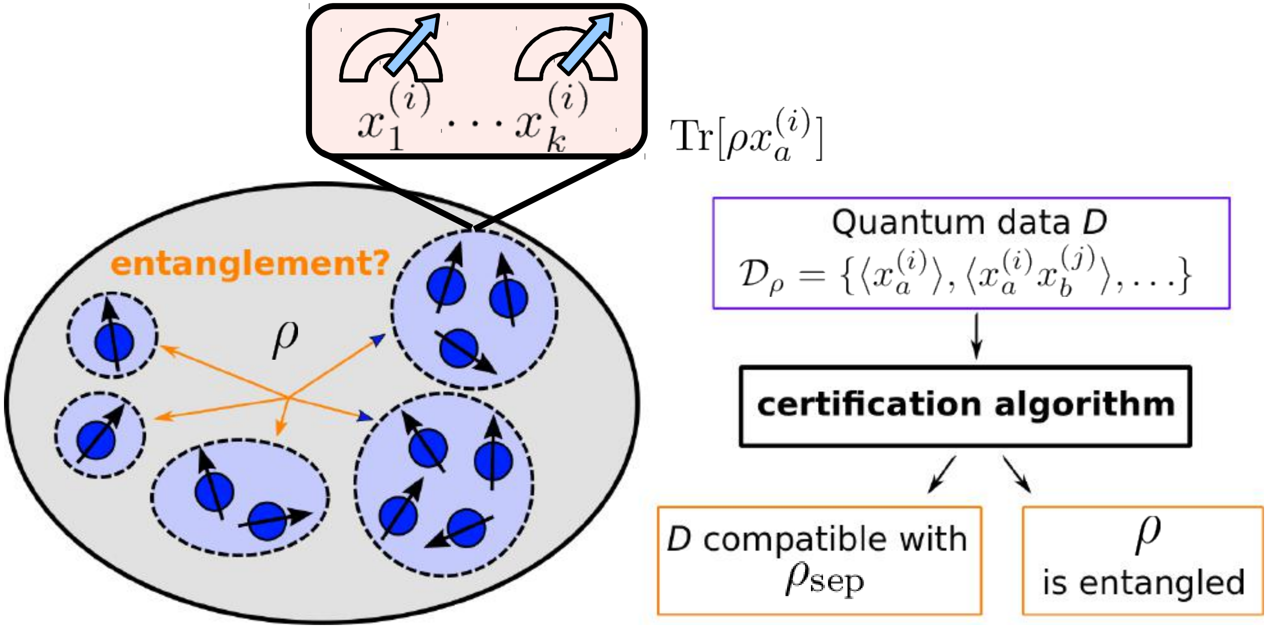

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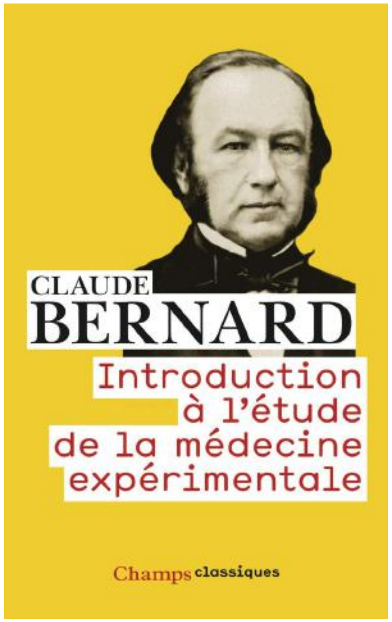

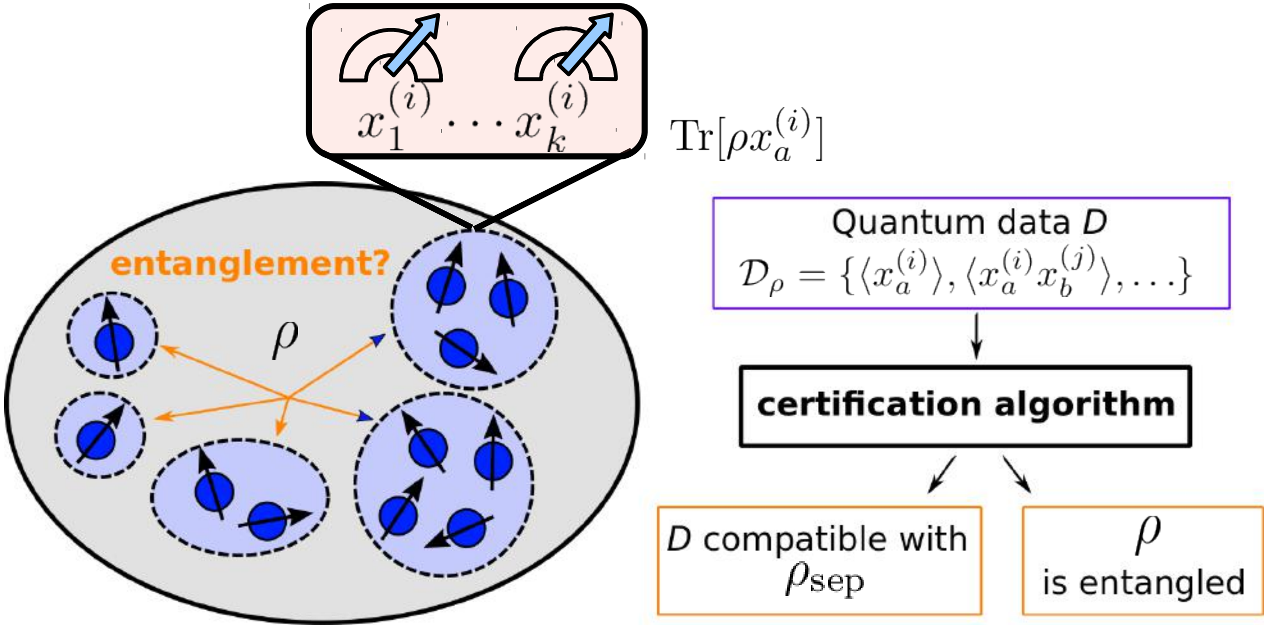

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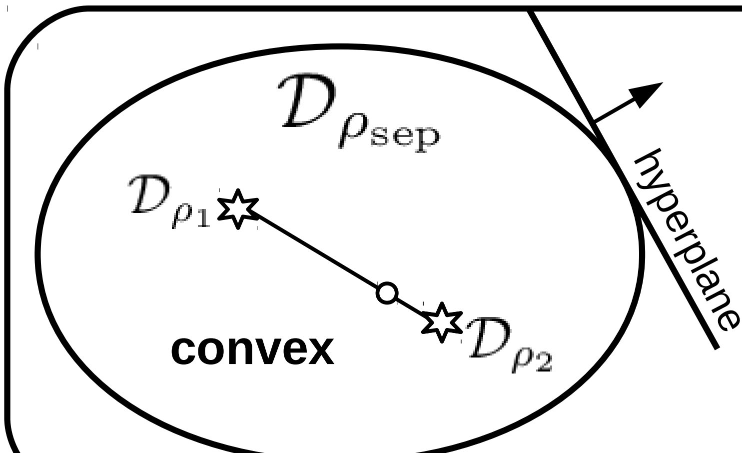


$\mathcal{D}_\rho$??
→ entangled

Entanglement witness
Bell's inequality

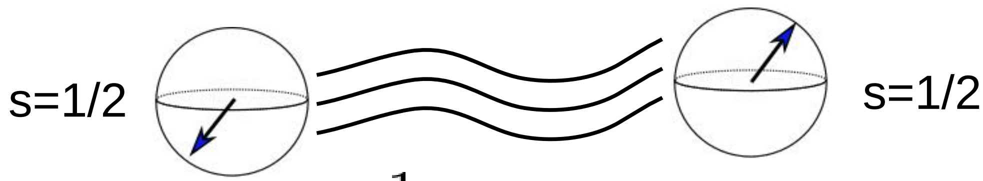
Gühne & Tóth
Physics Reports 2009
Brunner *et al*
Rev. Mod. Phys. 2014

Entanglement witness: Bell pair – spin singlet

$$\vec{J} = \vec{S}_1 + \vec{S}_2$$

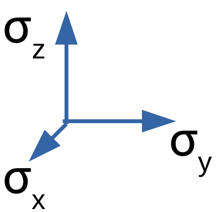
$$J_a|\psi\rangle = 0 \ (a \in \{x, y, z\})$$

Invariant under global rotations

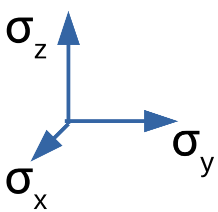


$$|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle]$$

Measure A



Measure B



σ_a: Pauli matrices

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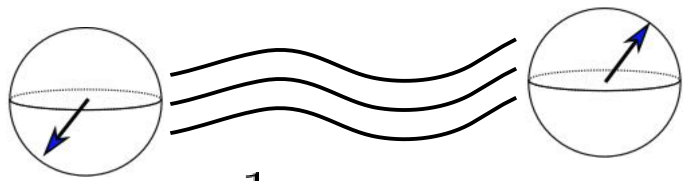
s=1/2  s=1/2

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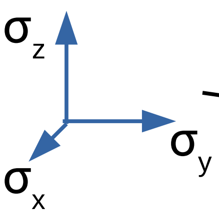
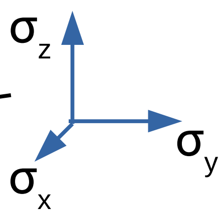


Correlations
$\langle \sigma_a^A \sigma_b^B \rangle$

Measure B



σ_a: Pauli matrices

$$\langle \sigma_x^A \sigma_x^B \rangle = \langle \sigma_y^A \sigma_y^B \rangle = \langle \sigma_z^A \sigma_z^B \rangle = -1$$

Compatible with separable state?

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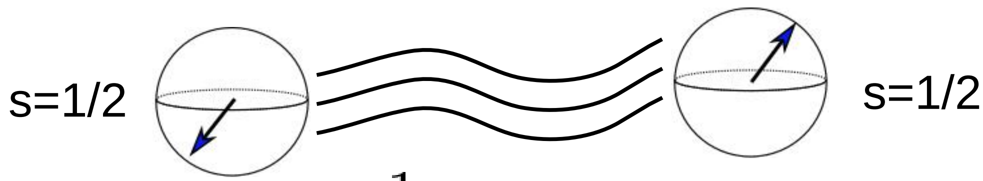

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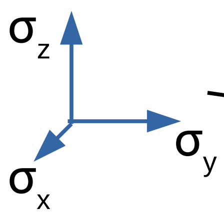

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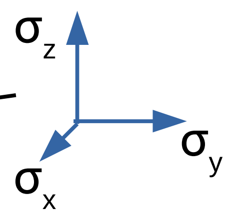

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Compatible with separable state?

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Separable states ρ_{sep} : $\displaystyle\sum_{a\in\{x,y,z\}} \sum_{\mathcal{C}} p(\mathcal{C})\langle \mathcal{C}|\sigma_a^A \sigma_a^B|\mathcal{C}\rangle = \langle \vec{n}^{(A)} \cdot \vec{n}^{(B)}\rangle \geq -1$ **But here: =-3**

Entanglement witness… violated by the data

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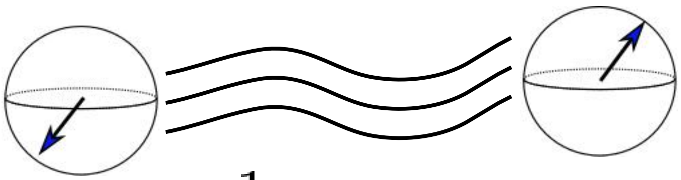

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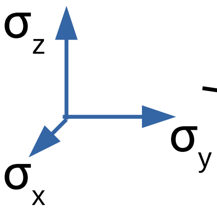

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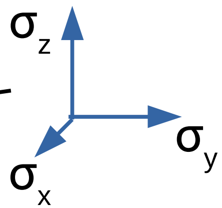

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Compatible with separable state?

Device-dependent

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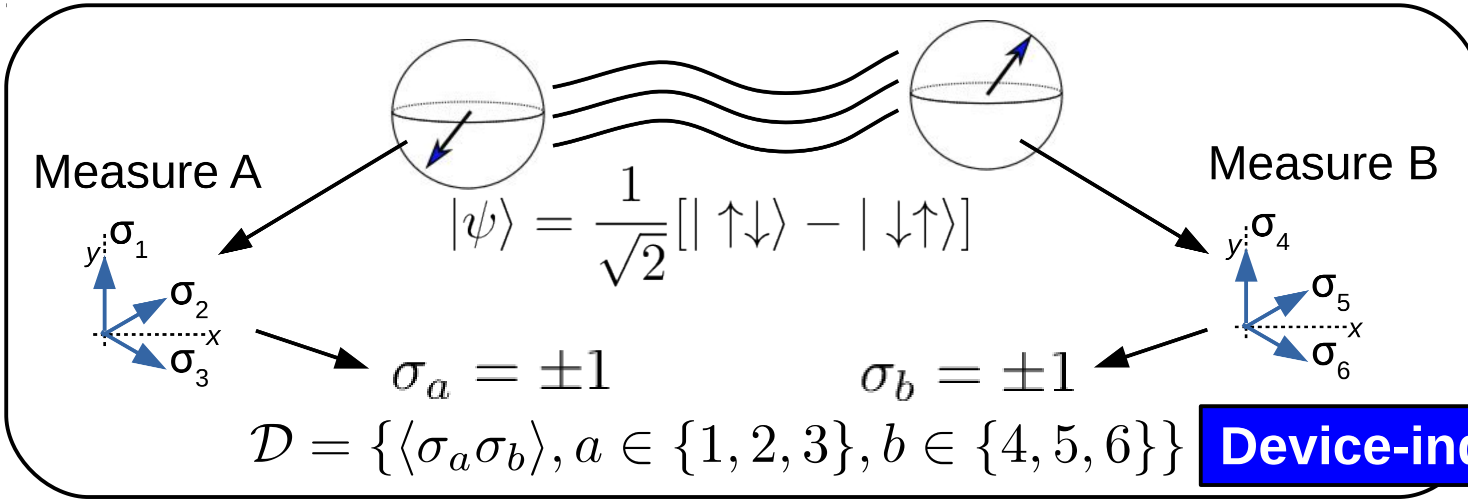
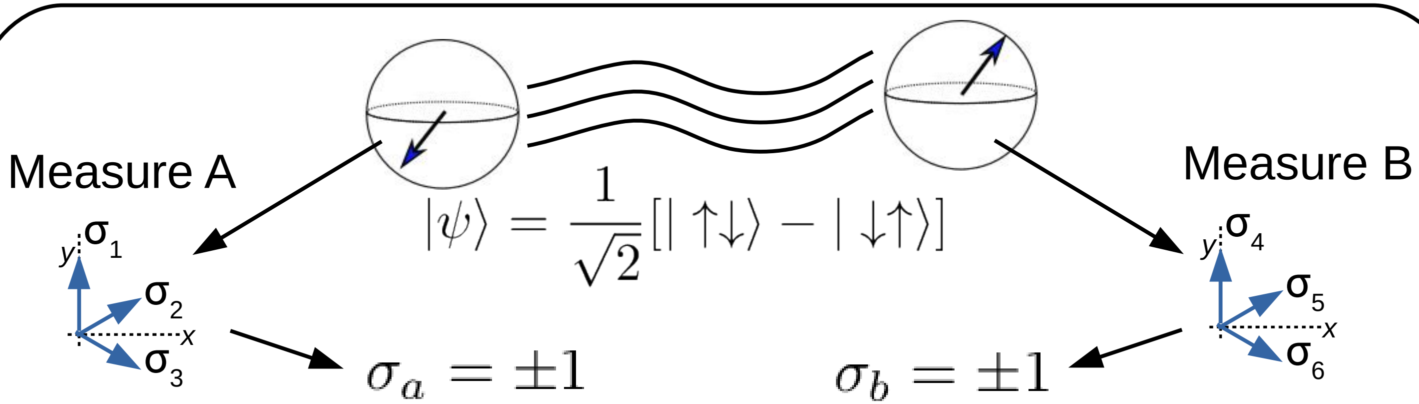
Bell's inequalities



Device-independent

Bell's inequalities



$$|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle]$$

$$\sigma_a = \pm 1 \qquad \sigma_b = \pm 1$$

$$\mathcal{D} = \{\langle \sigma_a \sigma_b \rangle, a \in \{1,2,3\}, b \in \{4,5,6\}\}$$

Device-independent

If the state is a product state:

$$p[\sigma_a, \sigma_b] = p_A(\sigma_a) p_B(\sigma_b)$$

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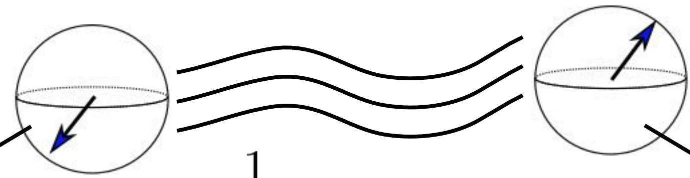

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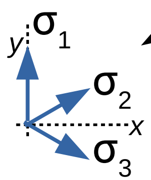

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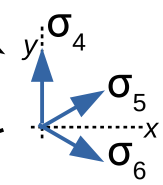

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Bell (Physics, 1964)

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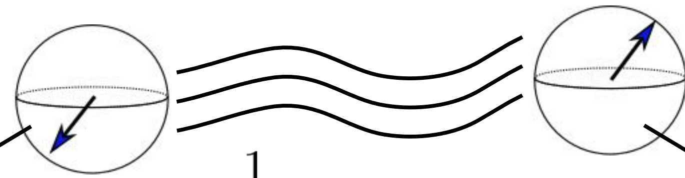
$$p[\sigma_a, \sigma_b] = \int_\lambda d\mu_\lambda \; p_A(\sigma_a|\lambda) p_B(\sigma_b|\lambda)$$

Local-variable model (LV) ***à la J. Bell***

Bell's inequalities



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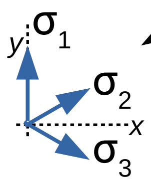

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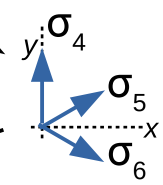

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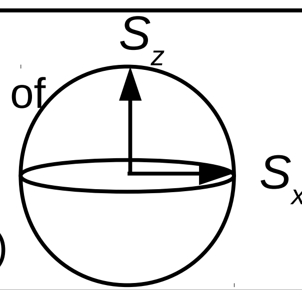
A. Fine (PRL 1982): Equivalent to the existence of a grand-probab. P({$\sigma_a^{(i)}$}) over all outcomes (including incompatible measurements) treated as classical variables (i.e. Ising spins)



$$[S_x, S_z] \neq 0$$
$$P(S_x, S_z) = ?$$

Bell's inequalities

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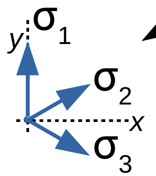

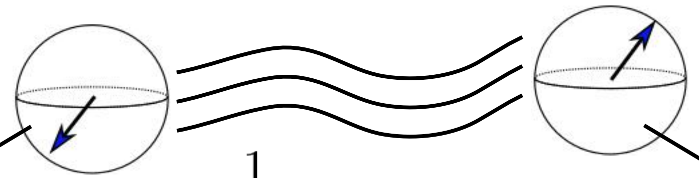

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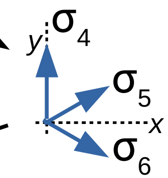

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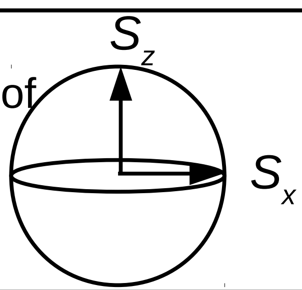

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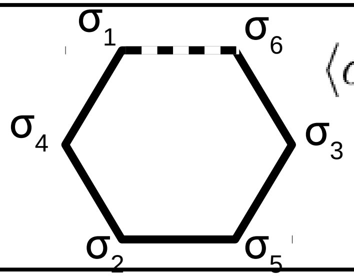
$$P(S_x, S_z) = ?$$



$$\langle \sigma_1 \sigma_4 \rangle + \langle \sigma_4 \sigma_2 \rangle + \langle \sigma_2 \sigma_5 \rangle + \langle \sigma_5 \sigma_3 \rangle + \langle \sigma_3 \sigma_6 \rangle - \langle \sigma_6 \sigma_1 \rangle \geq -4$$

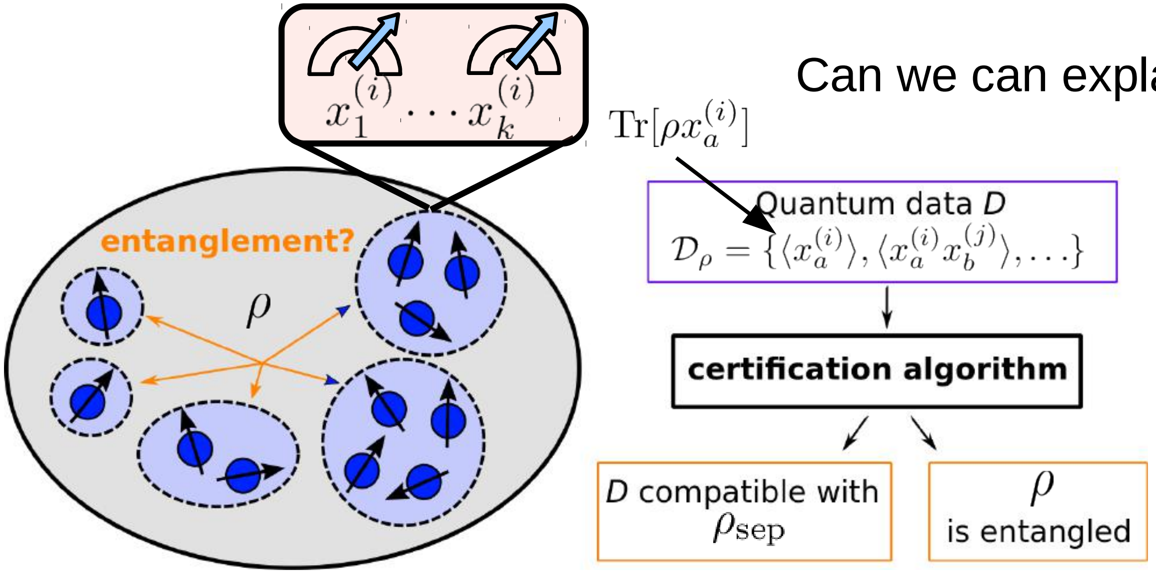
<u>But:</u> angles π/3, spin singlet: =-9/2 < -4

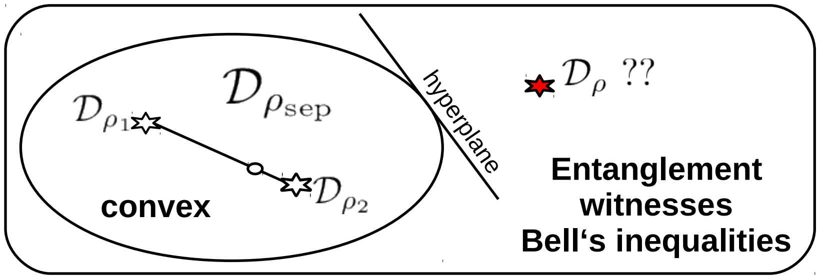
Bell's inequality

Braustein & Caves
Annals of Physics 1990

The problem: Entanglement certification

Can we can explain the data **without entanglement**?





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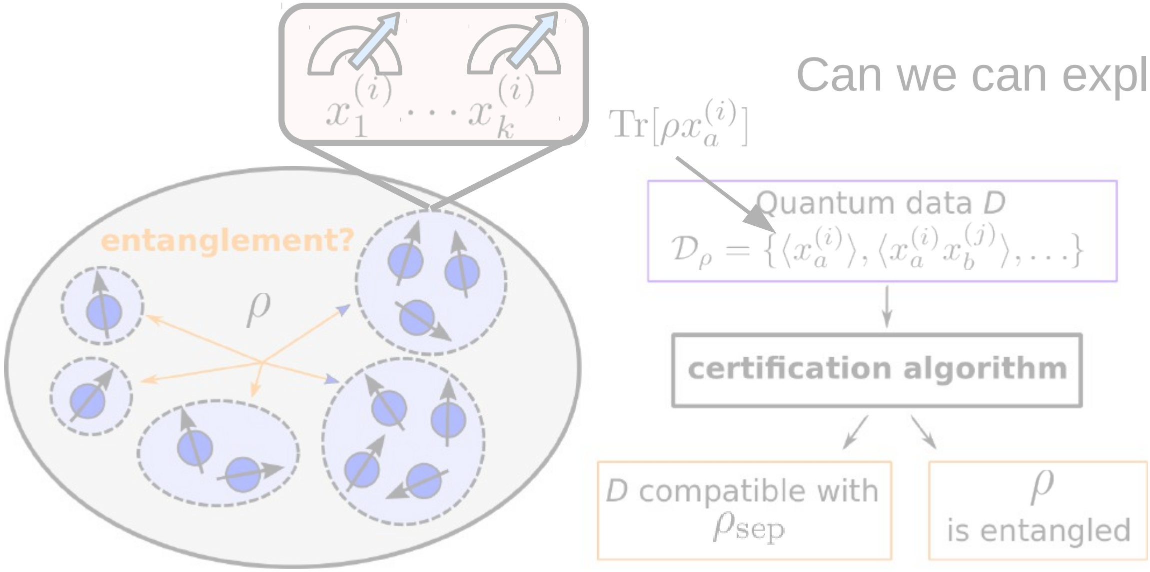
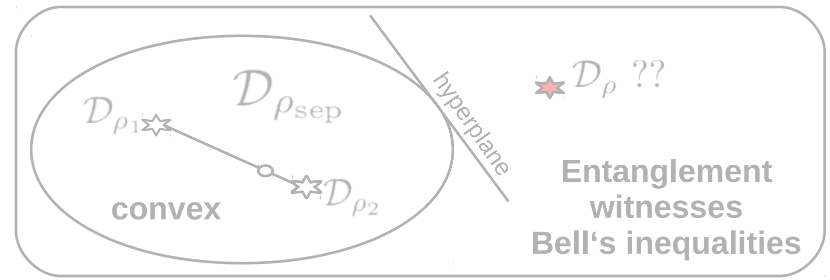
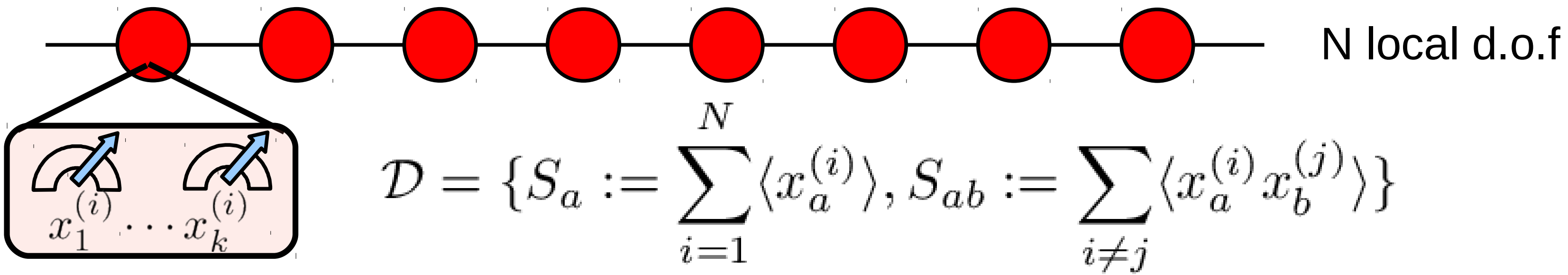




The issue: No scalable and data-driven method to certify entanglement
(either device-dependent, or device-independent)

- **device-independent**:
→ important for effective qubits (superconducting qubits, effective few-level atoms...)
- **scalability** is key (N>10)
- **data-driven**: → impossible to fully reconstruct the quantum state ρ (exp(N) meas.)
→ too many entanglement witnesses / Bell's inequalities (~exp(exp N))

Powerful simplification: Only 2-body correlations + symmetrize the data



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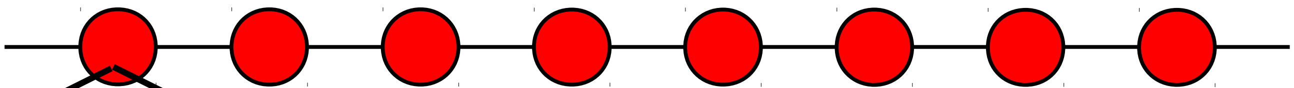
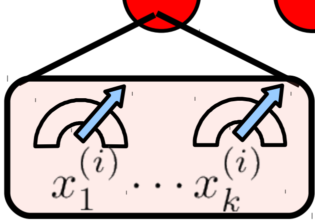


N local d.o.f



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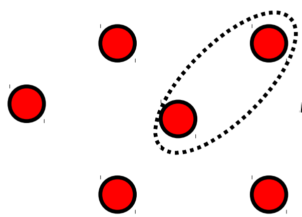
1) Bell's inequalities:

- Theory: Tura *et al*, Science 2014; Tura & Fadel, PRL 2017 (2 inputs – 2 outputs)
- Exp: Schmied *et al*, Science 2016; Engelsen *et al*, PRL 2017
- at quantum critical points: Piga, Aloy, Lewenstein & IF (PRL 2019)
- general data-driven approach: Müller, Aloy, Lewenstein, IF (in preparation)

2) Entanglement witnesses:

$$\rho_{2,\mathrm{av}} = \overline{\rho_{ij}}^{\text{all pairs}} \Leftrightarrow \{\langle J_a \rangle, \langle J_a J_b \rangle; a, b = x, y, z\}$$



$$\rho_{ij} = \mathrm{Tr}_{\mathrm{rest}}[\rho]$$

Tóth *et al*, PRA 2009
Vitagliano *et al*, PRL 2011

Collective spin: $\vec{J} = \sum_i \vec{S}^{(i)}$
(in general: SU(d) generators)

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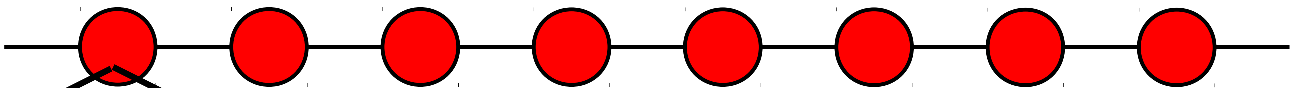

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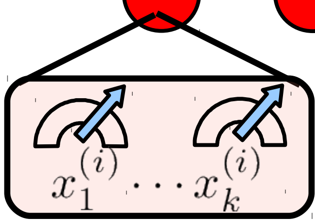

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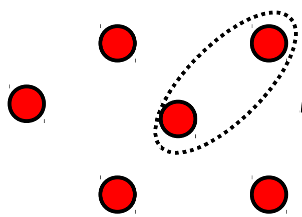

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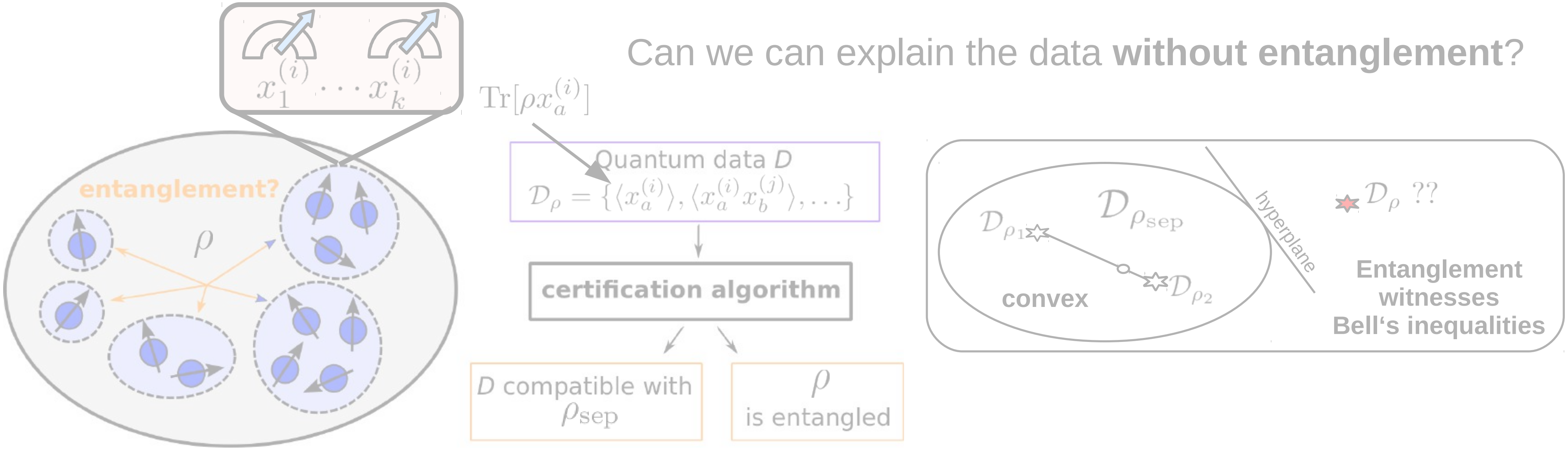
Tóth *et al*, PRA 2009
Vitagliano *et al*, PRL 2011

Too many inequalities: need data-driven approaches
Beyond permutation invariance?
Information in the hand of experimentalists,
but we don't know how to exploit it to certify entanglement

<u>See also:</u>
- Wang *et al*, PRL 2017
(1d translation invariance
+ finite-range correls.)
- Baccari *et al*, PRX 2017
(solves a relaxation for Bell's ineq.)

The problem: Entanglement certification



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Our approach: statistical inference

IF & T. Roscilde, arxiv: 2004.07796 (2020)

Statistical-inference approach to Bell's inequalities

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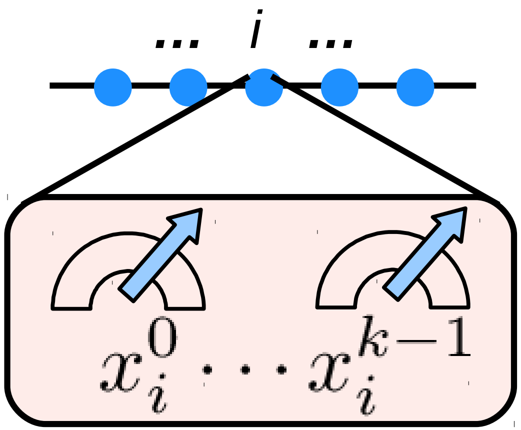

J. S. Bell, Physics 1964
A. Fine, Phys. Rev. Lett. 1982

Quantum data

$$\langle f_r(\sigma)\rangle_{\rm Q} \ (\sigma = \{x_i^a\})$$

$$\text{e.g.} \langle x_i^a\rangle_{\rm Q}, \langle x_i^a x_j^b\rangle_{\rm Q}, \ldots$$

Bell's **local-variable (LV) model (classical data)**:

Grand-probability distribution ***P*****$_{LV}$(σ)** s.t.

$$\langle f_r\rangle_{\rm Q} = \langle f_r\rangle_{\rm LV} \ (r = 1, \ldots R)$$

Always exists if the state is *not* entangled

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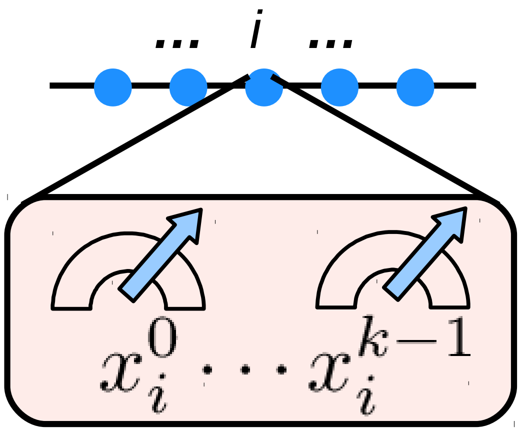

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Inverse problem: max entropy solution

$$P_{\mathrm{LV}}(\sigma) = Z^{-1} \exp[\sum_{r=1}^{R} K_r f_r(\sigma)]$$

Boltzmann distribution over a classical model

E.T. Jaynes, Phys. Rev. 1957

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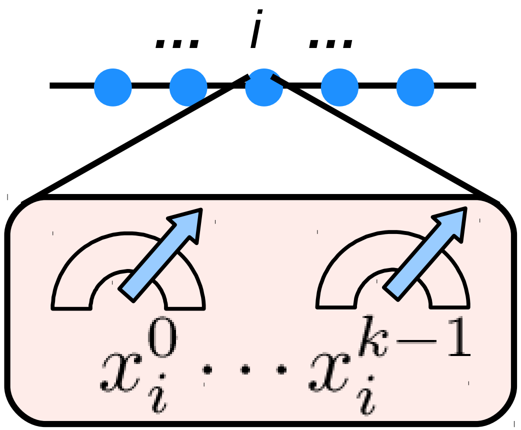

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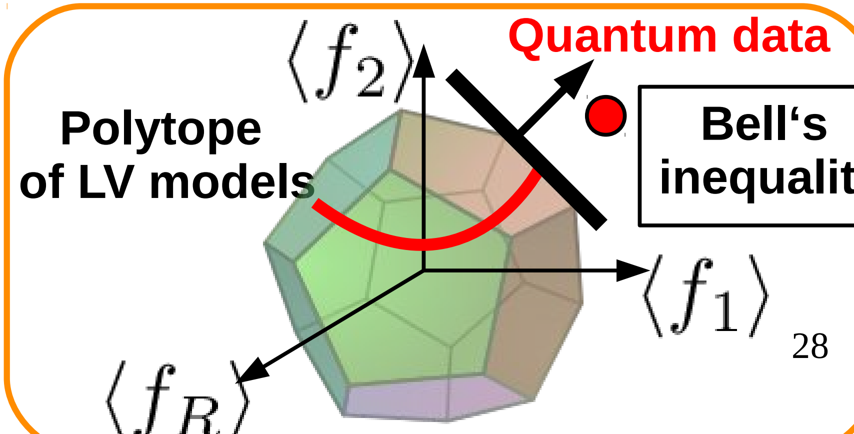
Update rule: **iterate**

$$K_r' = K_r - \epsilon(\langle f_r\rangle_{\rm LV} - \langle f_r\rangle_{\rm Q})$$

gradient-descent of the log-likelihood **(convex)**

Nguyen *et al.*, Adv. Phys. 2017

Trajectory of classical data converging to the quantum data



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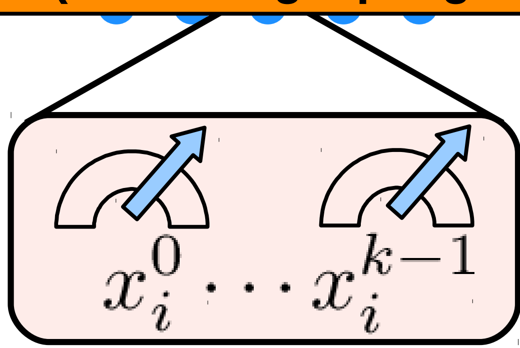
IF & T. Roscilde, arxiv: 2004.07796 (2020)

Generic: applies to any quantum data
Scalable: Monte-Carlo sampling of classical statistical models (unless e.g. spin glass)



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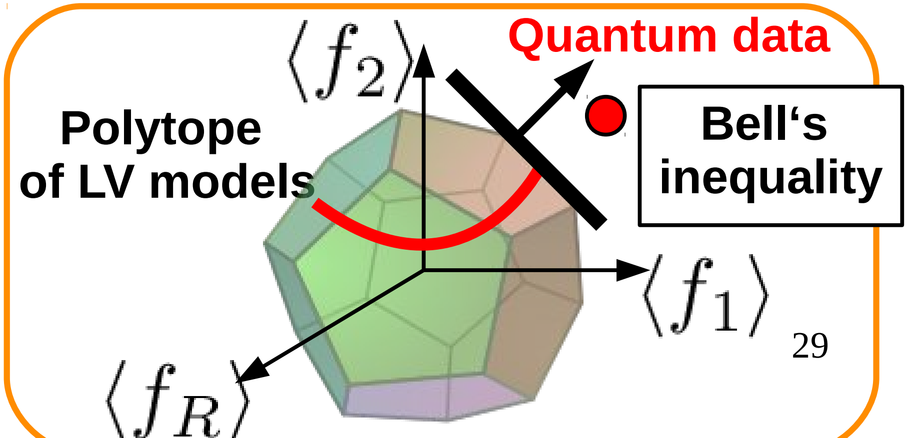
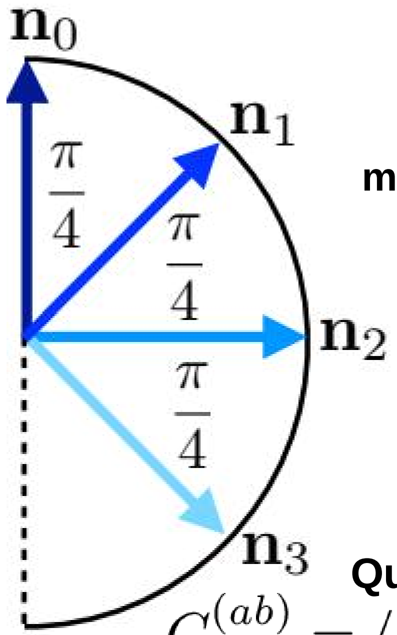


Application: Heisenberg antiferromagnets

$$\mathcal{H} = J \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j$$

S=1/2 quantum spins on a square lattice

Ground state: many-body singlet $S_{tot}=0$



Uniform measurement strategy (k=4)

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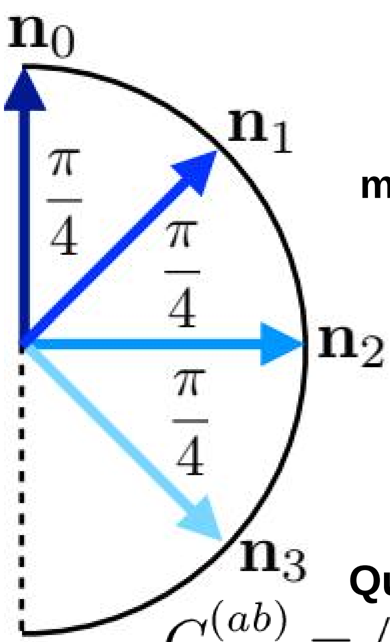

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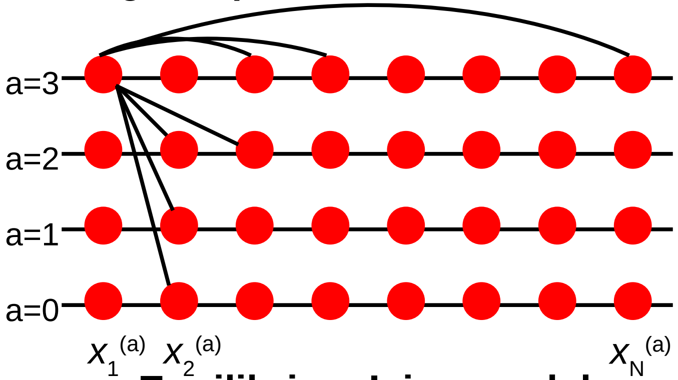
Fitting the quantum data with LV models



Equilibrium Ising model

$$H = -\sum_{i<j} \sum_{a,b} K_{ij}^{(ab)} x_i^{(a)} x_j^{(b)}$$

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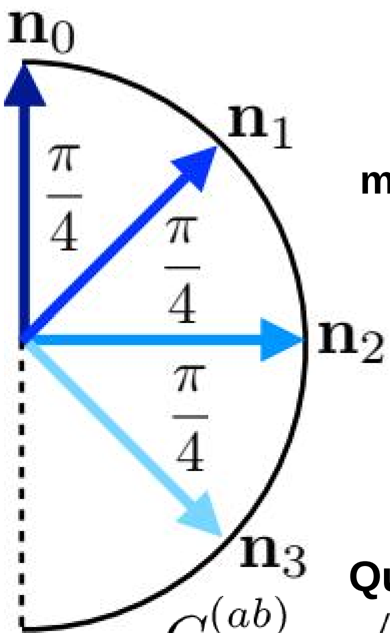

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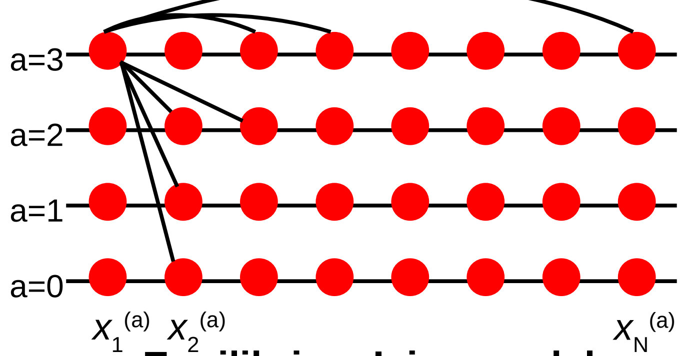

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Novel Bell inequality (permutationally invariant)

$$\langle B \rangle_{\text{LV}} = S_{11} + S_{12} + S_{22} + S_{23} + \cdots + S_{kk} - S_{k1} \geq -2N(k-1)$$

$$S_{ab} = \sum_{i \neq j} C_{ij}^{(ab)}$$

Many-body singlets: $\langle B \rangle_{\text{Q}} = -Nk[1 + \cos(\pi/k)]$

(maximal quantum violation – Self testing)

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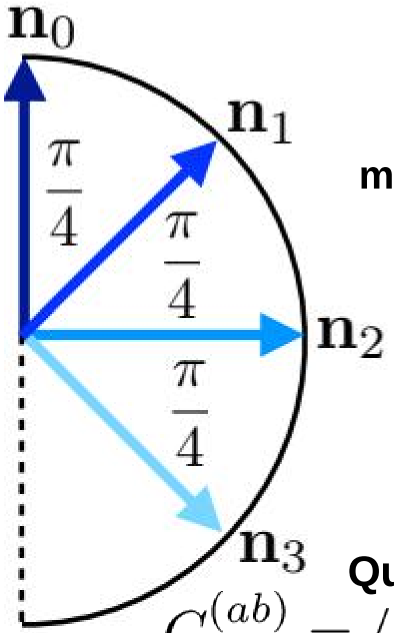

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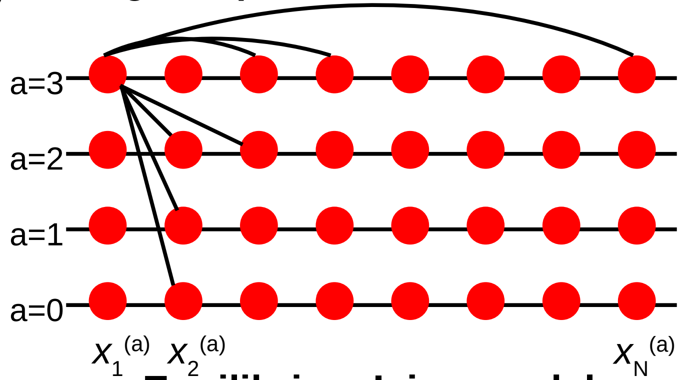

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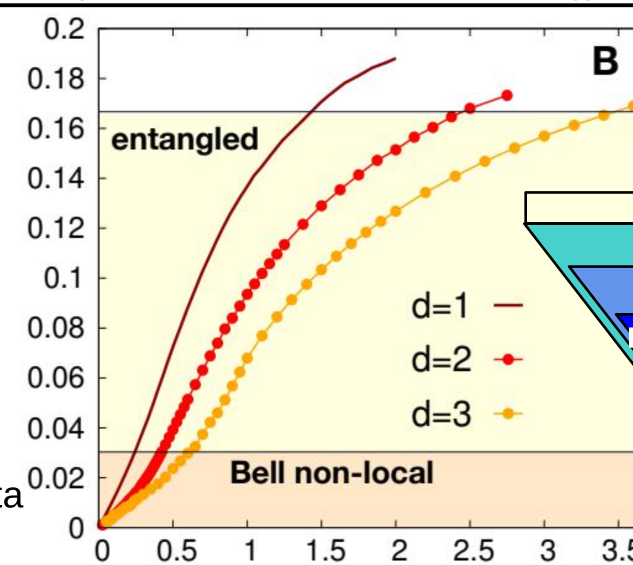
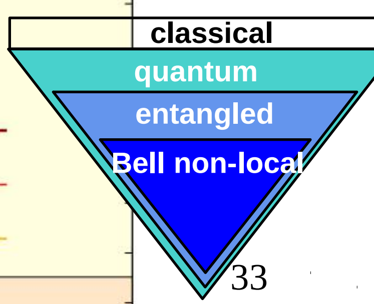
Assuming SU(2) invariance of the quantum state:

$$\langle \hat{J}_z^2 \rangle / N < \frac{1}{4} - \frac{k-1}{2k[1+\cos(\pi/k)]}$$ (Bell non-locality)

Compared to: $\langle \hat{J}_z^2 \rangle / N < \frac{1}{6}$ (entanglement, Tóth *et al.* PRA 2004)

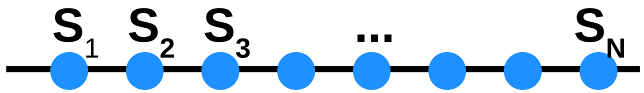




Similar structure to Schmied *et al* (Science 2016); emerges from the data
IF & Roscilde, arxiv: 2004.07796 (2020); IF & Acín, in preparation

Extension: data-tailored entanglement witnesses

N quantum spin-1/2

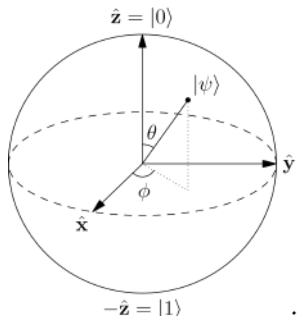


$$\rho_{\text{sep}} = \prod_{i=1}^{N} \int_{|\mathbf{S}_i|=1} d\mathbf{S}_i \; P(\mathbf{S}_1, \cdots \mathbf{S}_N) \; \rho_1(\mathbf{S}_1) \otimes \cdots \otimes \rho_N(\mathbf{S}_N)$$

with $\rho(\mathbf{S}_i) := |\psi(\mathbf{S}_i)\rangle\langle\psi(\mathbf{S}_i)|$

P **: statistical distribution over classical rotators**

Bloch sphere



$$|\mathbf{S}(\theta, \phi)|^2 = 1$$

$$|\psi(\mathbf{S})\rangle = \cos(\theta/2)|\uparrow\rangle + \sin(\theta/2)e^{i\phi}|\downarrow\rangle$$

Extension: data-tailored entanglement witnesses

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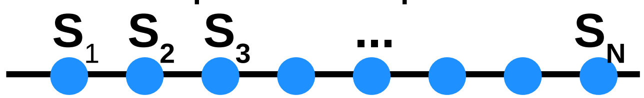

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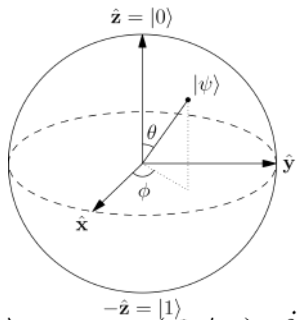

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Quantum data = 2-body correlation matrix:

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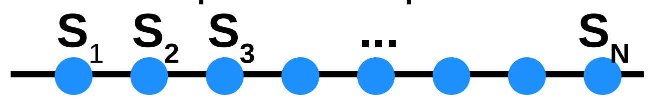
$$P(\{\mathbf{S}_i\}) \propto \exp[\sum_{i,j=1}^{N} \sum_{a,b \in \{x,y,z\}} K_{ij}^{ab} \; S_i^a S_j^b]$$

Solution: **Generalized XYZ model for classical rotators**

Extension: data-tailored entanglement witnesses

N quantum spin-1/2



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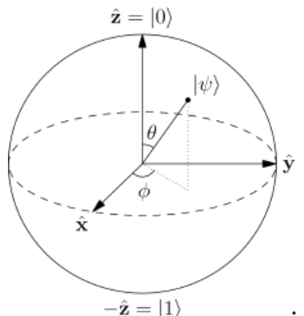
***P* : statistical distribution over classical rotators**

Bloch sphere



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Solution: **Generalized XYZ model for classical rotators**

Tomographically complete
(no other possible entanglement witness based on 2-body correlators)

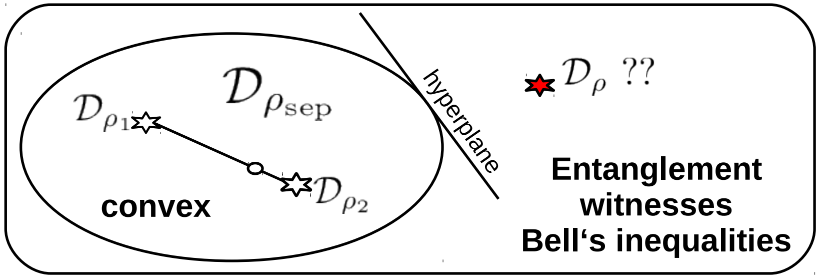
General case: local Hilbert space dim = d:
- 2d – 2 classical variables
- d^2 - 1 measurements (e.g. generators of SU(d))

Leads to the discovery of new EW for Heisenberg chains and quantum Ising chains (IF & Roscilde, in preparation)

Conclusions

The problem: Entanglement certification

Can we can explain the data **without entanglement**?



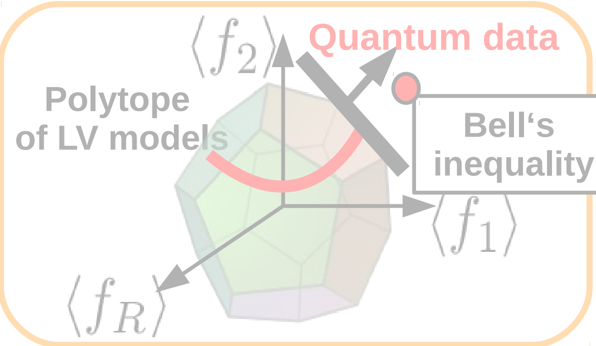
The issue: No scalable and data-driven method to certify entanglement

Our approach: statistical inference

Bell's LV model = classical spin model

Separable state = classical (lattice) field theory

Entanglement problem = Inverse statistical problem



IF & T. Roscilde, arxiv: 2004.07796 (2020)

Perspectives

- Apply to real experimental data (trapped ions, cold atoms, superconducting qubits)
- Alternative way to probe the birth and death of quantum correlations during dynamics
- Other „membership problems" of quantum info could be solved similarly

Quantum info ↔ Statistical physics

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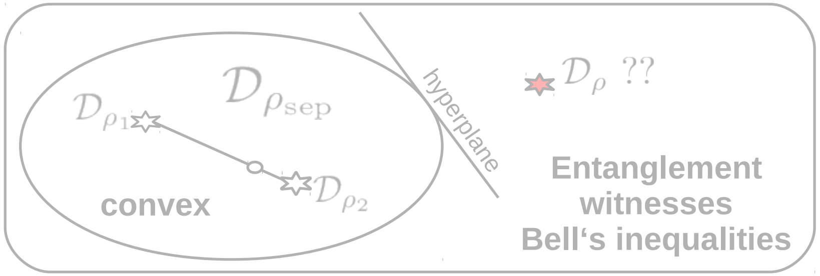

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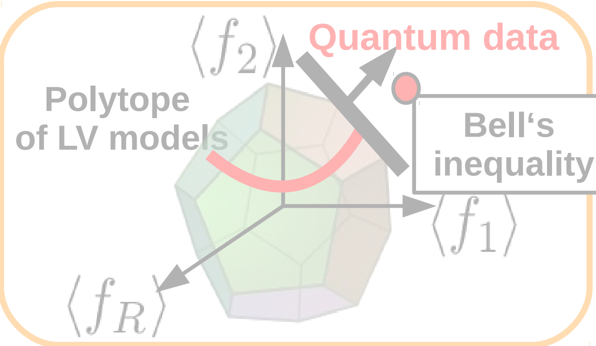

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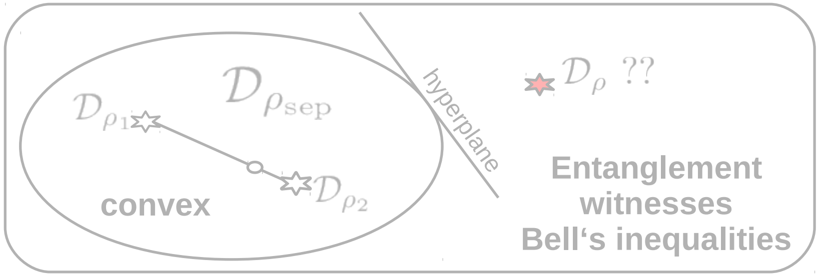

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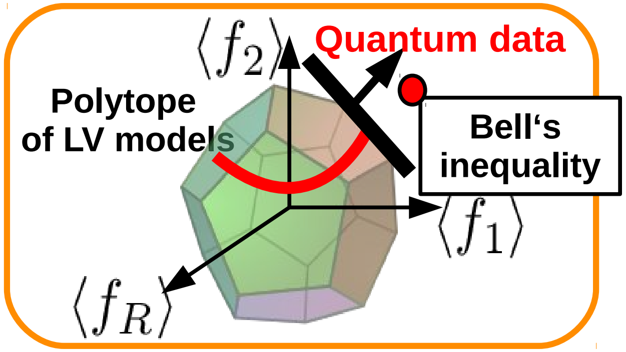

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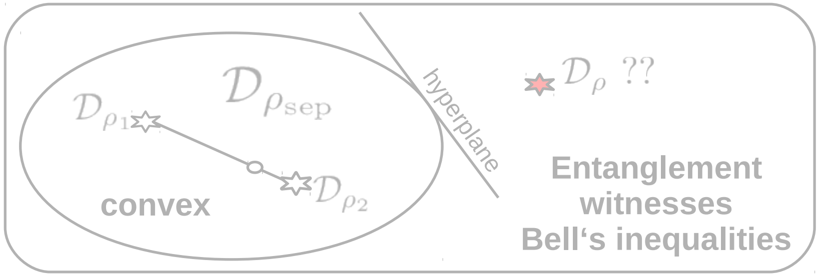

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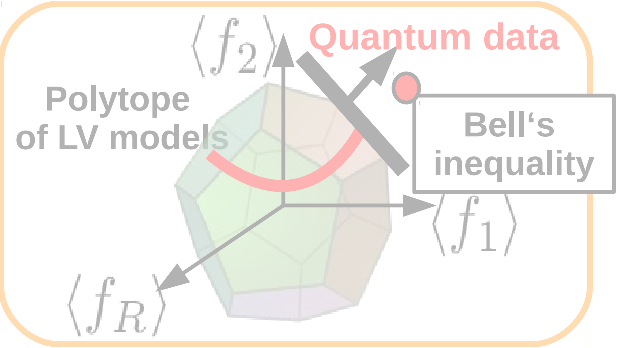

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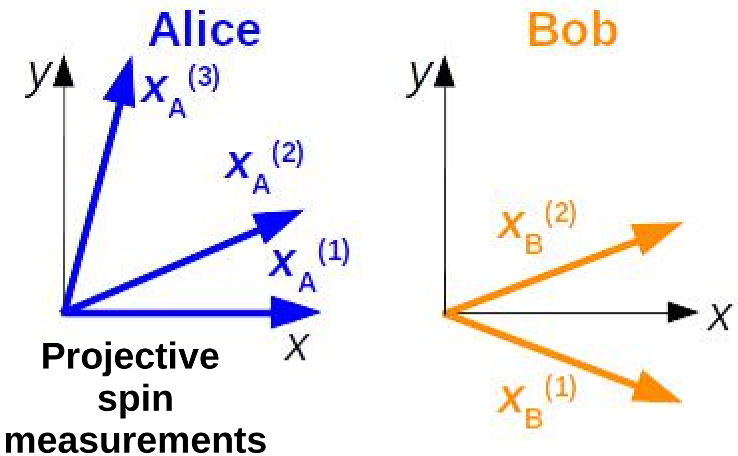
Entanglement versus Bell non-locality

$$|\Psi_-\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$$

Spin singlet



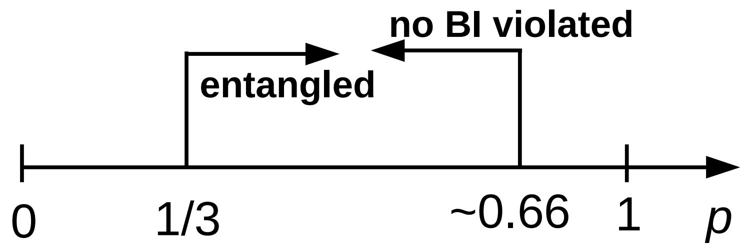
Quantum data:

$$C_{ab} = \langle x_A^{(a)} x_B^{(b)} \rangle_Q = -\vec{x}_A^{(a)} \cdot \vec{x}_B^{(b)}$$

Noisy singlet

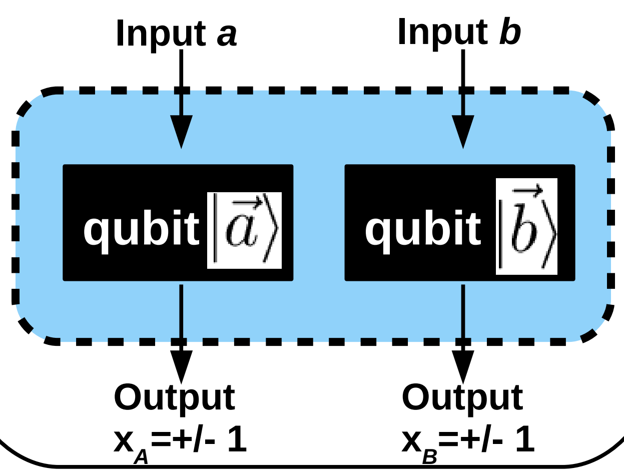
$$\rho = p|\Psi_-\rangle\langle\Psi_-| + (1-p)\frac{\mathrm{Id}}{4}$$

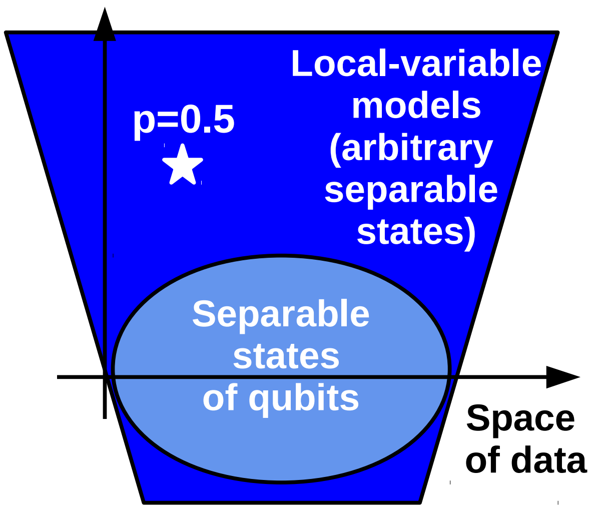
$$\rho_{\text{ent.}} \neq \sum_\lambda P(\lambda)\ \rho_A(\lambda) \otimes \rho_B(\lambda)$$



Werner, PRA 1989
Acín *et al*, PRA 2006

Device dependent





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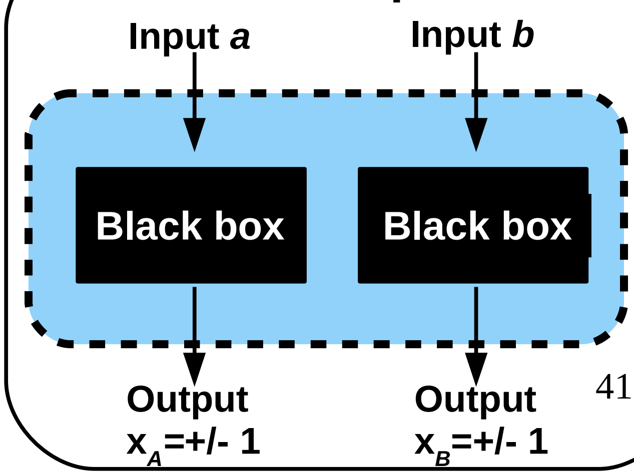
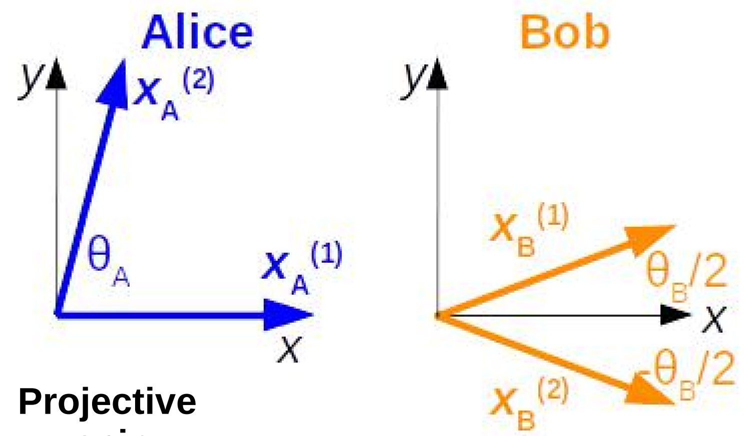


Illustration: the Bell pair

$$|\Psi_-\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$$

Spin singlet



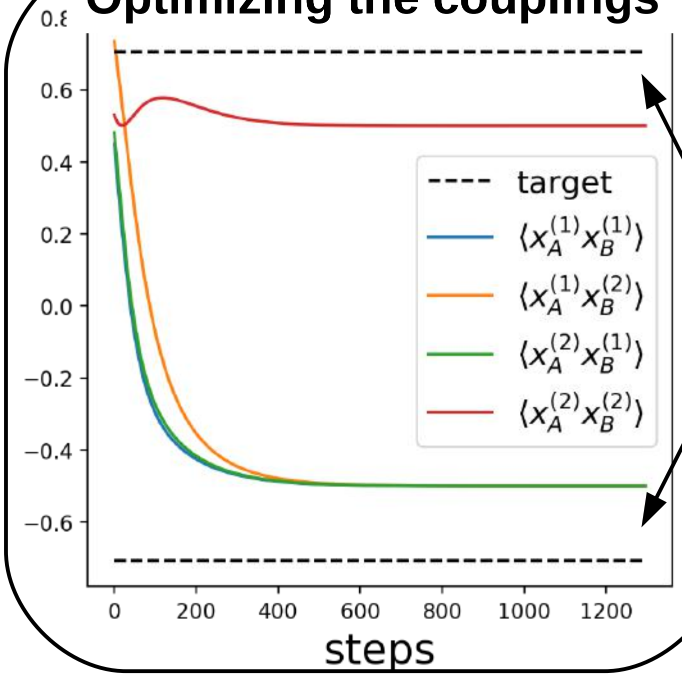
Projective spin measurements

$$(\theta_A, \theta_B) = (\pi/2, \pi/2)$$

Quantum data: (A. Aspect *et al.* PRL 1982)

$$C_{ab} = \langle x_A^{(a)} x_B^{(b)} \rangle_Q = -\vec{x}_A^{(a)} \cdot \vec{x}_B^{(b)}$$

Optimizing the couplings

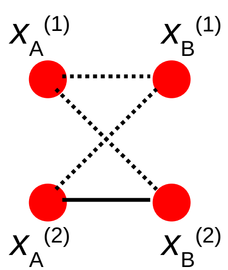


Mismatch: Coeffs of the Bell inequatliy (here: CHSH, Clauser *et al.* PRL 1969)

Equilibrium Ising model

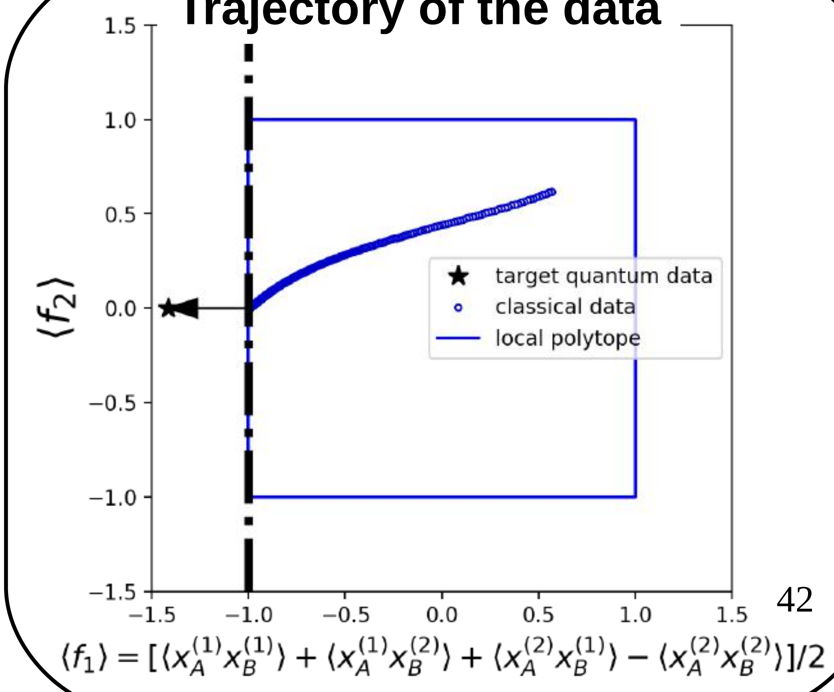
$$\sigma = (x_A^{(1)}, x_A^{(2)}, x_B^{(1)}, x_B^{(2)})$$

Ising variables (+/- 1)

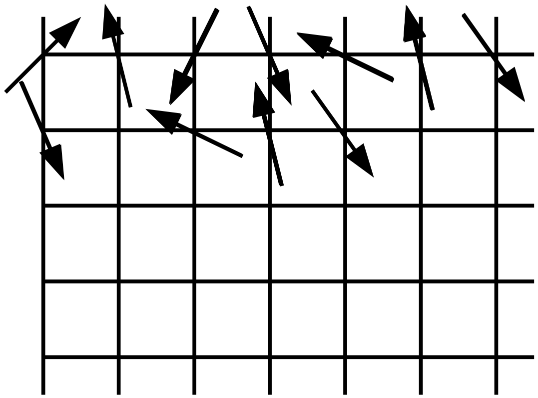


$$P_{\rm LV}(\sigma) = Z^{-1} \exp\left[\sum_{a,b=1}^{2} K_{ab} x_A^{(a)} x_B^{(b)}\right]$$

Trajectory of the data



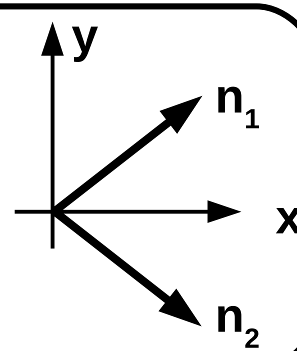
2d quantum Ising model at the quantum-critical point



LxL square lattice

$$\mathcal{H} = -J \sum_{<i,j>} S_i^z S_j^z - \Gamma \sum_i S_i^x$$

<u>Each quantum spin</u>
2 measurement directions
$\mathbf{n}_1$ - $\mathbf{n}_2$ along y

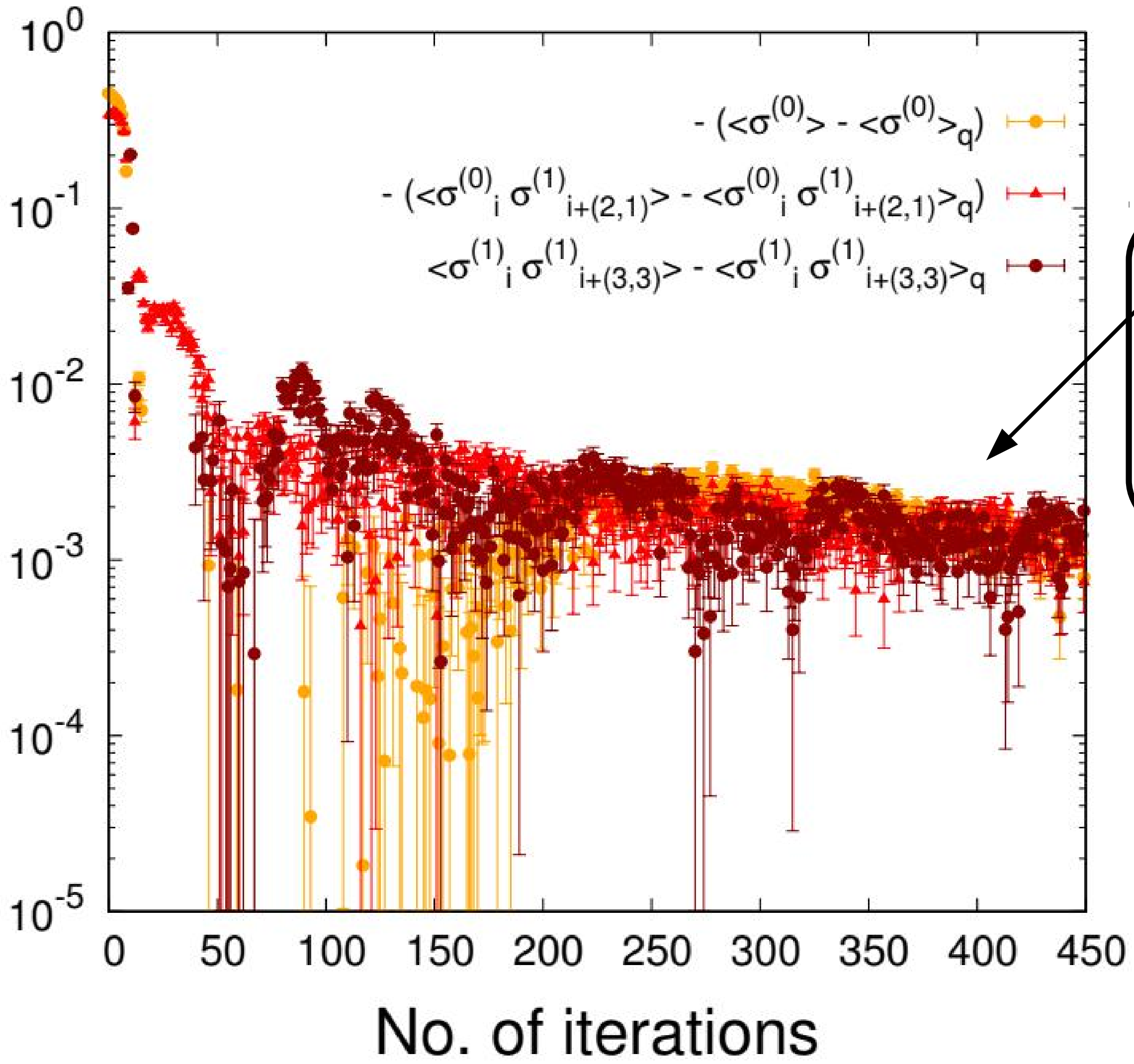


<u>Collect:</u> $\vec{x} = \left(\langle \sigma_i^{\mathbf{n}_a} \rangle, \langle \sigma_i^{\mathbf{n}_a} \sigma_j^{\mathbf{n}_b} \rangle;\ i \neq j;\ a, b \in \{0, 1\}\right)$

<u>Educated guess</u>: those data violate the Bell inequality derived in Tura *et al* (Science 2014) and Schmied *et al* (Science 2016), as a consequence of spin squeezing at the QCP (IF & Roscilde, PRL 2018; Piga, Aloy, Lewenstein & IF, PRL 2019).

Recovering Tura *et al*'s permutationally invariant Bell inequality

(Tura *et al,* Science 2014)



All differences converge to the same value!
(Despite the fact that individual averages are very different)

$$\mathcal{H}_{\text{Tura}} = -\sum_i \left(\sigma_i^{(0)} + \sigma_i^{(1)}\right) + \sum_{i<j} \left(\sigma_i^{(0)}\sigma_j^{(0)} + \sigma_i^{(1)}\sigma_j^{(1)} - \sigma_i^{(0)}\sigma_j^{(1)} - \sigma_i^{(1)}\sigma_j^{(0)}\right)$$