Limitations in quantum computing from resource constraints

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(Dated: July 7, 2020)

arXiv:2007.01966 (with new version next week)

Fast overview

- Large scale quantum computing requires fault tolerance

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- Fault tolerance assumes:
  noise per quantum gate does **not** depend on computer size

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- Fault tolerance assumes:
  noise per quantum gate does **not** depend on computer size
- **This work:**

  Studies consequences of relaxing this assumption

  What is the maximum accuracy one can reach given a noise scaling.

  How to find minimum resource (for instance: energy) to perform a calculation

Q.E.C & F.T.Q.C

Quantum Error Correction & Fault Tolerant Quantum Computing

Goal of quantum error correction

You have an **unknown** data that you want to protect against noise

You need to detect if an error occured and to correct it.

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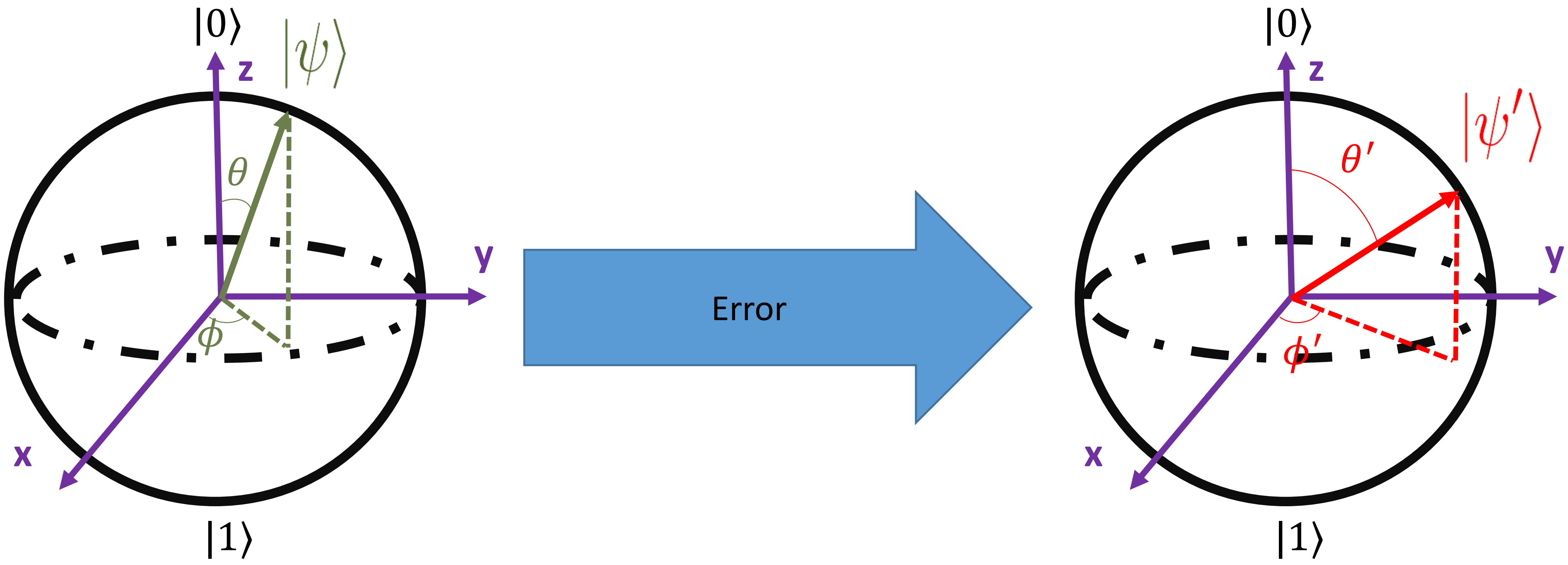
You have an **unknown** data that you want to protect against noise

You need to detect if an error occured and to correct it.

- Non cloning theorem: « you cannot copy a quantum state »
- Measurement are destructive

Stabilizer codes

Error seems continuous



Error discretization

An arbitrary error can be decomposed as a combination of X and Z errors.

We only need to know how to correct X & Z errors.

General framework: stabilizer codes

$$|\psi\rangle = a\,|0\rangle + b\,|1\rangle \quad \xrightarrow{\text{Encoding}} \quad |\psi_L\rangle = a\,|0\rangle_L + b\,|1\rangle_L$$

Logical qubit

General framework: stabilizer codes

$$|\psi\rangle = a\,|0\rangle + b\,|1\rangle$$

Encoding

$$|\psi_L\rangle = a\,|0\rangle_L + b\,|1\rangle_L$$

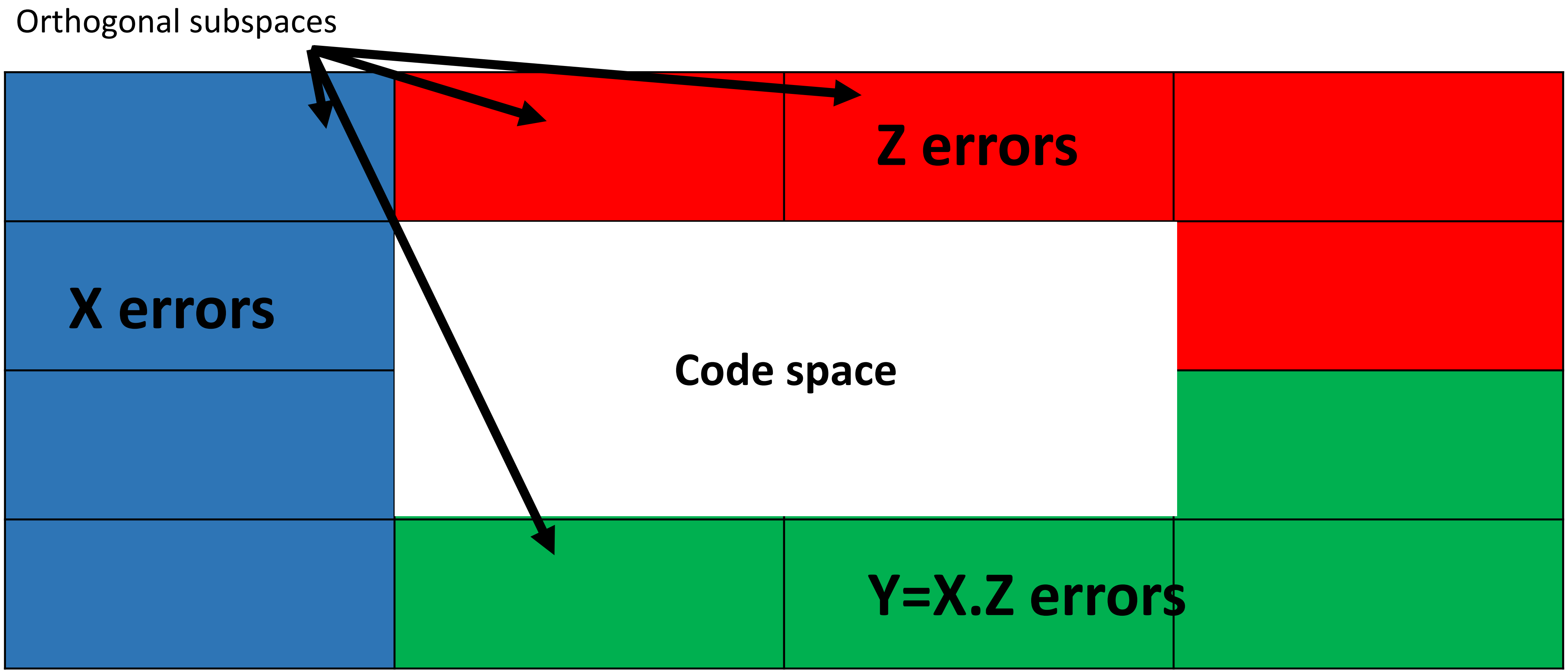
Logical qubit

1 physical qubit

More than 1 physical qubit
For Steane: 7 physical qubits

General framework: stabilizer codes

Stabilizer code of distance d=2t+1
=> Able to correct arbitrary errors occuring on t physical qubits



Fault tolerant quantum computing

The need for fault tolerance

Stabilizer code of distance 2t+1:

$$|\psi_L\rangle \xrightarrow{\text{Error on less than t qubits}} |\psi'_L\rangle \xrightarrow{\textbf{Perfect}\text{ correction}} |\psi_L\rangle$$

Initial logical qubit | Perturbed logical qubit | Corrected logical qubit

Quantum error correction assumes:

- less than t qubits have been flipped [1].
- correction is perfectly done [2]

The need for fault tolerance

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Initial logical qubit — Perturbed logical qubit — Corrected logical qubit

Quantum error correction assumes

- less than t qubits have been flipped [1].
- correction is perfectly done [2]

Fault tolerant quantum computing:

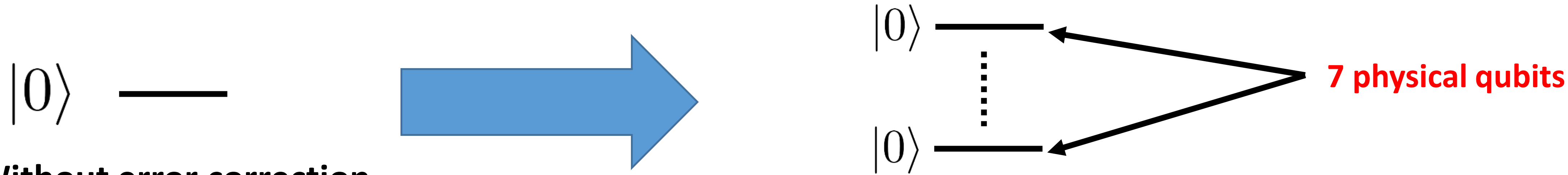
- How to design circuit such that [1] will be verified: **circuit that avoids error to propagate**
- Takes in account the fact Q.E.C is noisy (relax [2])

Fault tolerant Steane code

Short intro to concatenated code

Principle of error correction (concatenated codes):

Without error correction
« bare » gate



Without error correction
« bare » qubit

With error correction
« bare » qubit replaced by 7 physical qubits

With error correction
« bare » gate replaced by « A » physical gates
Acting on those qubits

Short intro to concatenated code

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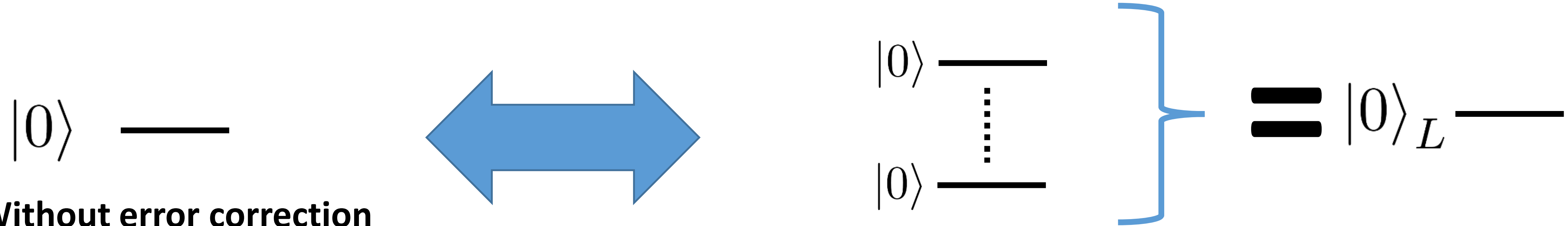


Without error correction
« bare » qubit

With error correction
« bare » qubit replaced by 7 physical qubits

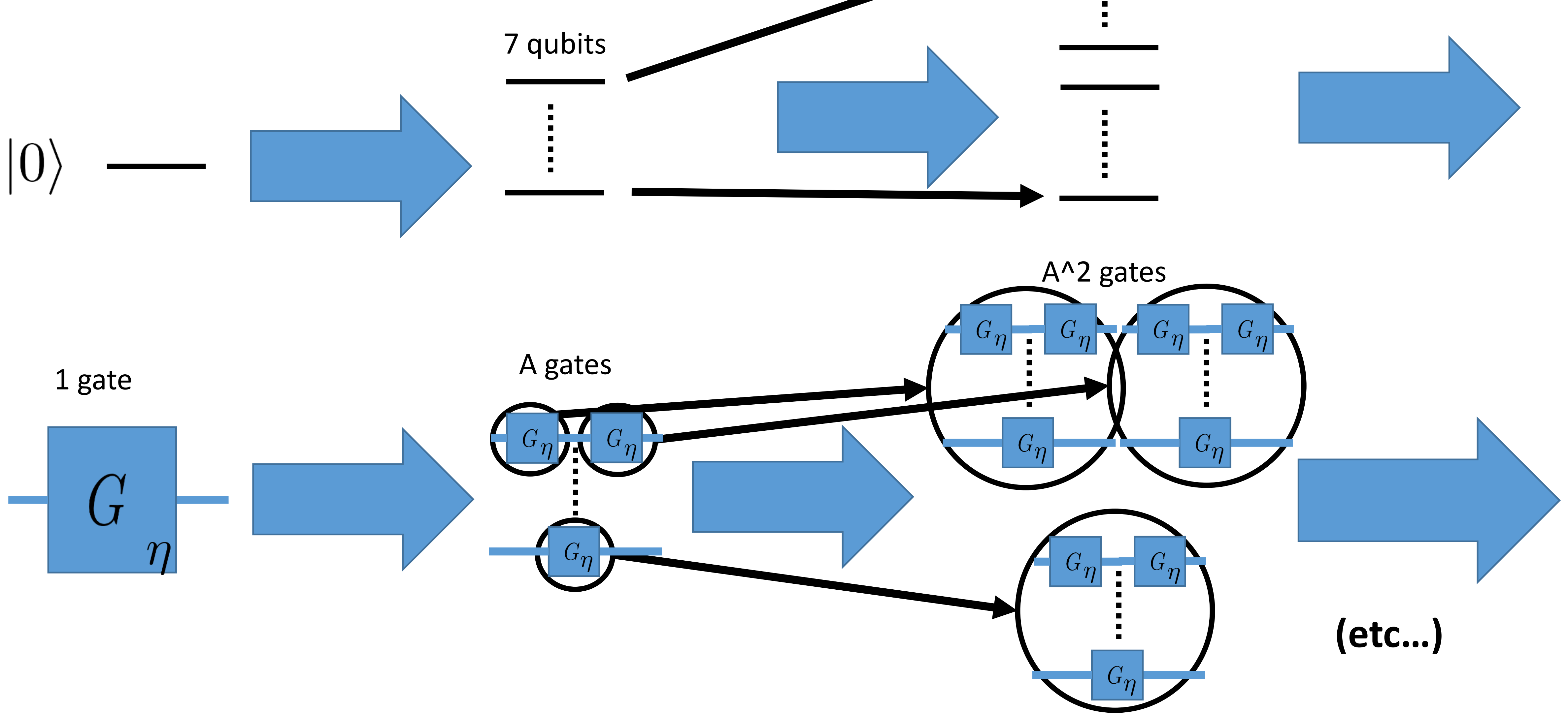
Same logical operation !

But noise now lower !

With error correction
« bare » gate replaced by « A » physical gates
Acting on those qubits

Short intro to concatenated code

<u>Principle of error correction (concatenated codes):</u>



Short intro to concatenated code

Probability of failure of a logical gate: Standard Fault tolerance

This term is a constant: physical error strength

Concatenation level (integer)

$$p(k) = \eta_{thr} \left(\frac{\eta}{\eta_{thr}} \right)^{2^{k}}$$

$$\eta_{thr} \approx 10^{-4}$$

If $\eta < \eta_{thr}$ Concatenating enough: gate failure can be put as close to 0 as wanted.

Short intro to concatenated code

<u>Probability of failure of a logical gate: Standard Fault tolerance</u>

$$p(k) = \eta_{thr} \left(\frac{\eta}{\eta_{thr}} \right)^{2^k}$$

$\eta > \eta_{thr}$ Q.E.C brings in more noise than what it can correct: **QEC looses competition**

$\eta < \eta_{thr}$ Q.E.C brings in less noise than what it can correct: **QEC wins competition**

$N(k) = A^k$ The number of physical resources (gates) grows exponentially with the level of concatenation

Our work

Considering a scale dependant noise

In our work we assume η grows when computer grows

Physical examples in which it might occur:

- Engineering design issues
  Example: Crosstalk: increases with computer size (frequency overcrowding)

- Limited resources:
  Example: Energetic, like limited cooling power. [Cost issue]

Considering a scale dependant noise

More precisely, we assume:

$\eta(k)$ strictly increasing and $\eta(+\infty) > \eta_{\mathrm{thr}}$

we will always work under those following assumptions in what follows.

$$\boxed{p(k) = \eta_{thr} \left(\frac{\eta(k)}{\eta_{thr}} \right)^{2^k}}$$

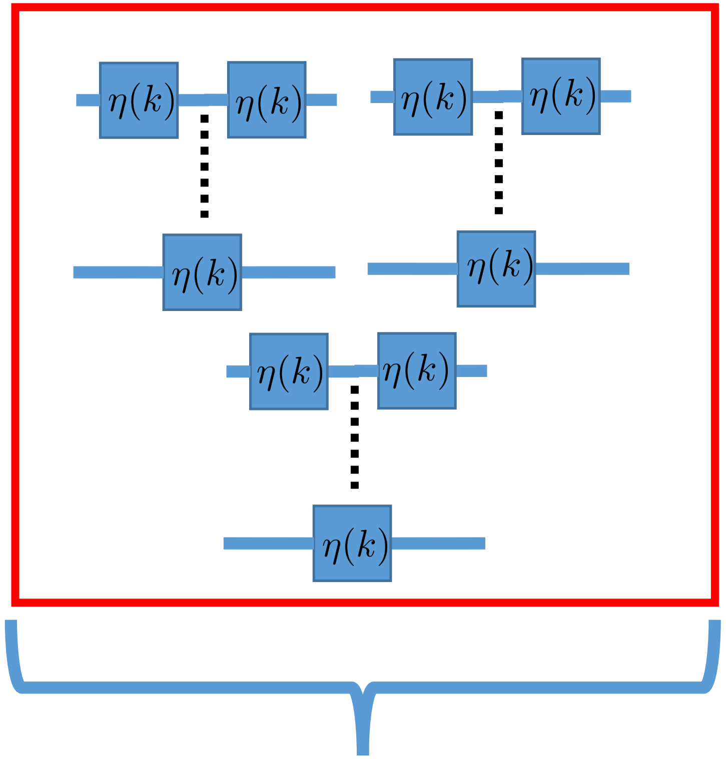
Considering a scale dependant noise

$\eta(k)$ **Physical** error strength

$p(k)$ **Logical** error strength

$$p(0) = \eta(0)$$

Error strength without concatenation
Thus logical error strength = physical error strength



Logical gate (k concatenations)
$p(k)$

Considering a scale dependant noise

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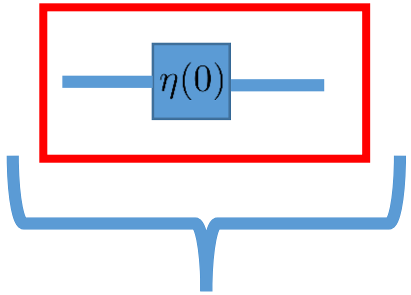


Logical gate = physical gate for 0 concatenation

$$p(0)$$

Considering a scale dependant noise

$$\left.\begin{array}{l}\eta(k) \nearrow \\ \eta(+\infty) > \eta_{thr}\end{array}\right\} \Rightarrow \exists \widetilde{k} | \eta(\widetilde{k}) \geq \eta_{thr}$$

The will exist a **maximum** concatenation level before situation gets degraded

$$p(k) = \eta_{thr} \left(\frac{\eta(k)}{\eta_{thr}} \right)^{2^k}$$

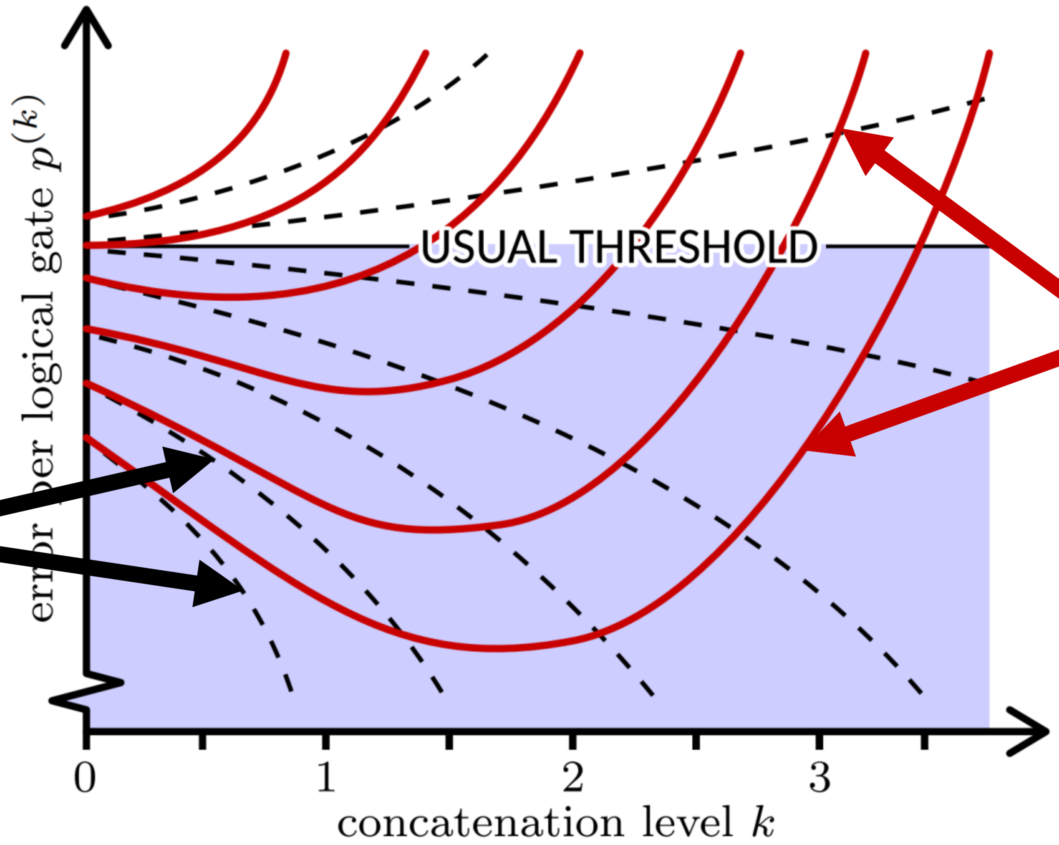
$\Longrightarrow$ **The quantum computed is limited in accuracy**

Considering a scale dependant noise

$$\eta^0 \equiv p(0)$$

$$\eta(\cancel{k}) = \eta^0$$

Standard Fault tolerance



$$\eta(k)$$

Fault tolerance with
Scale dependant noise

Intuitive scale dependent models

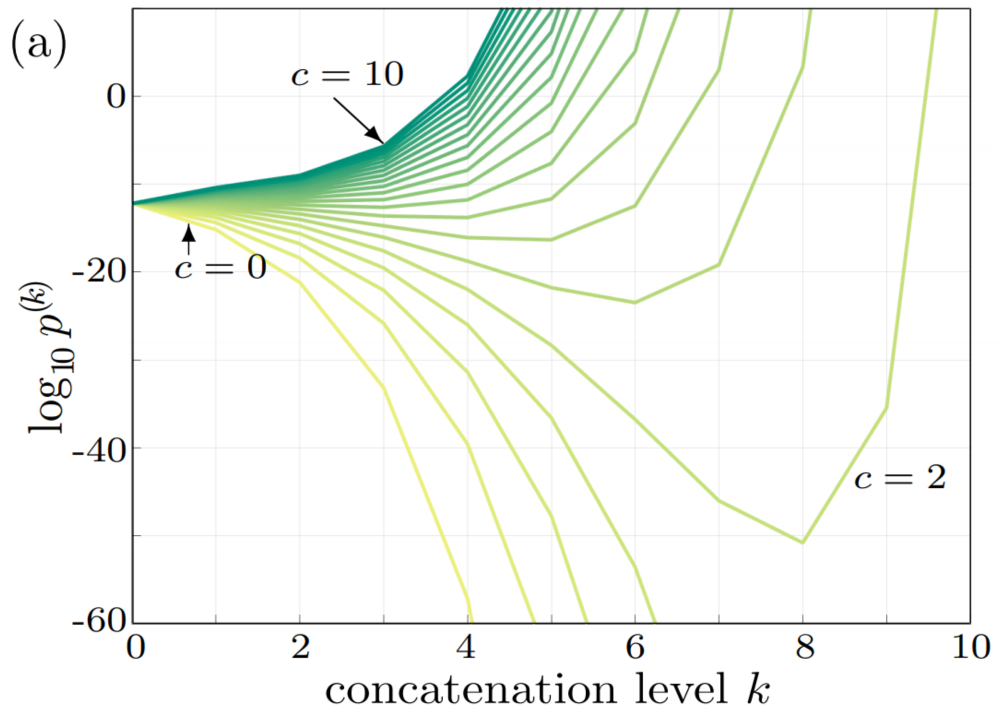
Pedagogic toy model

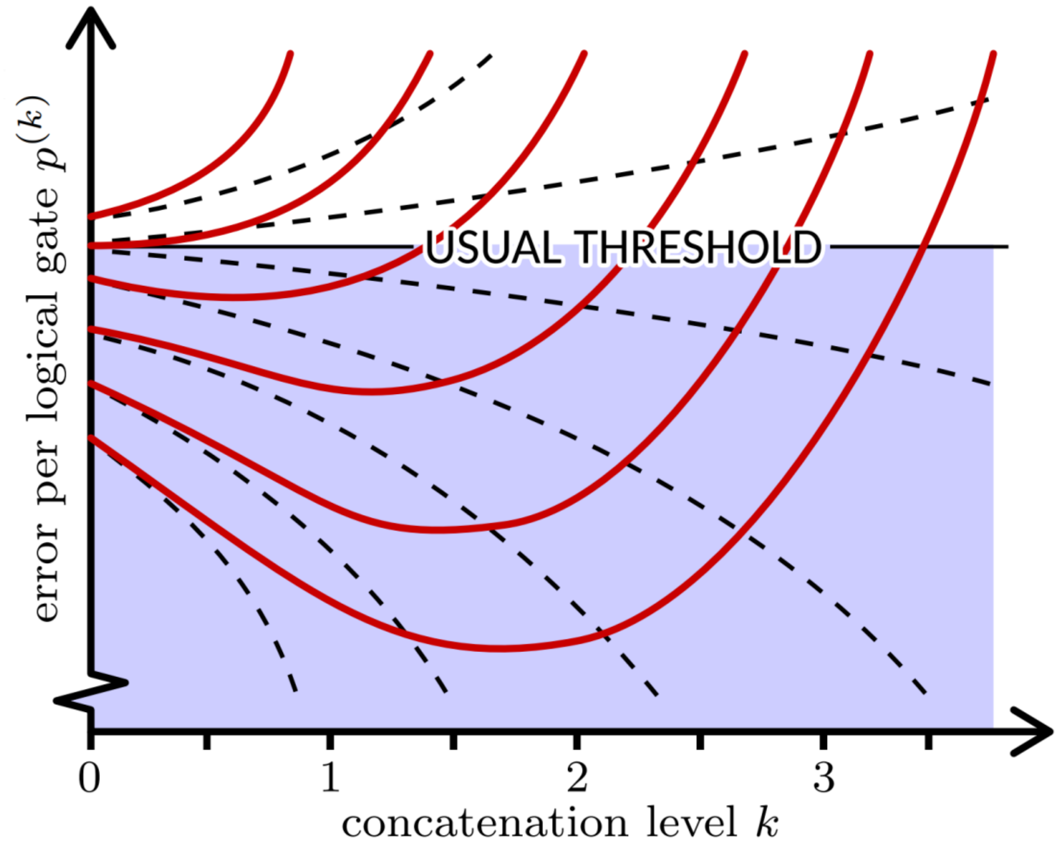
Physical noise growth linearly with k

$$\eta(k)$$

$$p(k) = \eta_{thr} \left(\frac{\eta^0(1+ck)}{\eta_{thr}} \right)^{2^k}$$

Pedagogic toy model

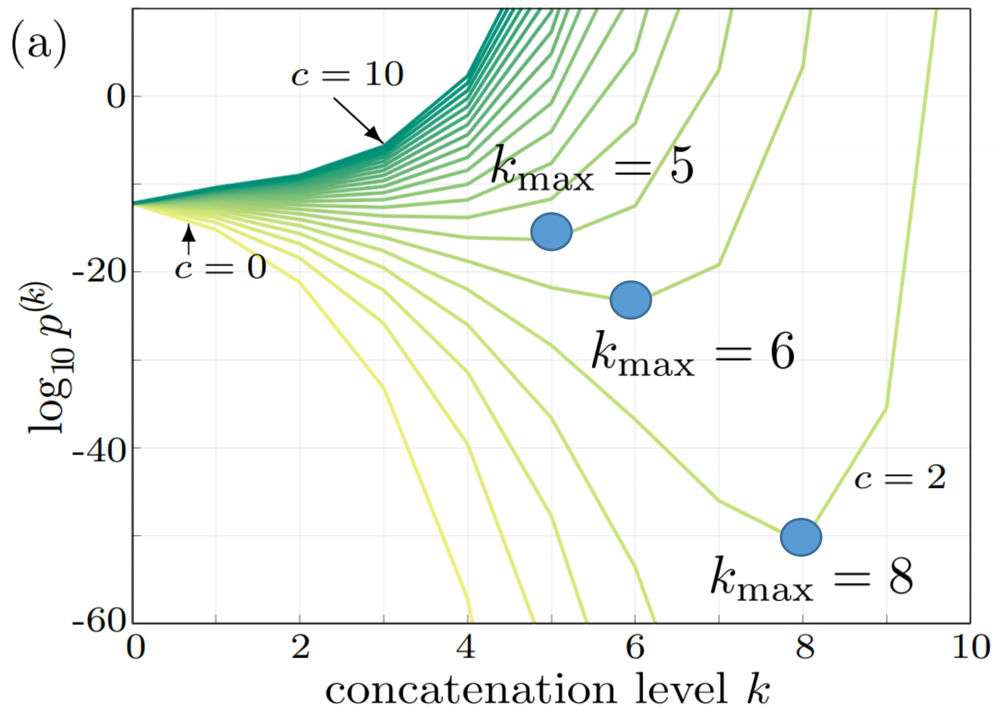


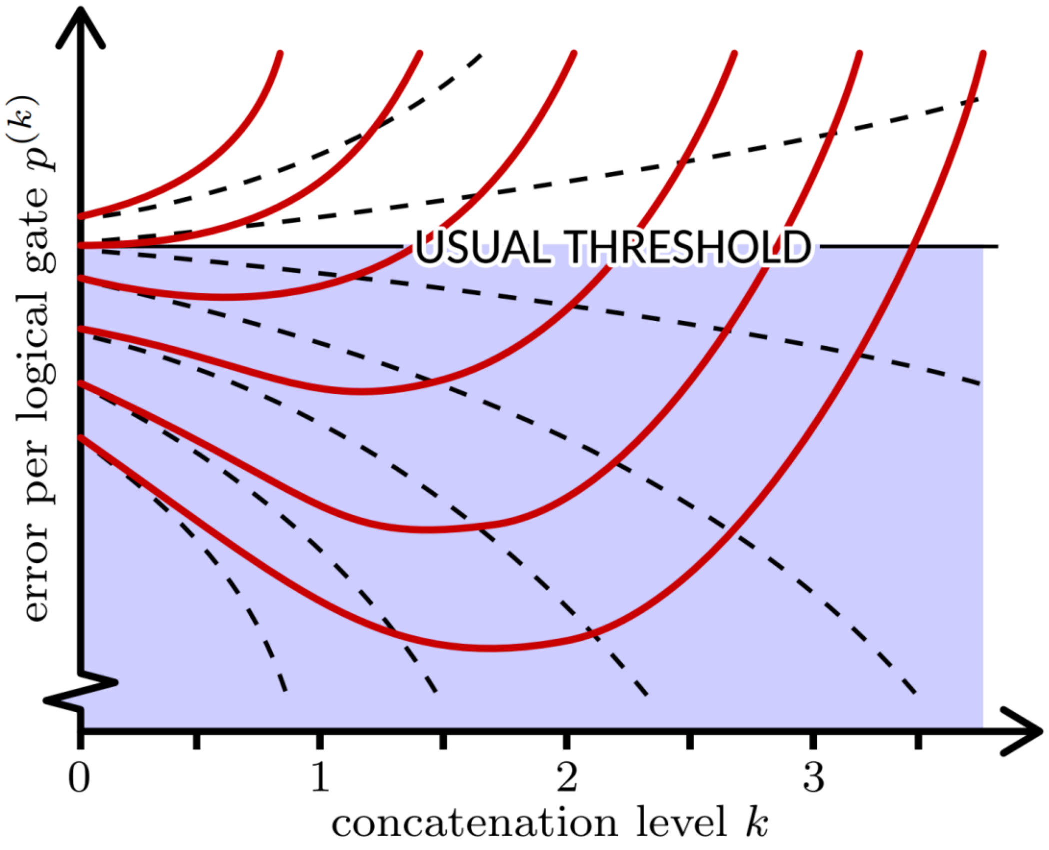


$$p(k) = \eta_{thr} \left(\frac{\eta^0 (1 + ck)}{\eta_{thr}} \right)^{2^k}$$

$$\eta^0 = 5 * 10^{-6}$$

Pedagogic toy model





$k_{\max}$ Maximum level of concatenation before situation gets degraded = concatenation level of max accuracy

$p(k_{\max})$ Maximum accuracy you can get to

$k_{\max} = +\infty$ In « standard » fault tolerance

Noise growing with number of physical elements

Let's assume:

$N =$ number of physical gates

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$\Rightarrow \eta^0 \equiv \eta(0)$ Noise strength if the computer had a single physical gate

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$\Rightarrow \eta = \eta^0 N^\beta$ Noise strength if the computer had a N physical gate

$(\beta > 0)$ Noise **grows** with the number of physical elements

Noise growing with number of physical elements

$N =$ number of physical gates

Considering one logical gate $N(k) = 1 * A^k$

$$\eta = \eta^0 N^\beta \Rightarrow \boxed{\eta(k) = \eta^0 A^{\beta k}}$$

$$(\beta > 0)$$

Noise growing with number of physical elements

No scale dependance: one (or more) concatenation usefull iff

$$\boxed{\eta(\cancel{k}) = \eta^0 < \eta_{thr}}$$

Scale dependance: $\eta(k) = \eta^0 A^{-\beta k}$ at least one concatenation usefull iff:

$$p(1) < p(0) \Leftrightarrow \boxed{\eta^0 < \eta_{thr} A^{-2\beta}}$$

Noise growing with number of physical elements

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$$p(1) < p(0) \Leftrightarrow \boxed{\eta^0 < \eta_{thr} A^{-2\beta}}$$

$$A = 575, \beta = 1 \Rightarrow A^{-2\beta} \approx 10^{-6}$$

Noise strength must be 6 order of magnitude lower than standard threshold to have concatenation usefull at all.

Noise growing with number of physical elements

This example said differently: if linear, **the growing rate must be very low**:

$$\eta^0 < 10^{-6}\eta_{thr} \Rightarrow \eta^0 < 10^{-10}$$

Noise growing with number of physical elements

This example said differently: if linear, **the growing rate must be very low**:

$$\eta^0 < 10^{-6}\eta_{thr} \Rightarrow \eta^0 < 10^{-10}$$

$$\boxed{\eta < 10^{-10} N}$$

To ensure possibly a **<u>single</u>** concatenation level.

Noise growing with number of physical elements

Noise: reduced double exponentially

But: number of physical elements: grows exponentially

$$p(k) = \eta_{thr} \left(\frac{\eta(k)}{\eta_{thr}} \right)^{2^k}$$

$$A^k$$

Noise growing with number of physical elements

$$p(k) = \frac{1}{B}(B\eta(k))^{2^k}$$

Noise: reduced double exponentially

But: number of physical elements: $\underbrace{\text{grows exponentially}}$ A^k

Implies: the noise must increase <u>very slowly</u> when you scale up

Noise growing with number of physical elements

In the paper:

We provide tools to estimate how far you can go for some typical noise scaling law.

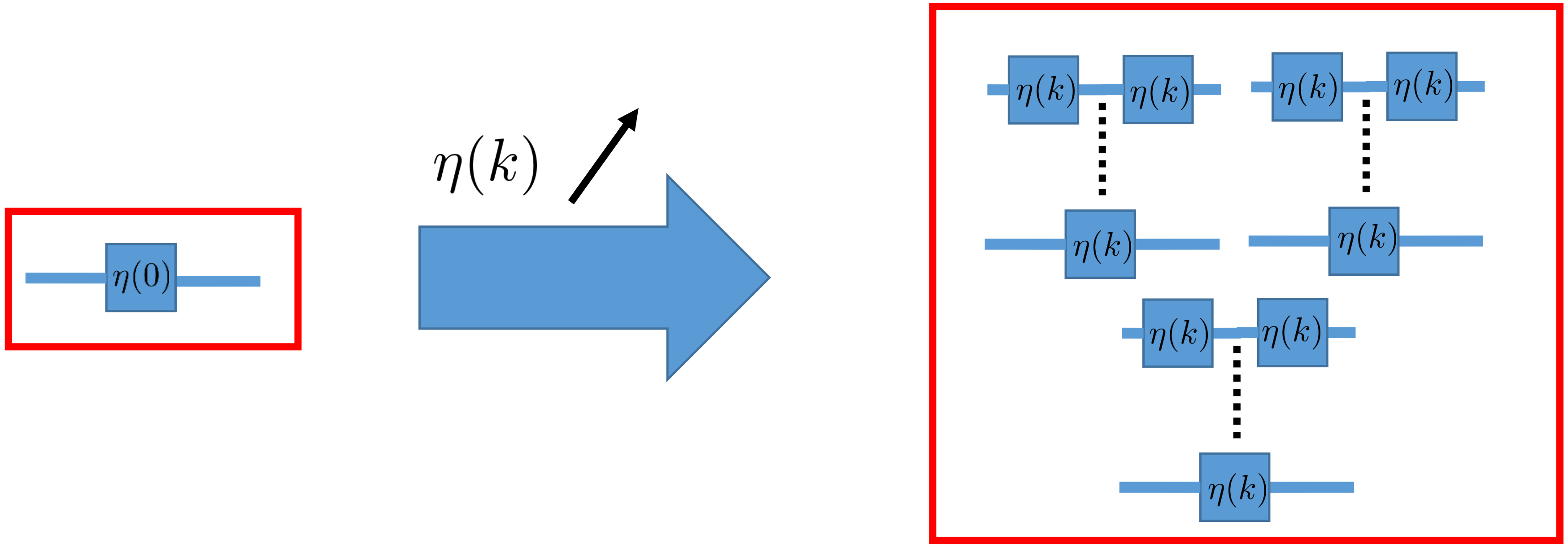
- $k_{\max}$
- $p(k_{\max})$

To estimate those for instance

Maximum accuracy reachable

Maximum concatenation level before situation gets worse (i.e logical error increases)

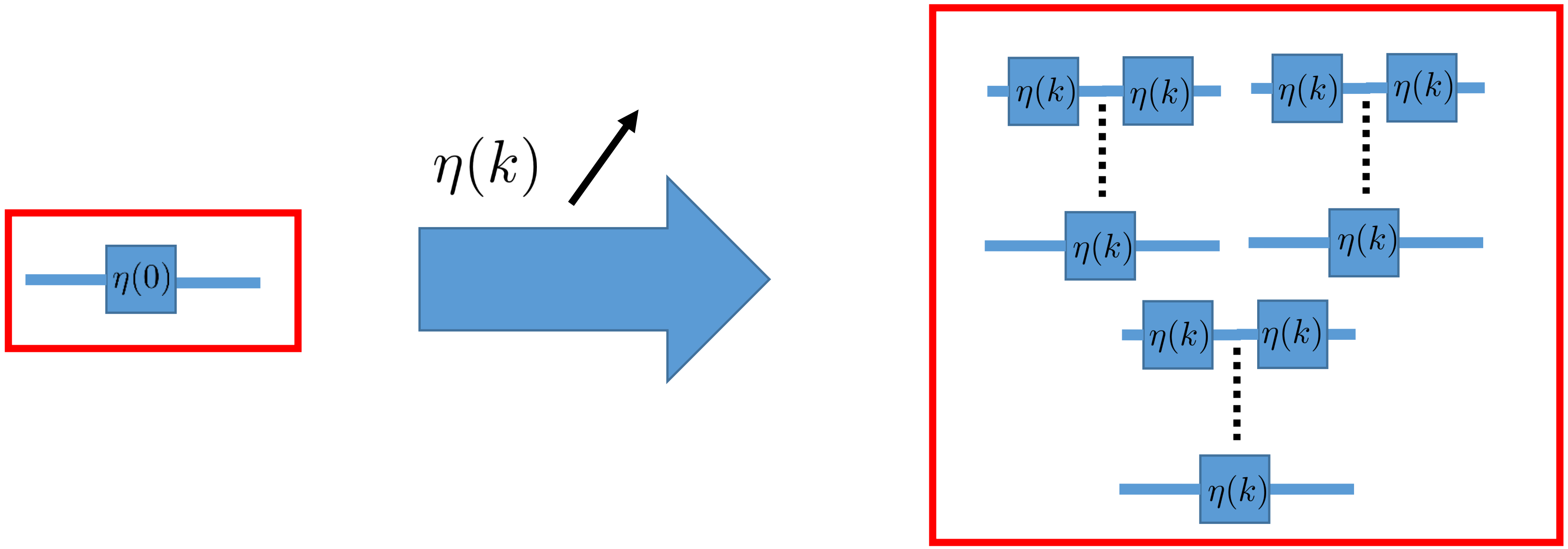
Noise growing with number of physical elements



The messages here:

- FTQC is **very sensitive** to how physical noise growth with computer size (because of fast growth of physical resources required)

Noise growing with number of physical elements



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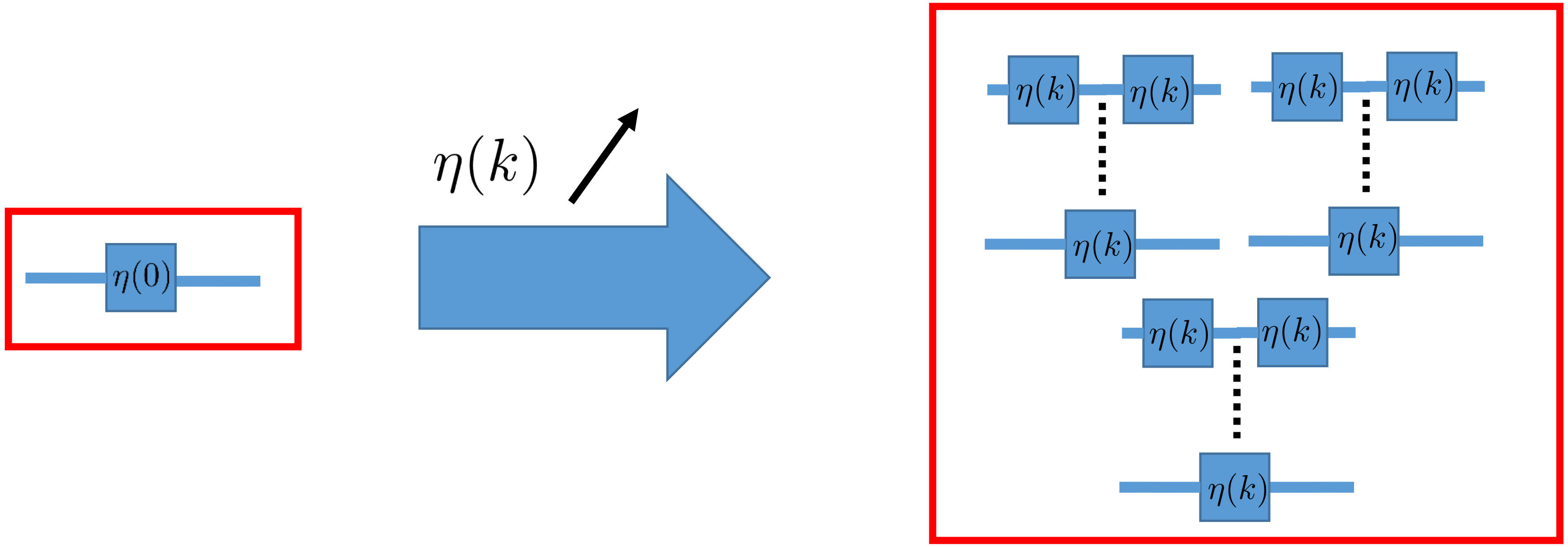
- FTQC is **very sensitive** to how physical noise growth with computer size (because of fast growth of physical resources required)
- For quantum computer designs: **accessing scaling law** seems an important requirement
- Scaling law provides **what are the limitations of a given technology**

Noise growing with number of physical elements



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- FTQC is **very sensitive** to how physical noise growth with computer size (because of fast growth of physical resources required)
- For quantum computer designs: **accessing scaling law** seems an important requirement
- Scaling law provides **what are the limitations of a given technology**
- We study those questions on simple but quantitative examples

Resource estimation

Resource sharing

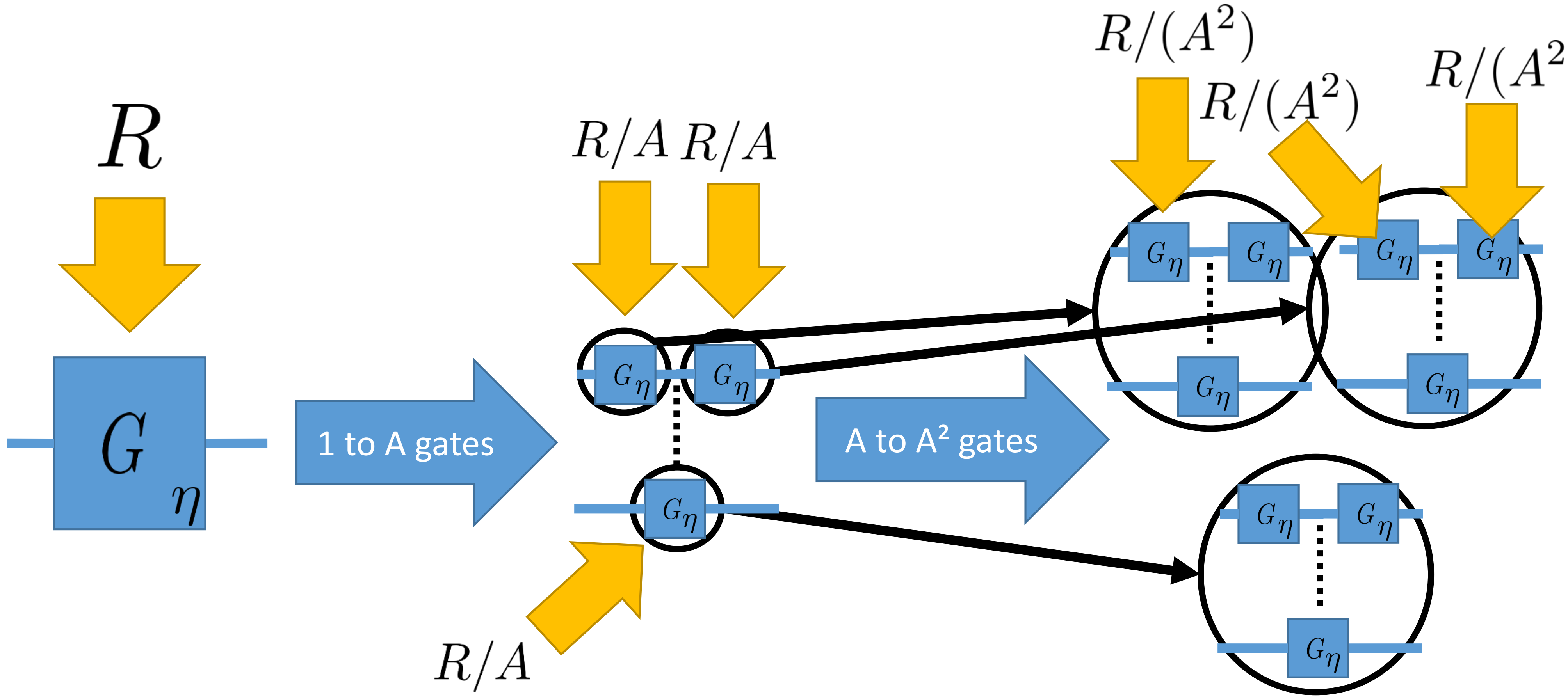
Let's assume:

Noise per physical gate depends on a resource: $$\eta = \alpha R^{-\beta}$$

Total amount of resources: R assumed to be **fixed**.

Resource sharing

Resource is then shared among physical gates



Resource sharing

Resource available per physical gate: $\dfrac{R}{A^k}$

$$\eta = \alpha R^{-\beta}$$

For one physical gate

I share the resource

$$R \to \frac{R}{A^k}$$

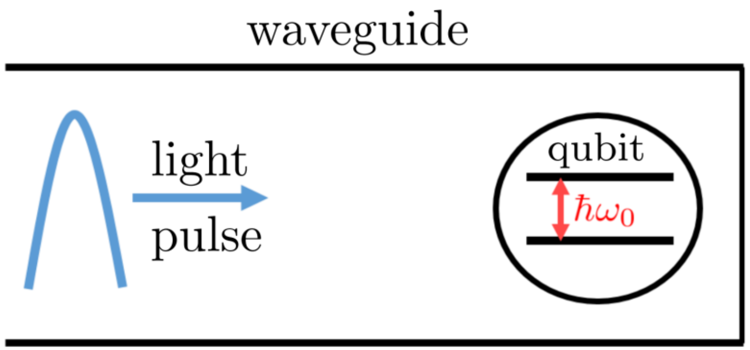
$$\boxed{\begin{aligned} \eta(k) &= \eta^0 A^{\beta k} \\ \eta^0 &\equiv \alpha R^{-\beta} \end{aligned}}$$

Limited resource => noise growth

The connection with previous explanations

Physical example

- Light-matter interaction



For simple example: we assume all physical gate are as noisy as π pulses

$$\dot{\rho} = -\frac{i}{\hbar}[H(t), \rho] + \mathcal{D}(\rho)$$

$$H(t) = -\frac{\hbar\omega_0}{2}\sigma_z + \frac{\hbar\Omega h(t)}{2}\left(|0\rangle\langle 1|e^{i\omega_0 t} + |1\rangle\langle 0|e^{-i\omega_0 t}\right)$$

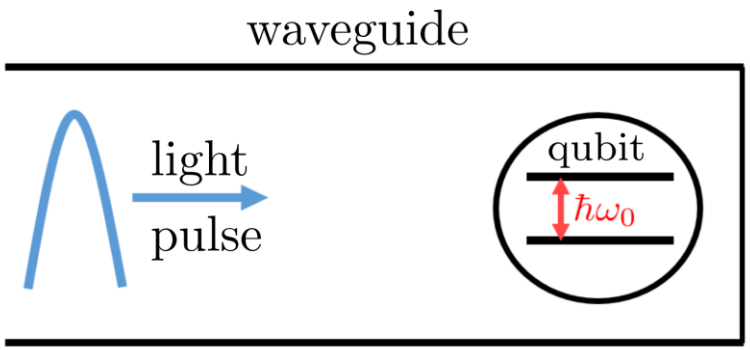
(Rabi oscillation)

$$\mathcal{D}(\rho) = \gamma(\sigma_-\rho\sigma_+ - \frac{1}{2}\{\rho, \sigma_+\sigma_-\})$$

(spontaneous emission)

Physical example

- Light-matter interaction



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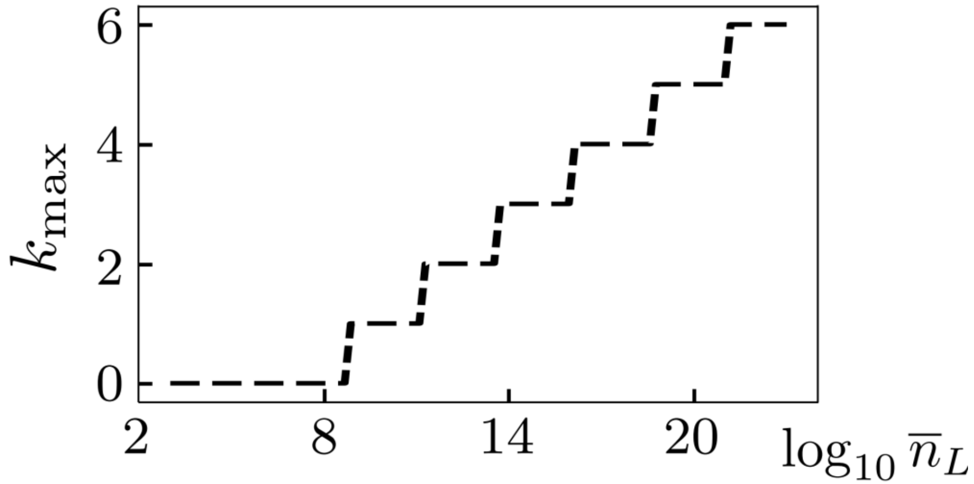
$$\mathcal{D}(\rho) = \gamma(\sigma_-\rho\sigma_+ - \frac{1}{2}\{\rho, \sigma_+\sigma_-\})$$

$$\eta(k) = \frac{\pi^2}{16\bar{n}_L}A^k$$

Number of photons for the logical gate

Physical example



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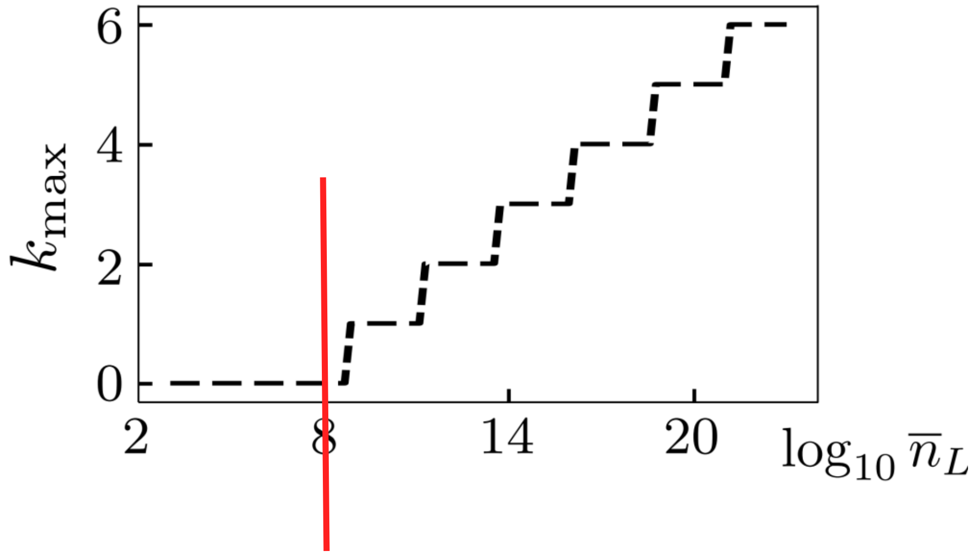
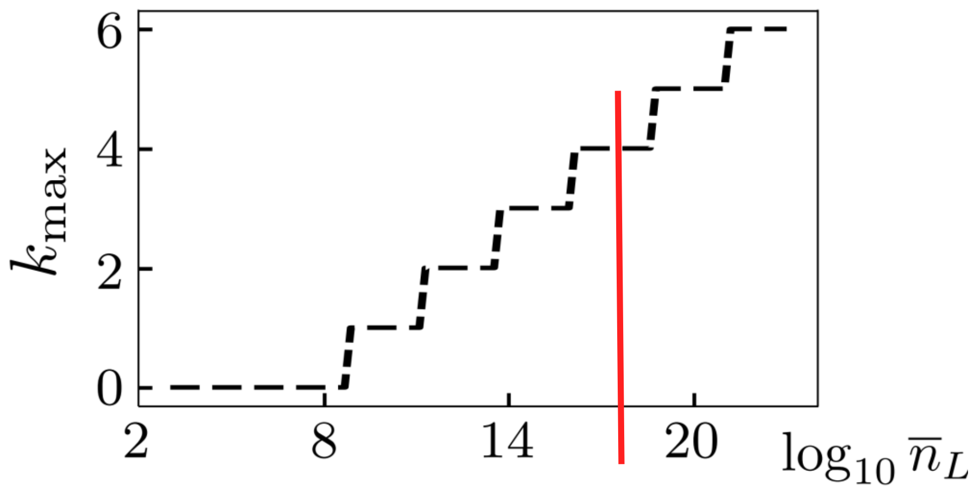


10^8 photons: I simply don't have enough resources to do error correction at all

$$\eta(k) = \frac{\pi^2}{16\overline{n}_L} A^k$$

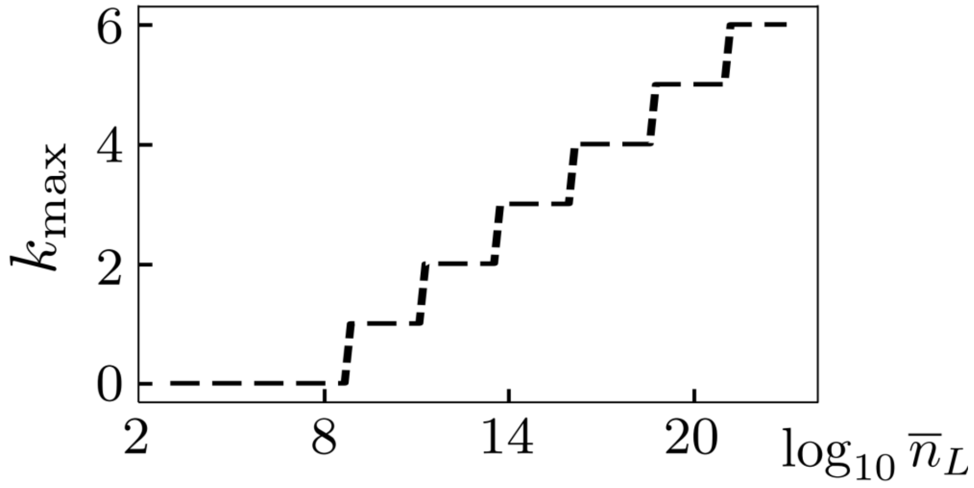
Physical example



10^18 photons: I can concatenate four times maximum

$$\eta(k) = \frac{\pi^2}{16\overline{n}_L} A^k$$

Physical example



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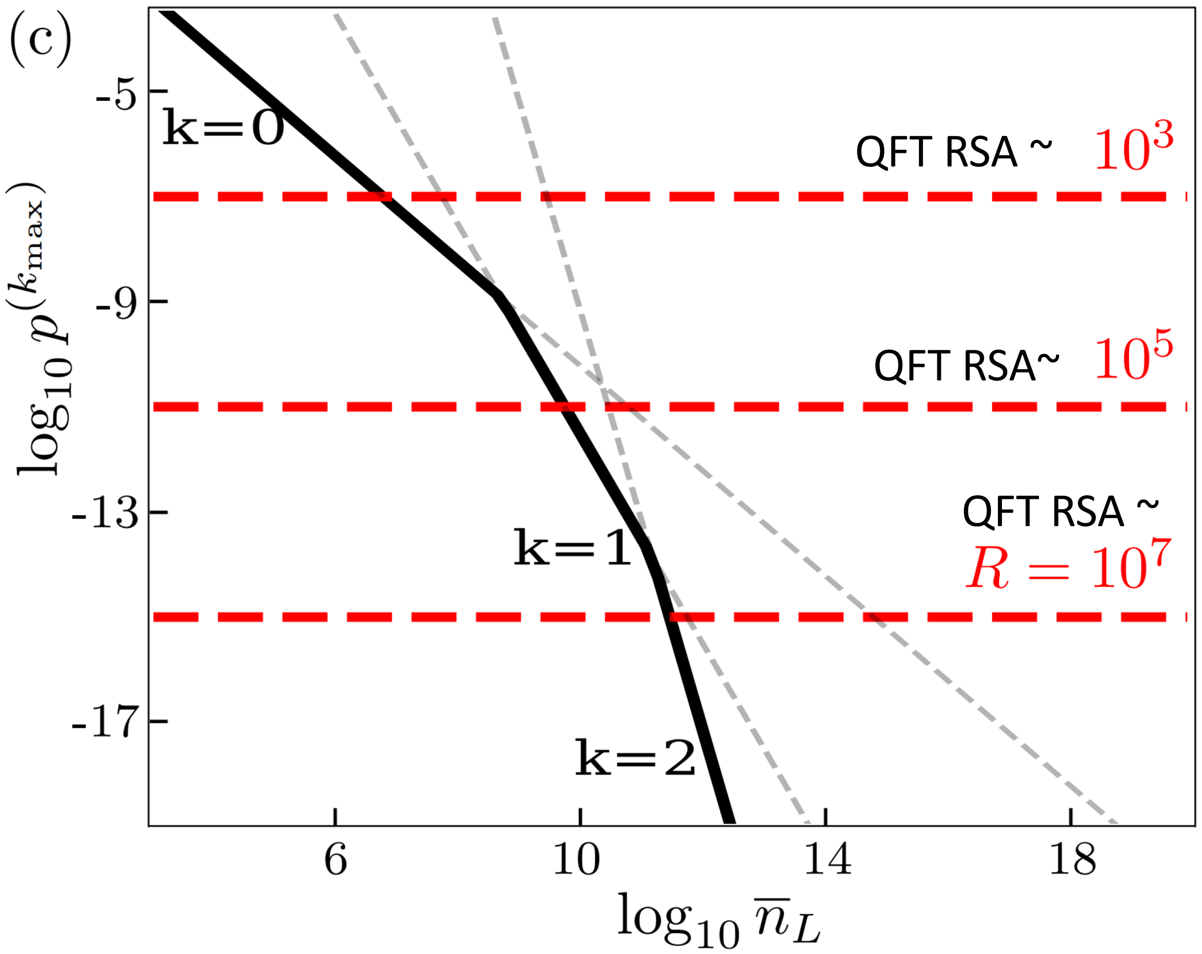

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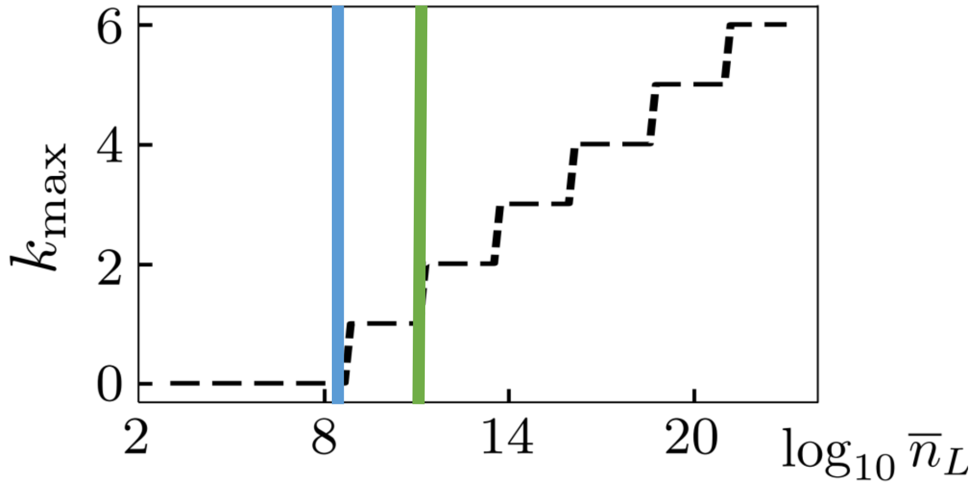

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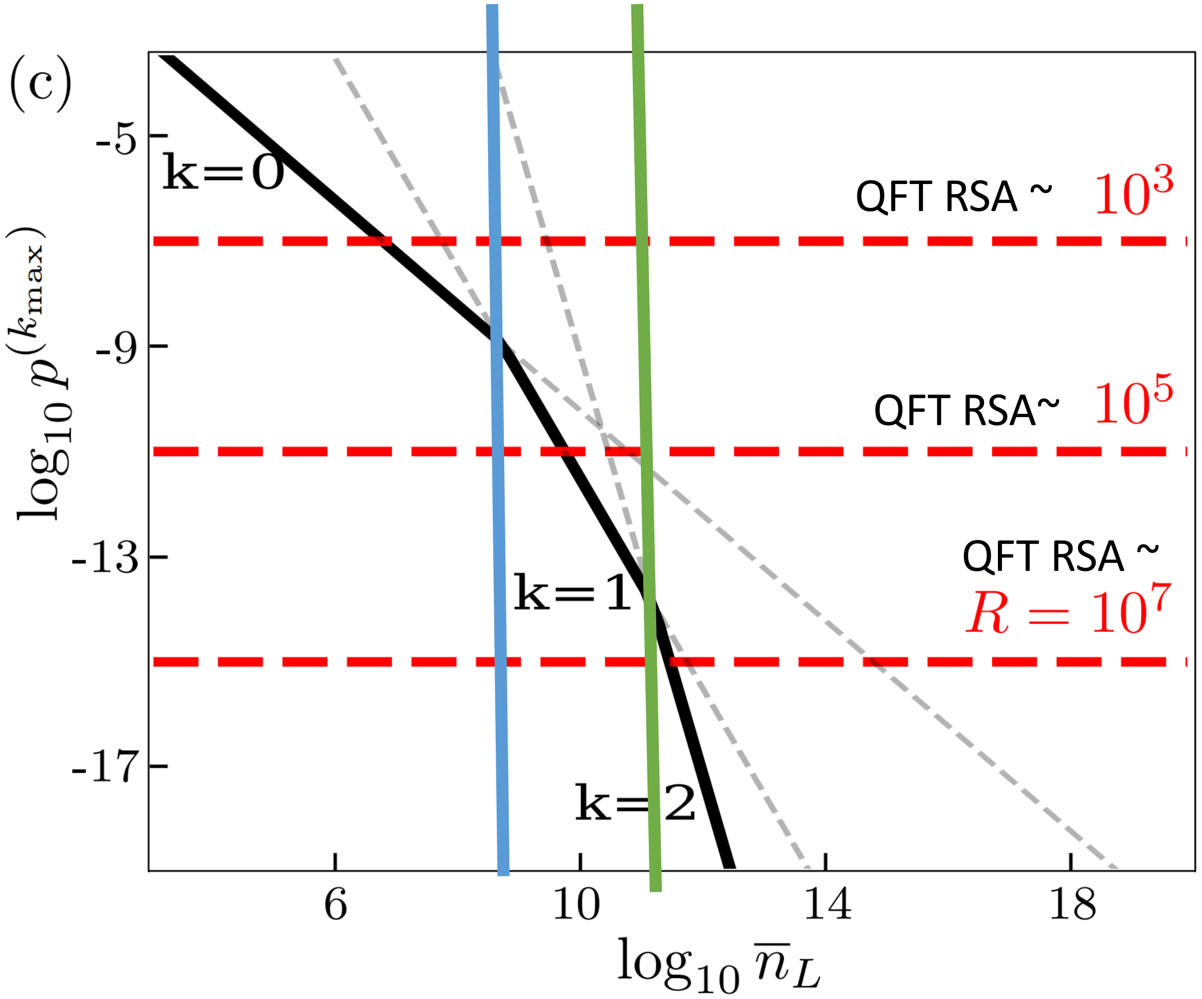

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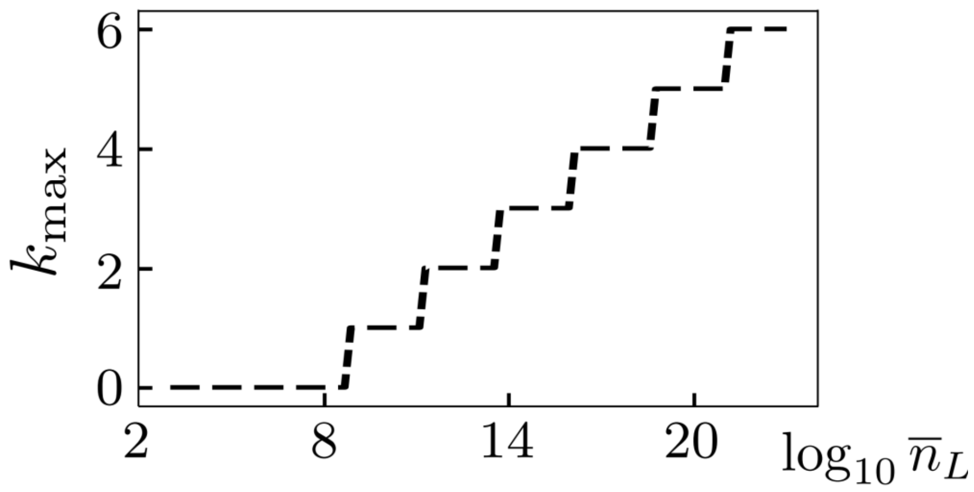
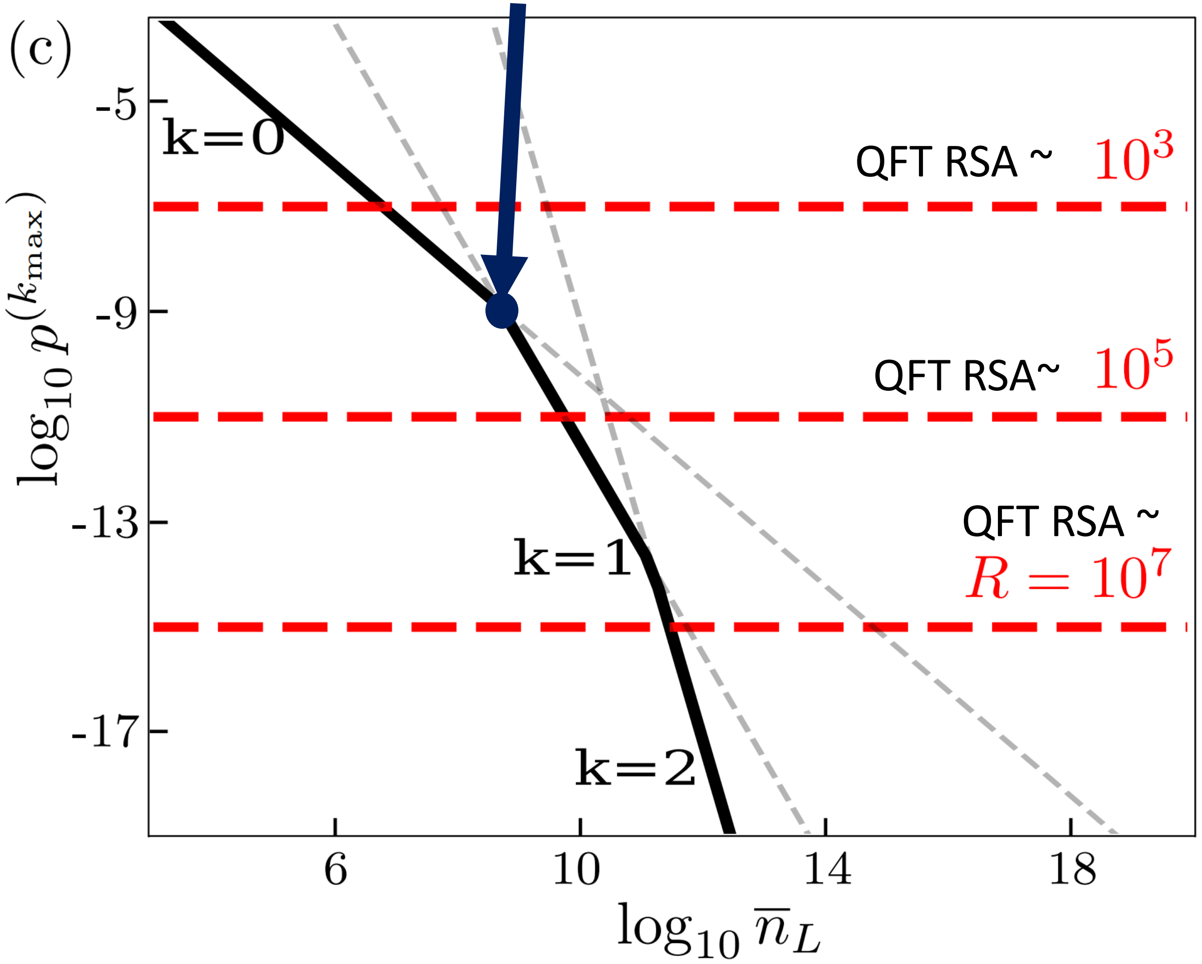


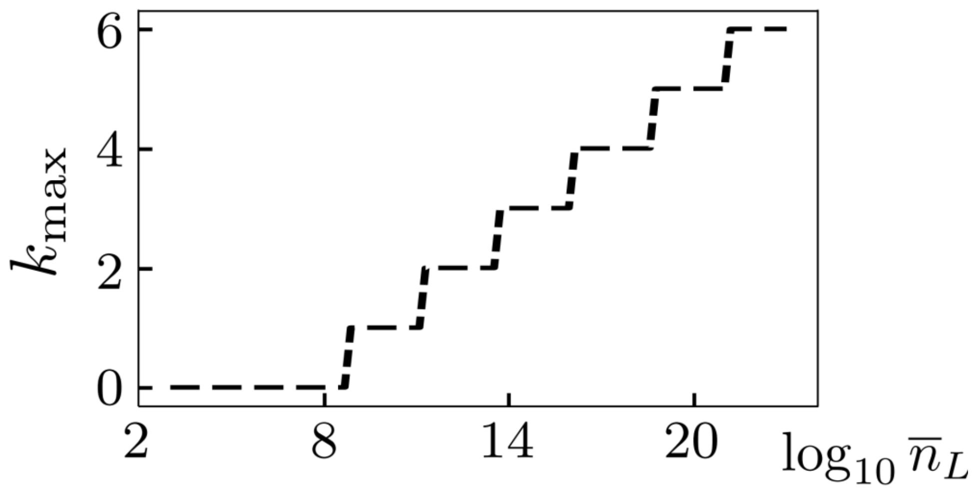
$$\eta(k) = \frac{\pi^2}{16\overline{n}_L} A^k$$

It starts to be interesting to concatenate
(Q.E.C starts to **win** competition)



If I don’t want to concatenate.
I could still reduce the noise… but slower !

Physical example



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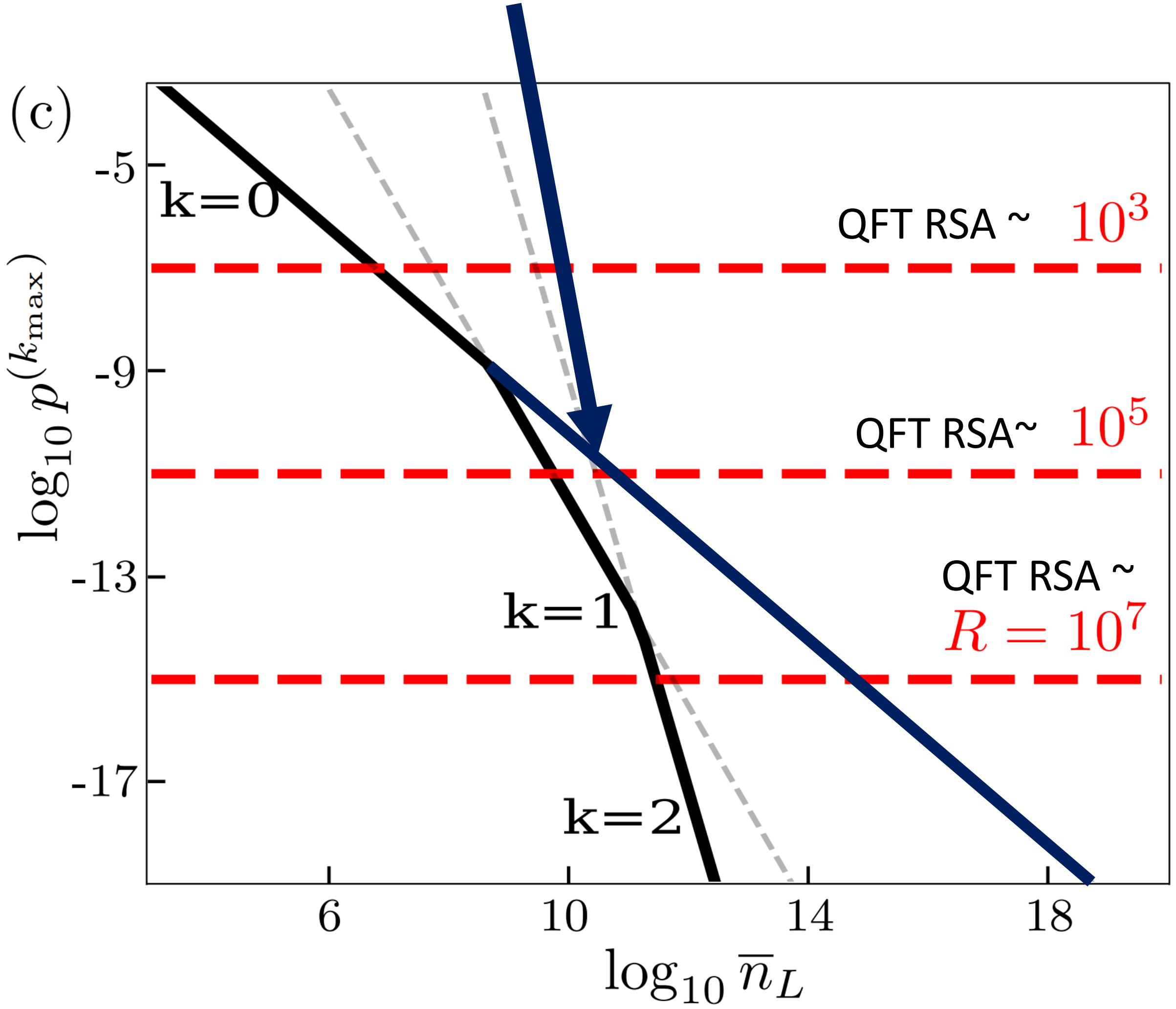

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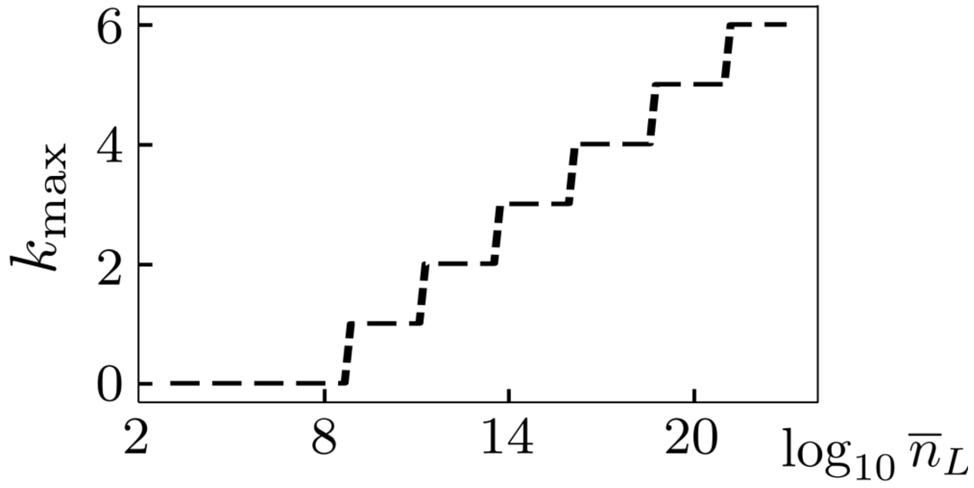
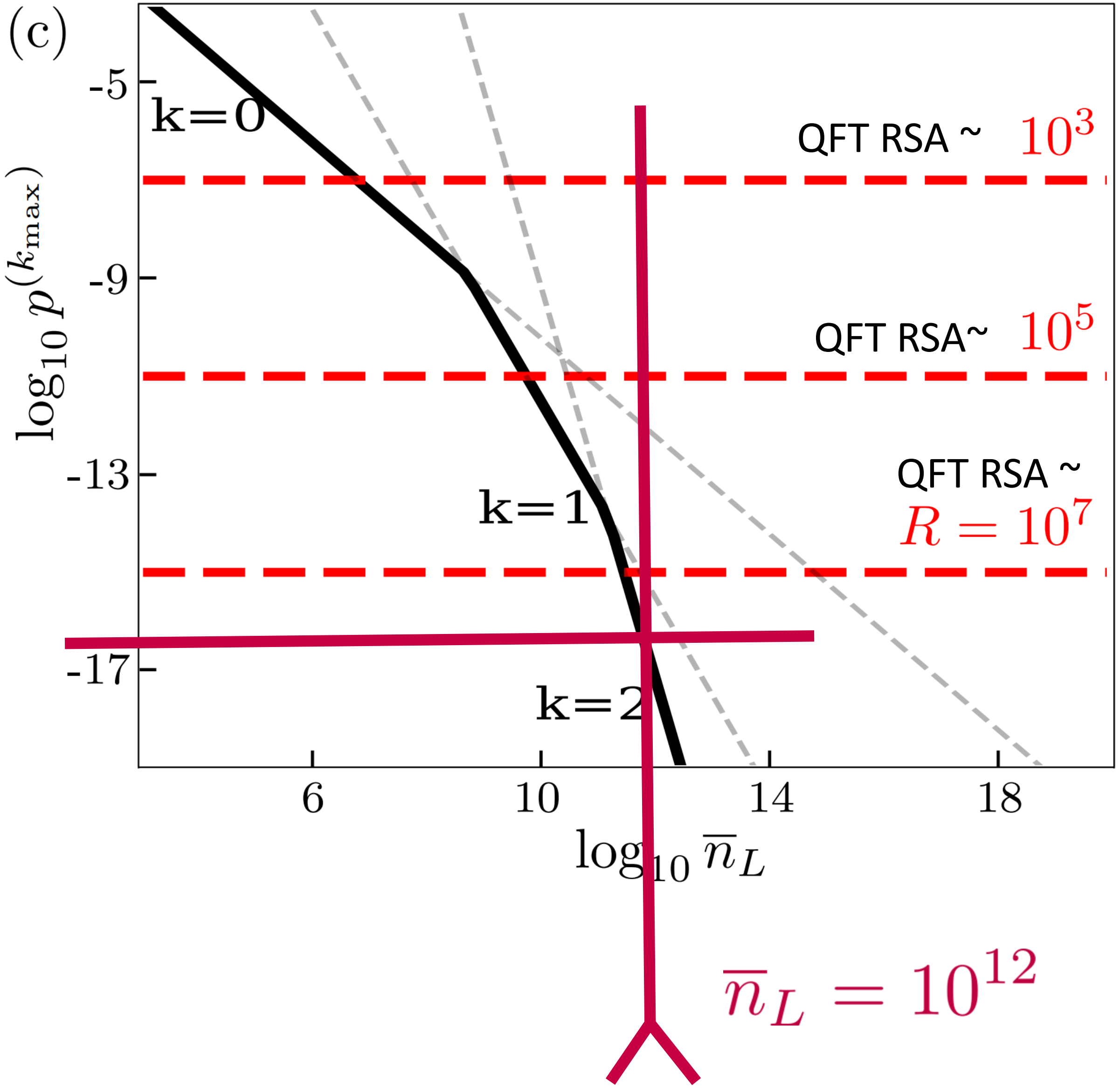


$p(k_{\max}) \approx 10^{-16}$



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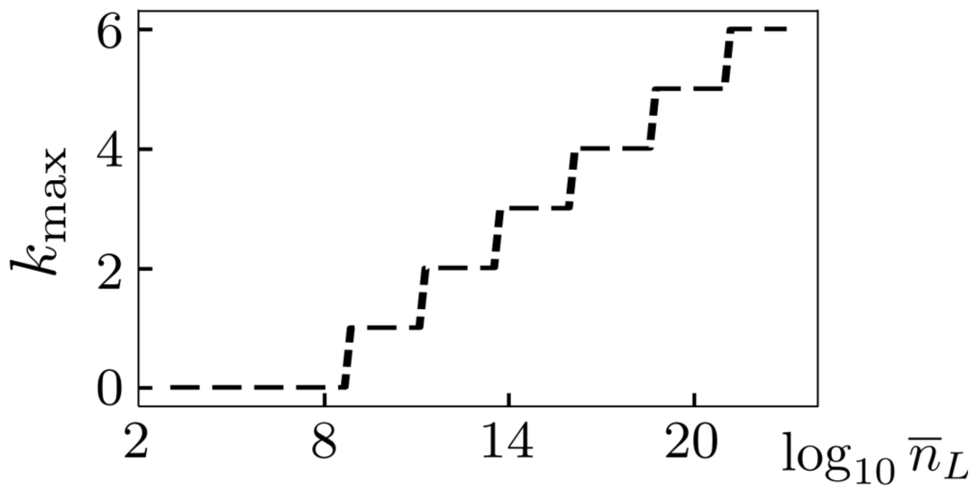
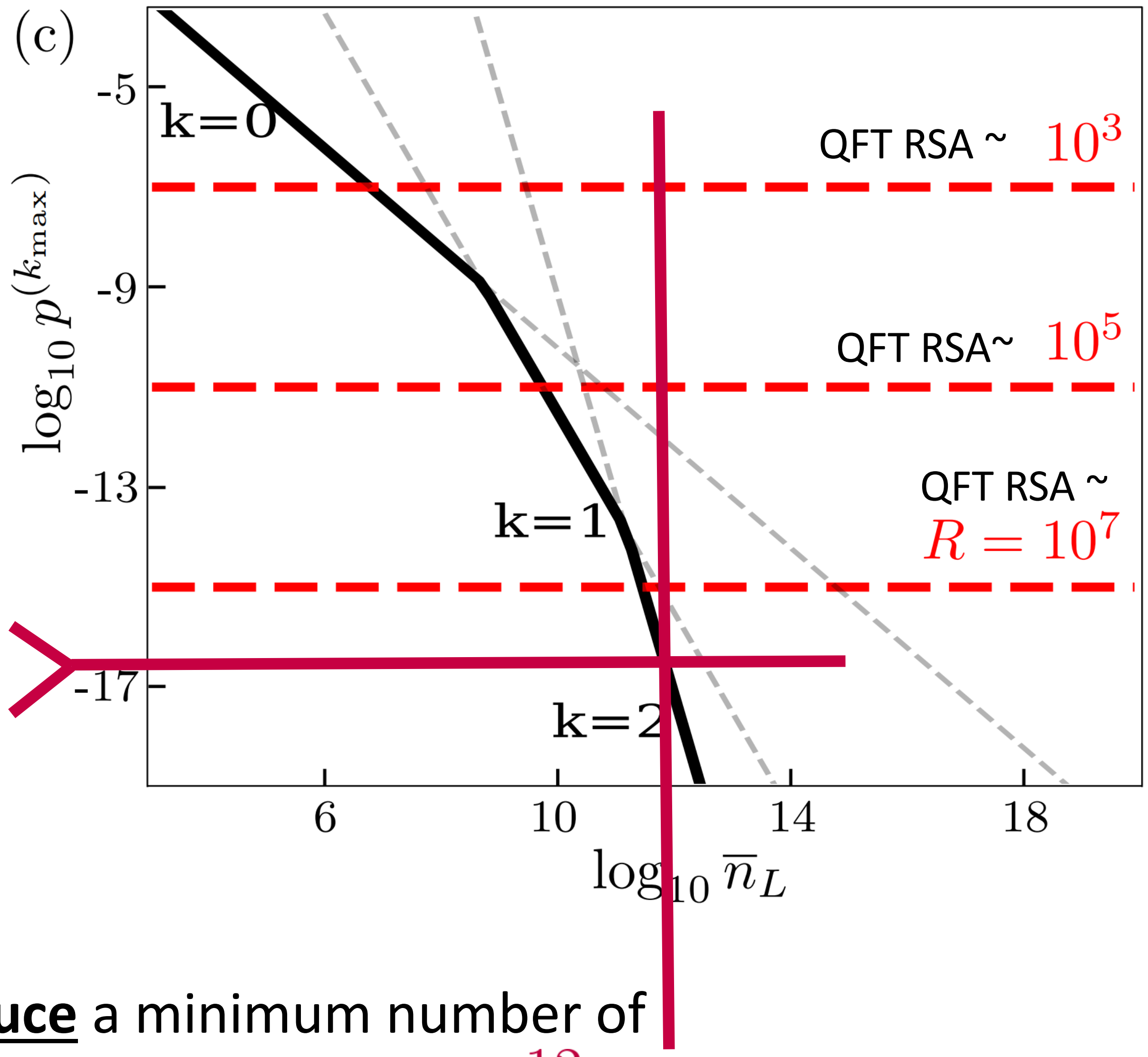


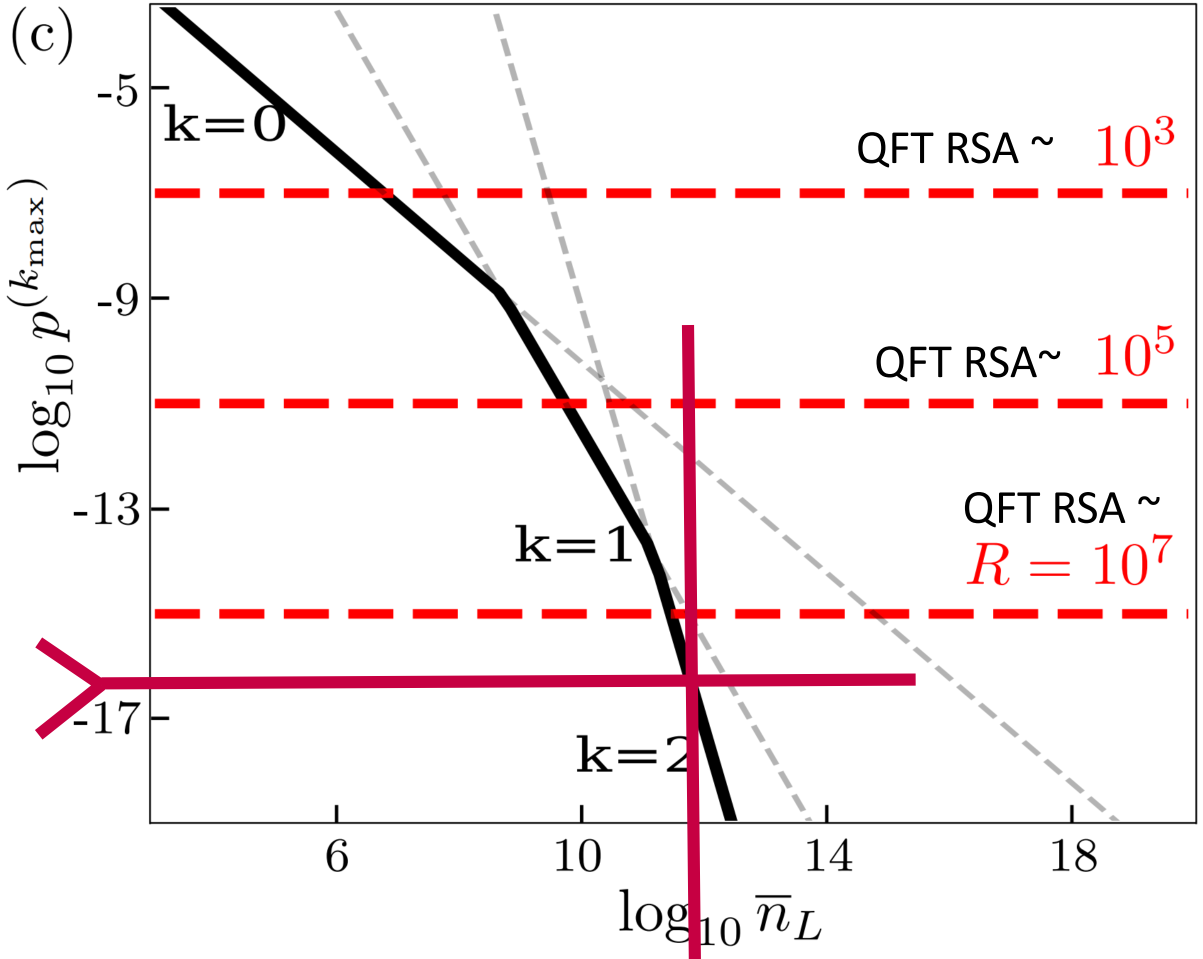


I **want** 10^{-16} of probability of failure for logical gate

I **deduce** a minimum number of photons being $\overline{n}_L = 10^{12}$

Physical example

I **want** 10^{-16} of probability of failure for logical gate



In principle:

We can estimate the minimum resources required to perform a targetted calculation

I **deduce** a minimum number of photons being $\overline{n}_L = 10^{12}$

Conclusions

- Resource cost estimation
  - Sharing resource => scale dependant noise => we can use it for resource estimation
  - Provides a good approach to study problems of energetic cost of large scale quantum computing

- F.T.Q.C with scale dependant noise
  - Noise growth must be **very** weak with computer size
  - **Important to understand how noise growth with computer size**
  - We provide tools to estimate the maximum accuracy one can get for a given noise growth