Impact of Non-Markovian Dynamics on the Foundations of Statistical Mechanics

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arXiv: 1311.1282 arXiv: 1311.5409

GDR: Quantum Information - Foundations & Applications

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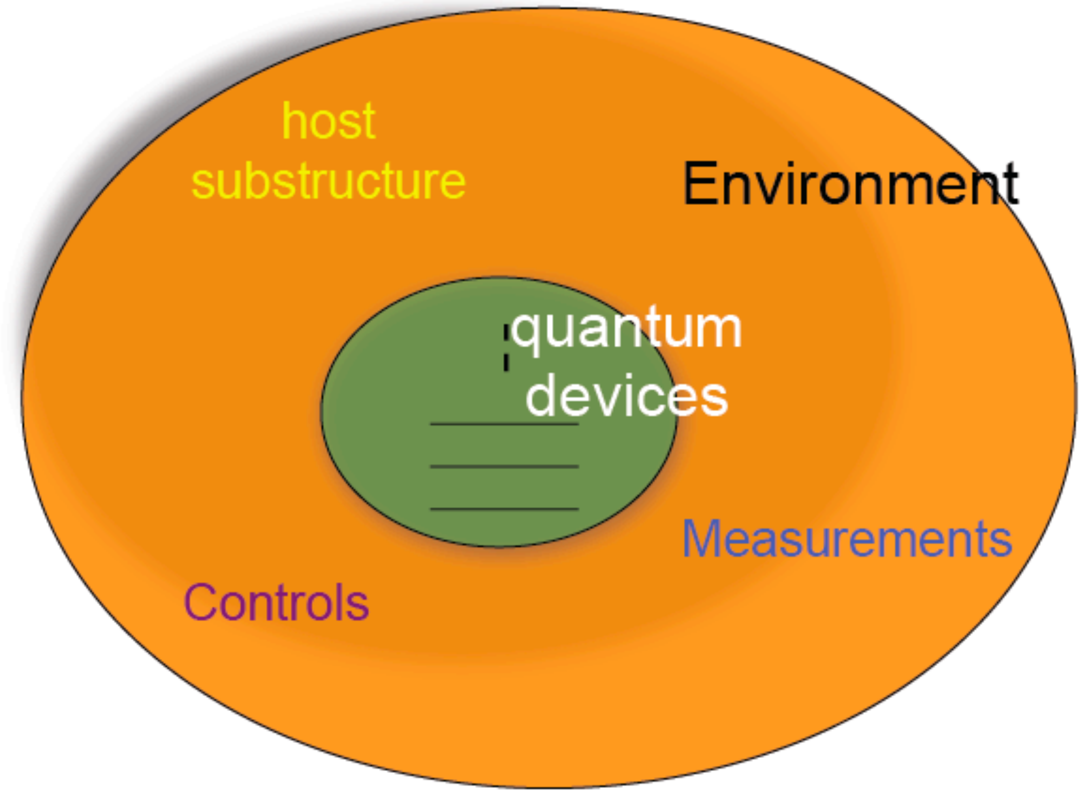
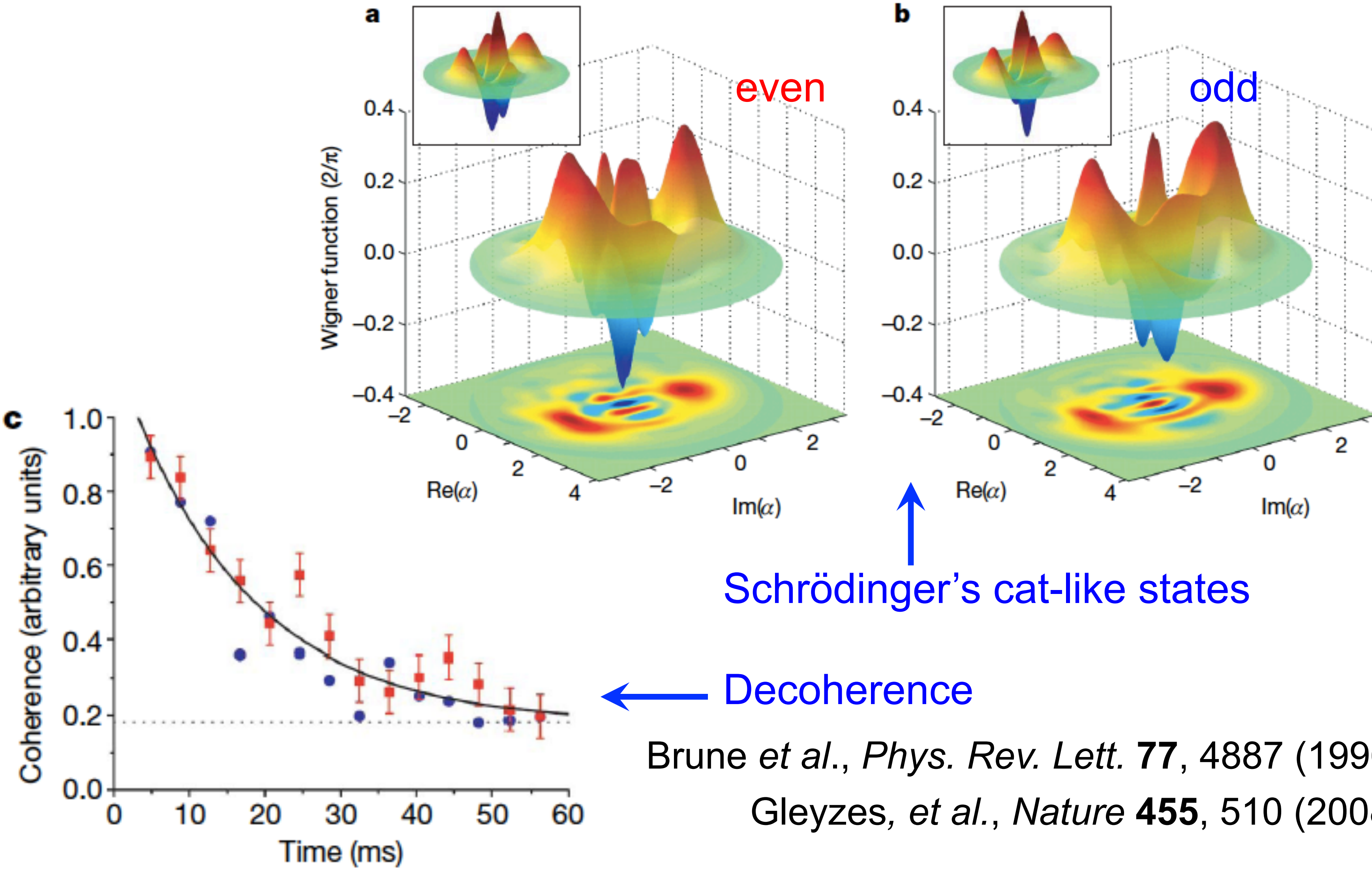
All realistic quantum devices for QIP



→ open quantum systems

→ decoherence

Decoherence of Schrödinger's cat-like state:



Brune *et al.*, *Phys. Rev. Lett.* **77**, 4887 (1996)

Gleyzes, *et al.*, *Nature* **455**, 510 (2008)

Investigation of open quantum systems in last two decades made profound impacts on many aspects:

- Quantum Information & Quantum Computing
- Nanoscience and Nanotechnology
- Mesoscopic physics, quark-gluon plasmas, quantum gravity, quantum biology, ……
- Foundations of Quantum Mechanics
- Foundations of Statistical Mechanics
- ……

Investigation of open quantum systems in last two decades made profound impacts on many aspects:

- Quantum Information & Quantum Computing
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- Foundations of Quantum Mechanics
- Foundations of Statistical Mechanics
- ……

- Statistical Mechanics was established on the equilibrium hypothesis:
  - over a sufficiently long time, a given system can always reach thermal equilibrium with its environment, and the statistical distribution does not depend on its initial state

L. D. Landau & E. M. Lifshitz, *Statistical Mechanics* (1969), p5

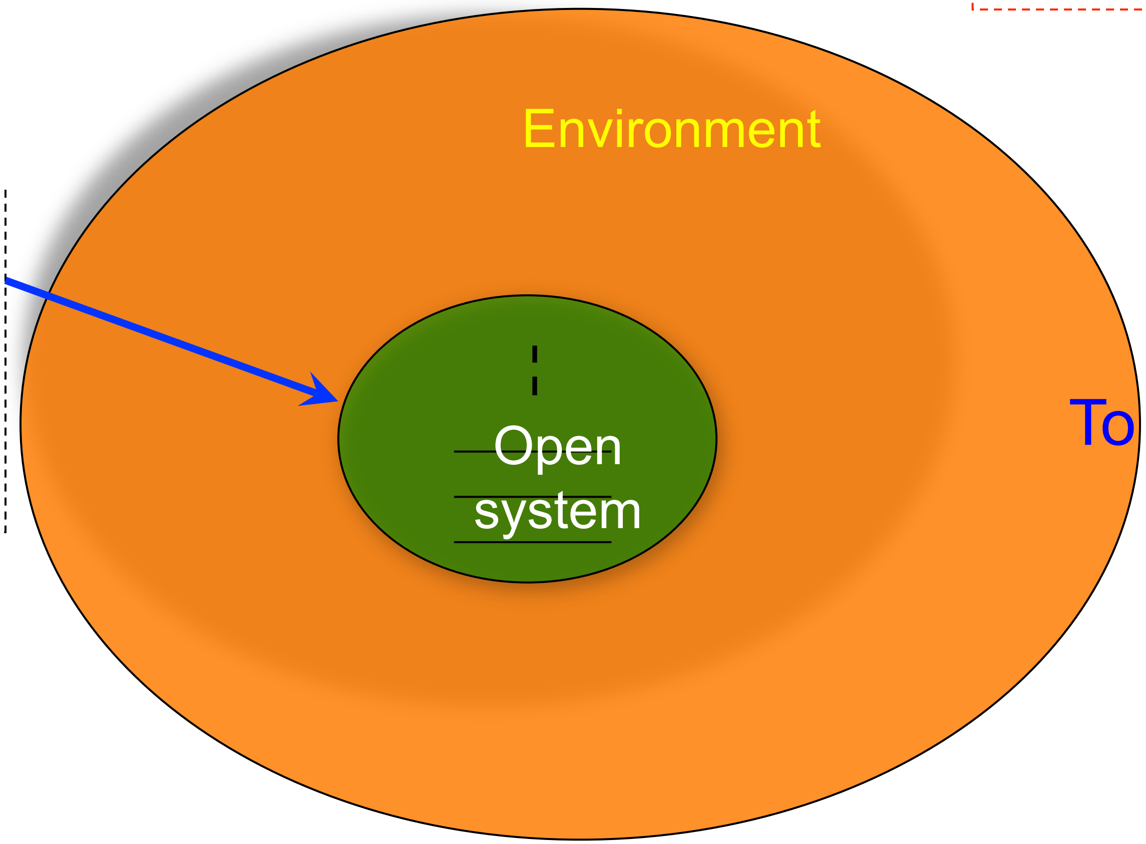
- It is a time-honored problem (since Ludwig Boltzmann 1844~1906), investigating the foundations of statistical mechanics has been focused on questions:
  - how does macroscopic irreversibility emerge from microscopic reversibility?
  - how does the system relax to thermal equilibrium with its environment?

K. Huang, *Statistical Mechanics* (1987), p189

Environment: a reservoir or a thermal bath,

- contains infinite number of degrees of freedom
- initially in a thermal equilibrium state: $\rho_E(t_0) = \dfrac{1}{Z} e^{-\beta H_E}$

Exchange of particles, energies and information between S-E



To find $\rho_S(t_s) = \mathrm{Tr}_E[\rho_{\mathrm{tot}}(t_s)]$

System:

- contains a few degrees of freedom
- initially in an arbitrary quantum state: $|\psi(t_0)\rangle = \sum_n c_c |n\rangle$

System-Environment Couplings:

- The exact master equation is derived if the system-environment couples via particle-particle exchanges:

$$H_{SB} = \sum_{\alpha k i} [V_{\alpha k i} a_i^\dagger b_{\alpha k} + V_{\alpha k i}^* b_{\alpha k}^\dagger a_i]$$

- the system contains arbitrary *N* energy levels

- environment can contain many different reservoirs

- each reservoir is specified by the spectral density:

$$\boldsymbol{J}_{\alpha i j}(\omega) = 2\pi \sum_{k} V_{\alpha i k} V_{\alpha j k}^* \delta(\omega - \epsilon_k)$$

Dynamics of open quantum systems: determined by master equation:

- Formally it can be written as (Nakajima-Zwanzig master equation)

$$\frac{d\rho(t)}{dt} = \int_{t_o}^{t} d\tau \underline{\mathcal{K}(t-\tau)} \rho(\tau)$$

non-Markovian dynamics due to memory effect

- Practically one is unable to derive such an equation explicitly and exactly for an arbitrary quantum system.

S. Nakajima, *Prog. Theo. Phys*. **20**, 948 (1958)
R. Zwanzig, *J. Chem. Phys*. **33**, 1338 (1960)

Overview on master equation:

- 1928, Pauli proposed phenomenologically the first master equation.
- Since then, attempts were made with various approaches ……
- Except for the Quantum Brownian Motion (QBM), there is still the lack of a satisfactory answer.

- W. Pauli, *Festschrift zum 60. Geburtstage A. Sommerfelds* (Hirzel, Leipzig, 1928)
- J. Schwinger, *J. Math. Phys*. **2**, 407 (1961).
- R. P. Feynman, F. L. Vernon, *Ann. Phys. (N.Y.)* **24**, 118 (1963).
- A. O. Caldeira, A. J. Leggett, *Ann. Phys. (N.Y.)*, **149**, 374 (1983)
- B. L. Hu, J. P. Paz, Y. H. Zhang, *Phys. Rev. D* **45**, 2843 (1992)
- ………….

➢ For over a century, not having such a rigorous master equation remains as the primary obstacle to understand the foundations of statistical mechanics !

K. Huang, *Statistical Mechanics* (1987)

Recent developed exact Master Equation:

$$\dot{\rho}(t) = -i\big[H'_S(t), \rho(t)\big]$$
$$+ \sum_{ij} \Big\{\gamma_{ij}(t)\left[2a_j\rho(t)a_i^\dagger - a_i^\dagger a_j\rho(t) - \rho(t)a_i^\dagger a_j\right]$$
$$+ \widetilde{\gamma}_{ij}(t)\big[a_i^\dagger\rho(t)a_j + a_j\rho(t)a_i^\dagger - a_j^\dagger a_i\rho(t) - \rho(t)a_j a_i^\dagger\big]\Big\}$$

where

$$\boldsymbol{\omega}'_S(t) = \frac{i}{2}\left[\dot{\boldsymbol{u}}(t,t_0)\,\boldsymbol{u}^{-1}(t,t_0) - \mathrm{H.c.}\right],$$
$$\boldsymbol{\gamma}(t) = -\frac{1}{2}\left[\dot{\boldsymbol{u}}(t,t_0)\,\boldsymbol{u}^{-1}(t,t_0) + \mathrm{H.c.}\right],$$
$$\widetilde{\boldsymbol{\gamma}}(t) = \dot{\boldsymbol{v}}(t,t) - \left[\dot{\boldsymbol{u}}(t,t_0)\,\boldsymbol{u}^{-1}(t,t_0)\,\boldsymbol{v}(t,t) + \mathrm{H.c.}\right]$$

Tu and WMZ, *Phys. Rev. B* **78**, 235311 (2008)
Jin, Tu, WMZ & Yan, *New J. Phys*. **12**, 083013 (2010)
Lei and WMZ, *Ann. Phys. (N.Y.)* **327,** 1408 (2012)
WMZ, Lo, Xiong & Nori, *Phys. Rev. Lett.***109**, 170402 (2012)

Nonequilibrium Green functions:

1. $a(t) = u(t, t_0)a(t_0) + \xi(t)$
2. $v(t', t) = \langle \xi^\dagger(t)\xi(t') \rangle$

with

spectral density

$$\frac{d}{dt}u(t, t_0) + i\varepsilon u(t, t_0) = \int_{t_0}^{t} \int d\tau \frac{d\omega}{2\pi} \underline{J(\omega)} e^{i\omega(t-\tau)} u(\tau, t_0)$$

$$v(t', t) = \int_{t_0}^{t'} d\tau \int_{t_0}^{t} d\tau' \int \frac{d\omega}{2\pi} \underline{J(\omega)}\, \underline{\overline{n}(\omega, T)} e^{i\omega(\tau-\tau')} u(t', \tau') u^\dagger(t, \tau)$$

initial thermal particle distribution of the reservoir

- nonequilibrium single-particle Green functions are fully determined by the spectral density $J(\omega)$. WMZ, Lo, Xiong & Nori, *Phys. Rev. Lett.* **109**, 170402 (2012)

  - which coincides with the conclusion reached by A. L. Leggett *et al.* that "*for any problem in which a thermal equilibrium statistical average is taken over the initial states of the environment and a sum over the final states, complete information about the effect of the environment is encapsulated in the single spectral density*"

A. J. Leggett, *et al.*, *Rev. Mod. Phys.* **59**, 1 (1987)

Universality of non-Markovian dynamics

- very different open systems with very different environmental spectral densities have the same structure of the non-Markovian solution:

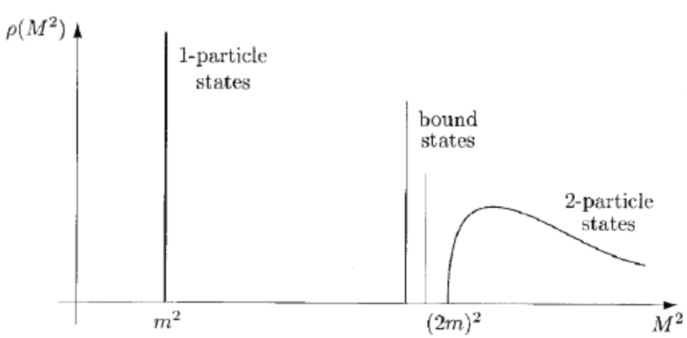
$$u(t-t_0) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mathcal{D}(\omega) \exp\{-i\omega(t-t_0)\}$$

where the environmental-modified spectrum:

$$\mathcal{D}(\omega) = \underbrace{\sum_j \mathcal{Z}_j \delta(\omega - \omega'_j)}_{\text{localized modes}} + \underbrace{\frac{J(\omega)}{[\omega - \varepsilon_s - \Delta(\omega)]^2 + J^2(\omega)/4}}_{\text{continuous spectrum part}}$$

$\varepsilon_s \longrightarrow$ localized modes

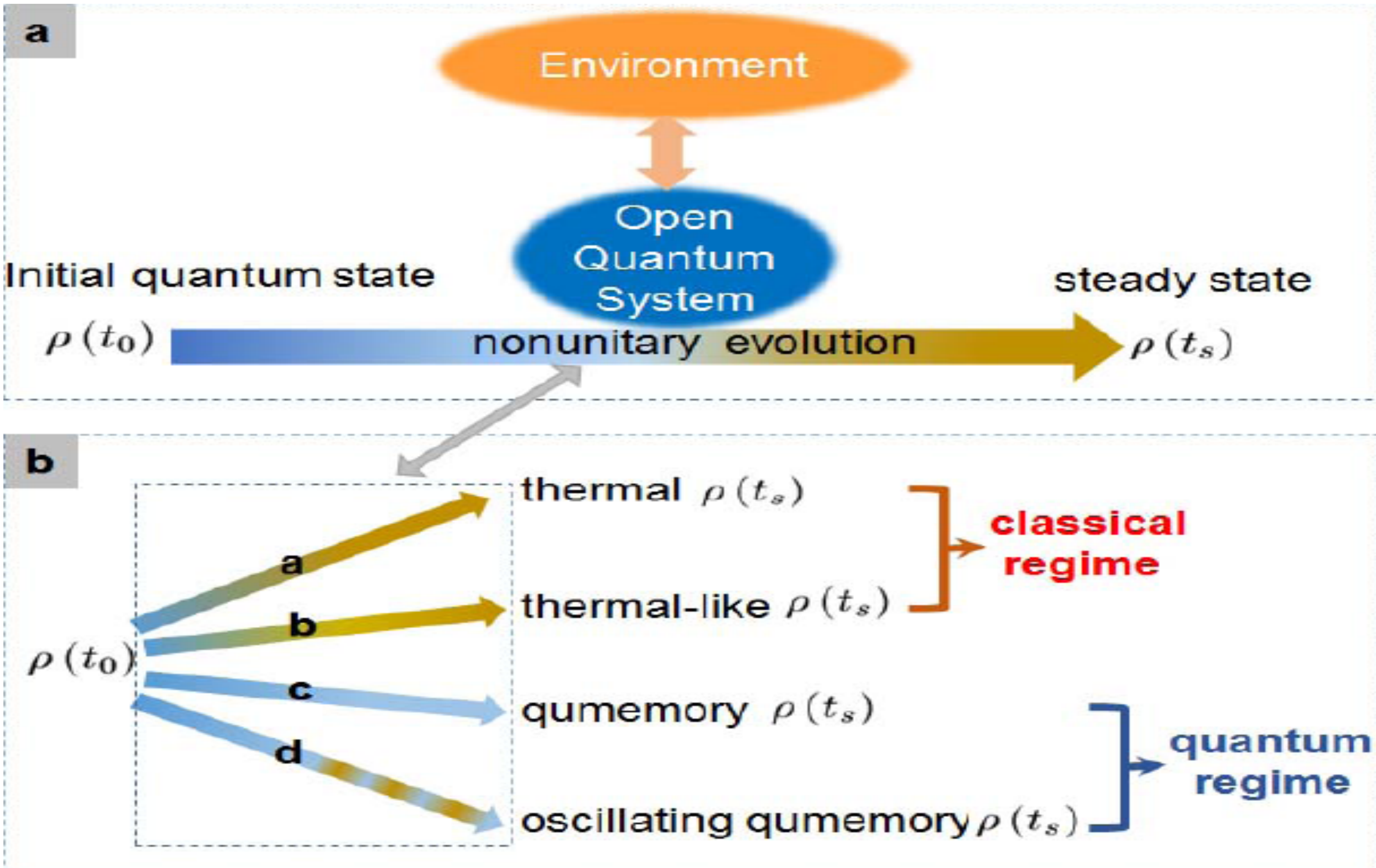
continuous spectrum part



can be directly measured in experiments !!

WMZ, Lo, Xiong & Nori, *Phys. Rev. Lett.* **109**, 170402 (2012)

- Solving the exact master equation with an arbitrary initial state, we found four decoherence scenarios:



arXiv: 1311.1282 (2013)

- Consider a boson system (similar solution can be found for fermion systems too), with a general initial state:

$$\boxed{\rho(t_0) = \sum_{m,n=0}^{\infty} c_m^* c_n |m\rangle\langle n|}$$

$$\boxed{|\psi(t_0)\rangle = \sum_n c_c |n\rangle}$$

- Solving the exact master equation, we have

$$\boxed{\rho\left(t\right) = \sum_{m,n=0}^{\infty} c_m^* c_n \sum_{k=0}^{\min\{m,n\}} d_k \mathcal{A}_{mk}^{\dagger}\left(t\right) \widetilde{\rho}\left(v\left(t,t\right)\right) \mathcal{A}_{nk}\left(t\right)}$$

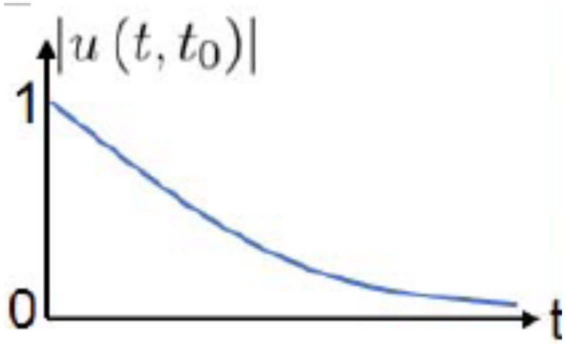
where

$$\widetilde{\rho}(v(t,t)) = \sum_{n=0}^{\infty} \frac{[v(t,t)]^n}{[1+v(t,t)]^{n+1}} |n\rangle\langle n|$$

$$\mathcal{A}_{mk}^{\dagger}(t) = \frac{\sqrt{m!}}{(m-k)!\sqrt{k!}} \left[\frac{u(t,t_0)}{1+v(t,t)} a^{\dagger}\right]^{m-k}, \quad d_k = \left[1 - \frac{|u(t,t_0)|^2}{1+v(t,t)}\right]^k$$

arXiv: 1311.1282 (2013)

- For a widely distributed spectral density (memory-less or Markov limit):



$$u(t_s, t_0) \to 0$$

$$v(t_s, t_s) \to n(\omega_S, T)$$

- The steady state of the system:

$$\boxed{\rho(t_0) \to \rho(t_s) = \sum_{n=0}^{\infty} \frac{[n(\omega_S, T)]^n}{[1 + n(\omega_S, T)]^{n+1}} |n\rangle\langle n|}$$

thermal state

– a rigorous proof how the system relaxed to thermal equilibrium with its environment !!

arXiv: 1311.1282 (2013)

- For other spectral densities, as long as the system-environment coupling is not too strong:

non-Markovian

$$u(t_s, t_0) \to 0$$

$$v(t_s, t_s) \to n(t_s)$$

$$n(t_s) = \int d\omega \mathcal{D}(\omega) n(\omega, T)$$

- The steady state of the system:

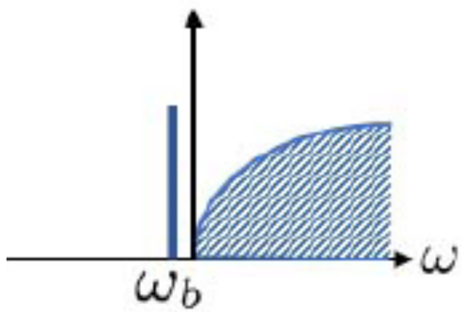
$$\boxed{\rho(t_0) \to \rho(t_s) = \sum_{n=0}^{\infty} \frac{[n(t_s)]^n}{[1 + n(t_s)]^{n+1}} |n\rangle\langle n|}$$

thermal-like state

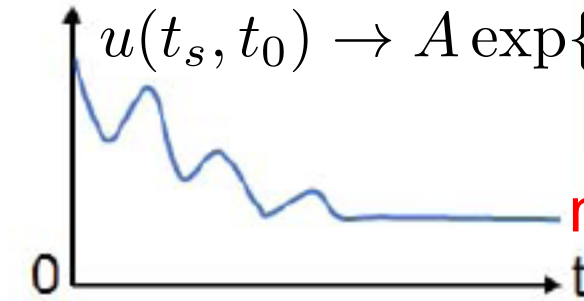
– the equilibrium hypothesis of statistical mechanics remains intact !!

arXiv: 1311.1282 (2013)

- For limiting-distributed spectral densities with strong system-environment coupling:



$$u(t_s, t_0) \to A \exp\{-i\omega_b(t_s - t_0)\}$$



strong non-Markovian

- The steady state of the system:

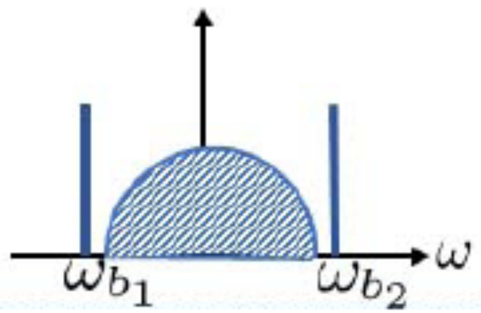
$$\rho\left(t_s\right) = \sum_{m,n=0}^{\infty} c_m^* c_n \sum_{k=0}^{\min\{m,n\}} d_k \mathcal{A}_{mk}^{\dagger}\left(t_s\right) \widetilde{\rho}(t_s) \mathcal{A}_{nk}\left(t_s\right)$$

qumemory state

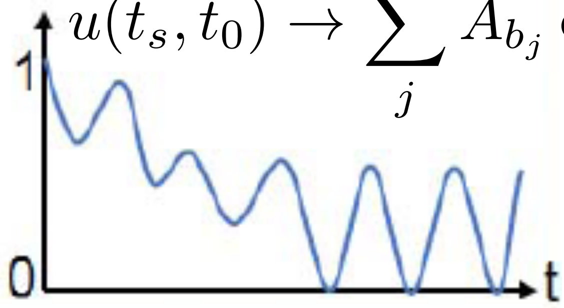
– maintain the initial-state information and partial quantum coherence, → quantum memory state or simply call it as a qumemory state !!

arXiv: 1311.1282 (2013)

- For spectral densities having band structures:



$$u(t_s, t_0) \rightarrow \sum_j A_{b_j} \exp\{-i\omega_{b_j}(t_s - t_0)\}$$



strong non-Markovian

- The steady state of the system:

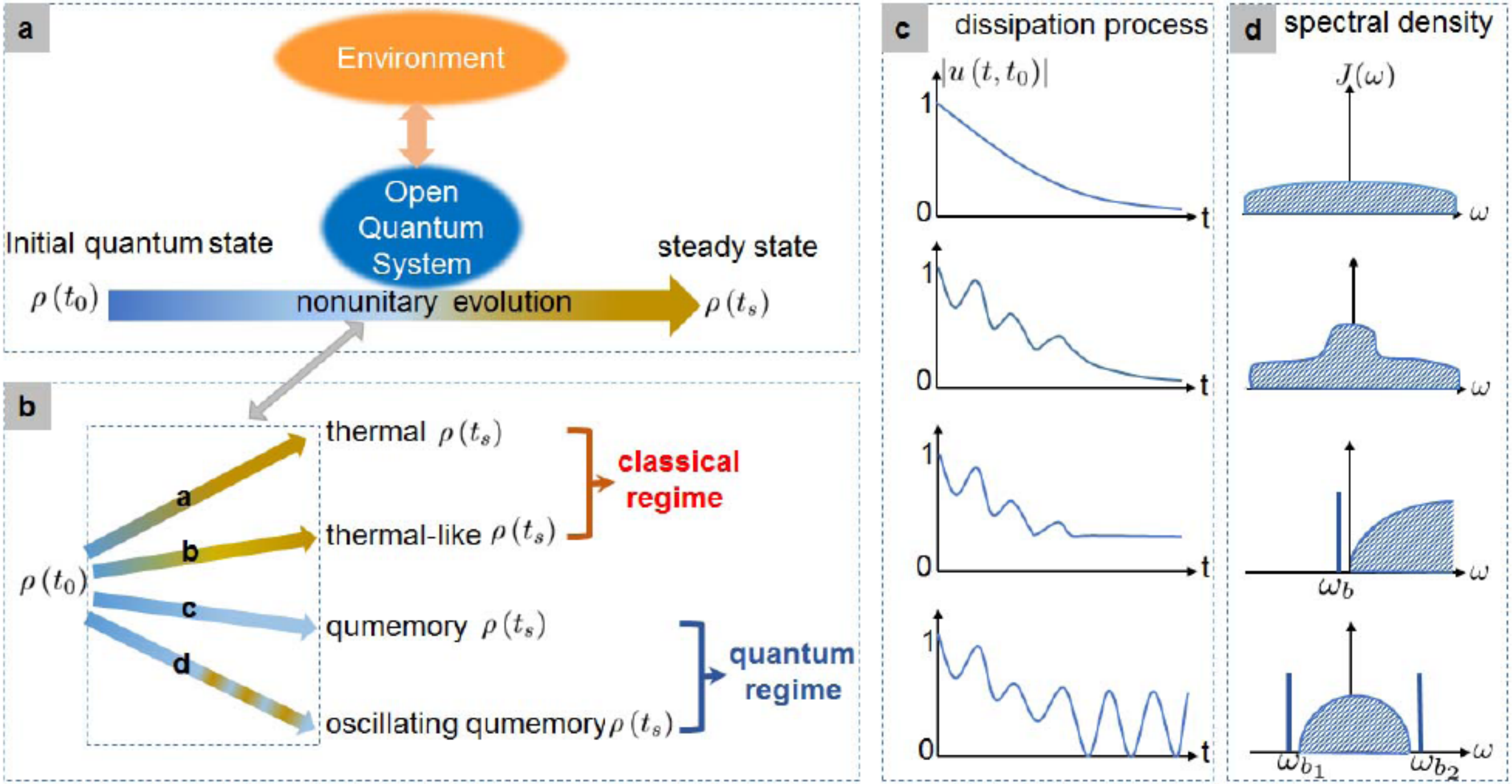
$$\rho\left(t_s\right) = \sum_{m,n=0}^{\infty} c_m^* c_n \sum_{k=0}^{\min\{m,n\}} d_k \mathcal{A}_{mk}^{\dagger}\left(t_s\right) \widetilde{\rho}(t_s) \mathcal{A}_{nk}\left(t_s\right)$$

oscillating qumemory state

– maintain the initial-state information and produces stationary quantum oscillations, → oscillating qumemory state !!

arXiv: 1311.1282 (2013)

Impact of Non-Markovian Dynamics on the Foundations of Statistical Mechanics:



arXiv: 1311.1282 (2013)

- qumemory and oscillating qumemory states:
  - maintain some or all initial state information (including quantum coherence).
  - generate intrinsic quantum oscillations among the localized modes.
  - do not reach thermal equilibrium with its environment.
    - challenge to the Foundations of statistical mechanics.
    - provide a dynamically-generated decoherence-free states:

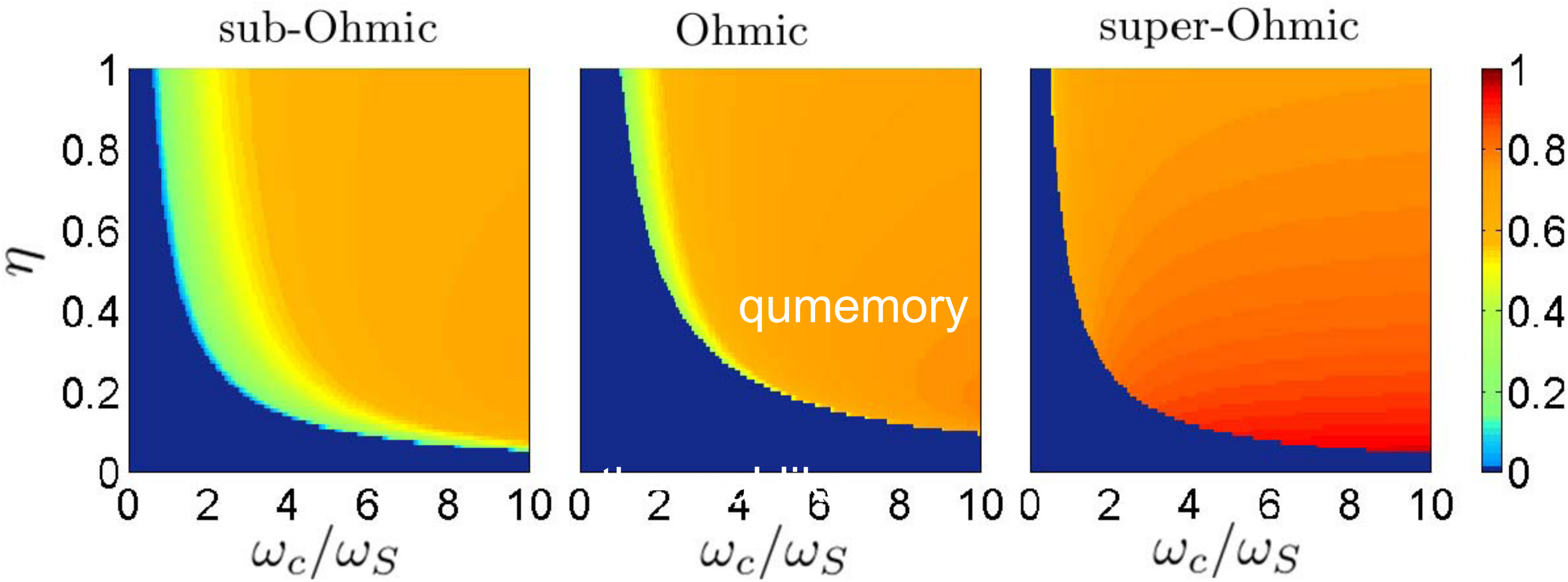
      which is different from the common concept of decoherence-free subspace, and it is a kind of generalization of the dynamical-stabilized decoherence-free states that recently proposed

Xiong, WMZ & Braun, Phys. Rev. A 86, 032107 (2012)

➢ Practical realization:

– consider a general Ohmic-type spectral density.

$$J(\omega) = 2\pi\eta\omega\left(\omega/\omega_c\right)^{s-1}\exp\left(-\omega/\omega_c\right)$$

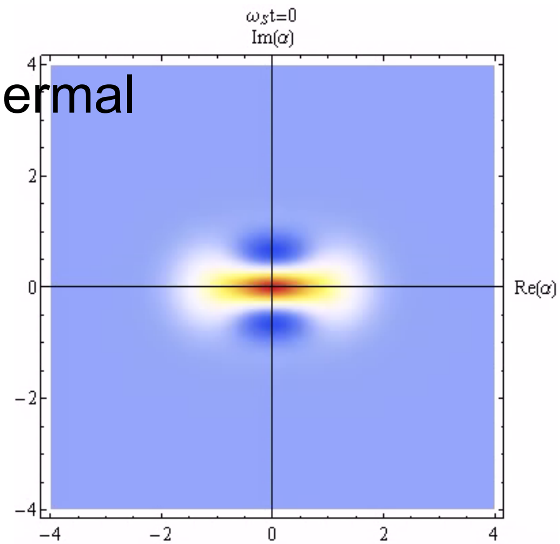


– boundary line: $\eta_c(\omega_c) = \omega_S/[\omega_c\Gamma(s)]$

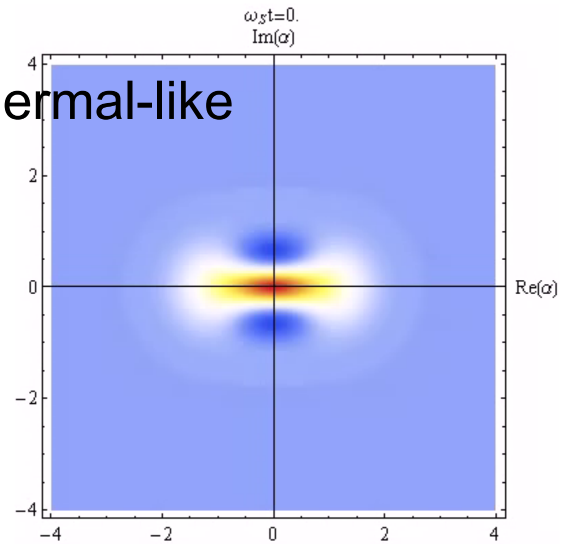
arXiv: 1311.1282 (2013)

Time evolution of initial Schrodinger's cat-like state:

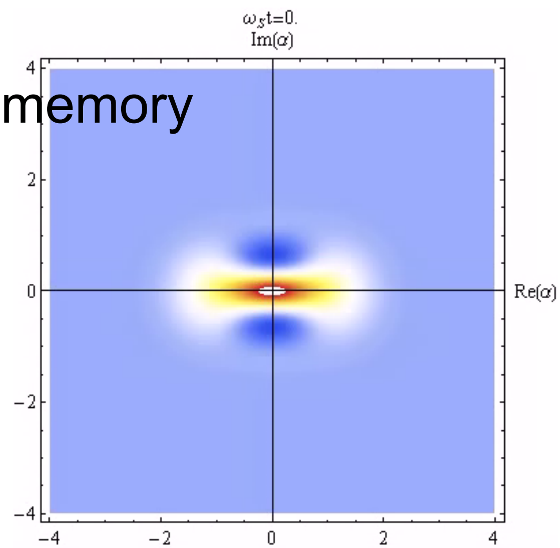
thermal



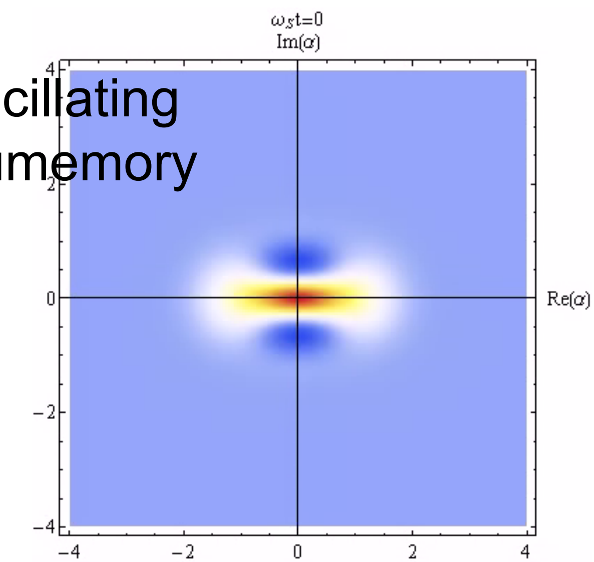
thermal-like



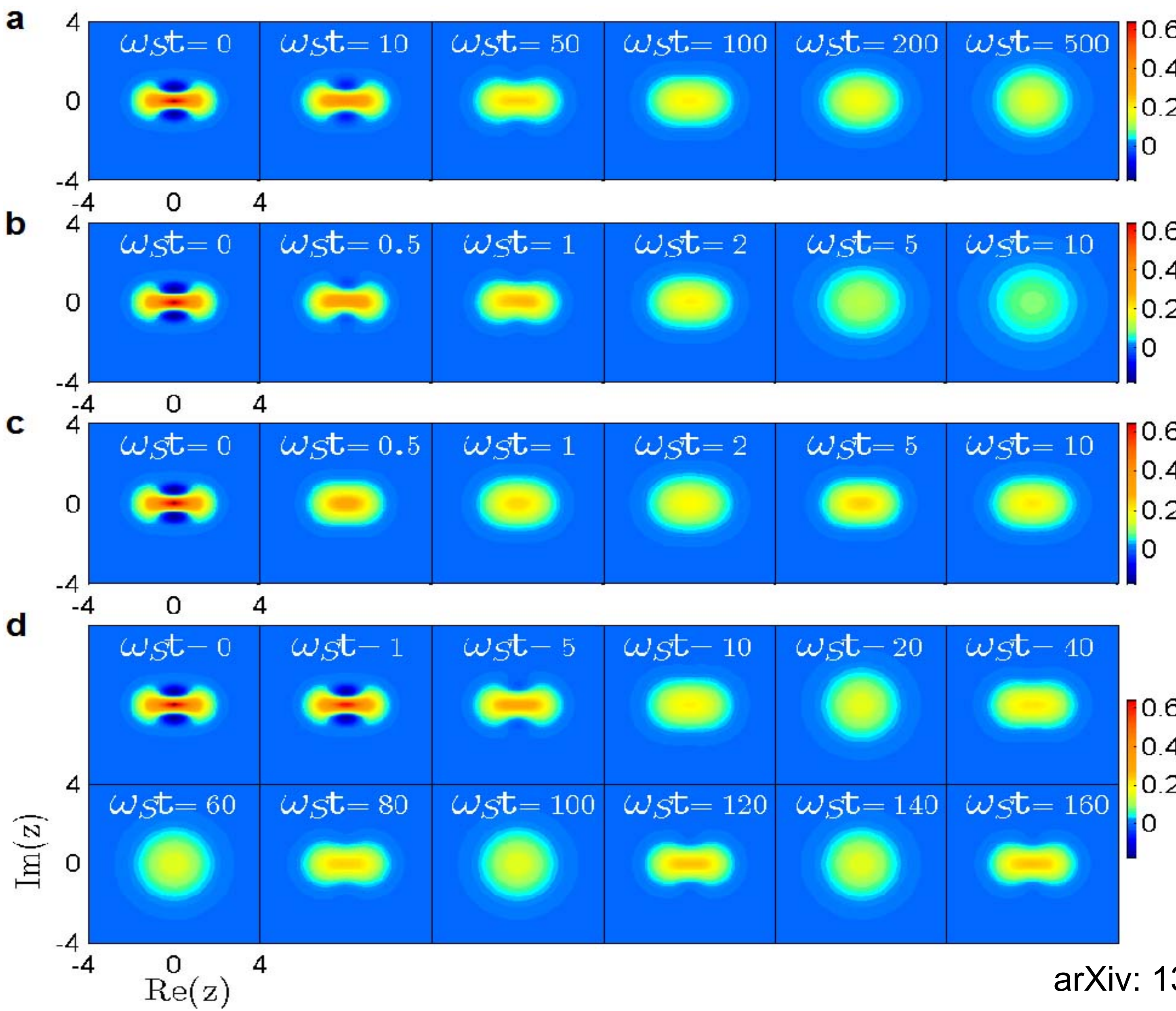
qumemory



oscillating
qumemory



- Four decoherence scenarios for Schrödinger’s cat-like state:



thermal

thermal-like

qumemory

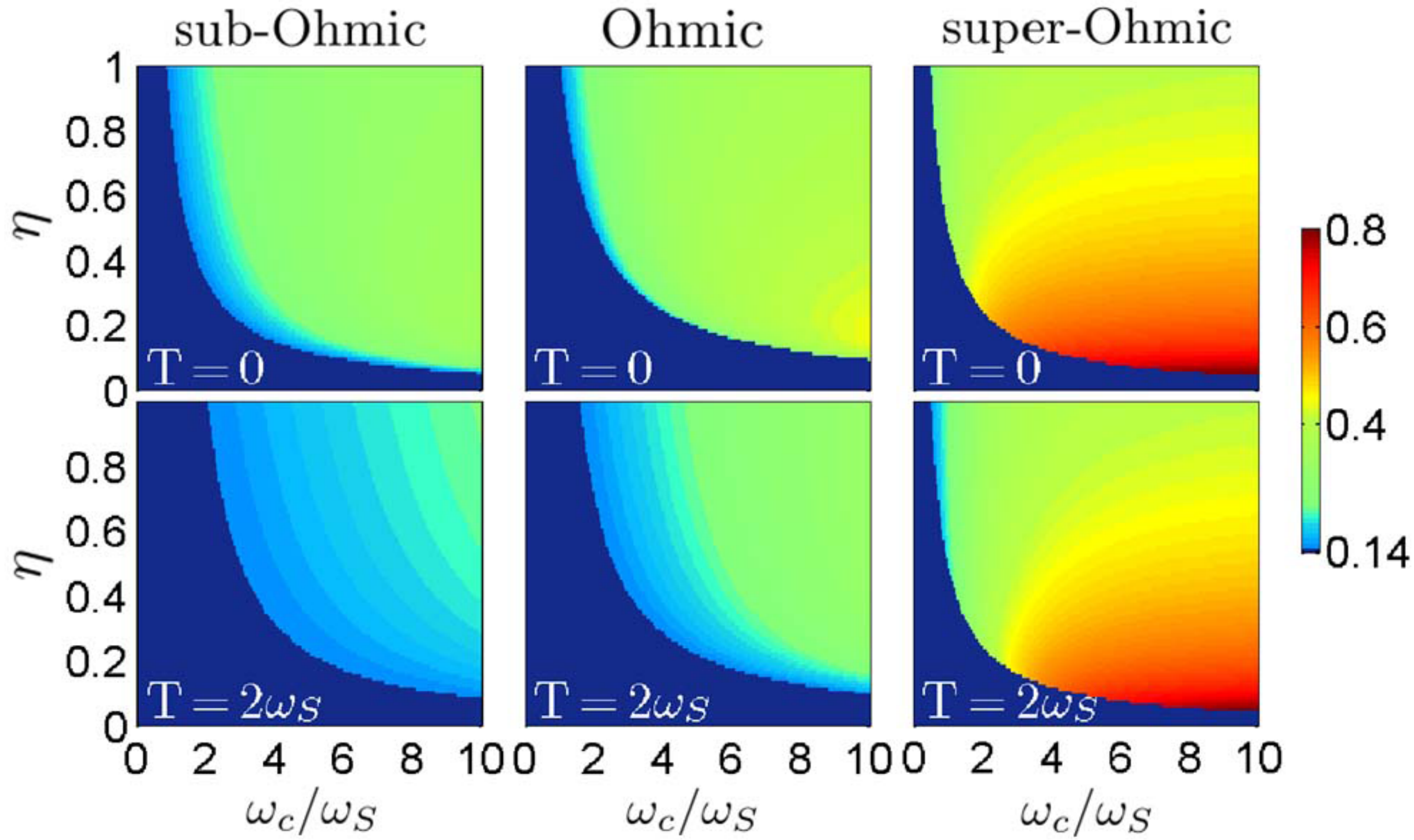
oscillating qumemory

arXiv: 1311.1282 (2013)

- Steady-state values of *fringe visibility*:

$$\text{Fringe visibility:} = \exp\left\{ -2\left|\alpha_0\right|^2 \left(1 - \frac{\left|u\left(t,t_0\right)\right|^2}{1+2v\left(t,t\right)}\right)\right\}$$

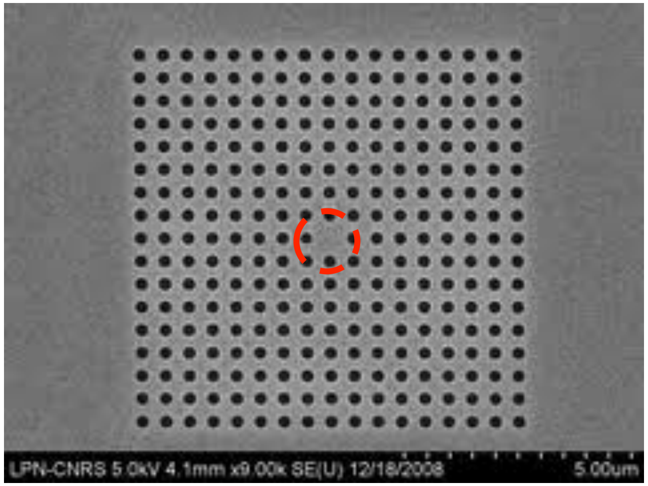
Lei & WMZ, *Phys. Rev*. A **84**, 052116 (2011)



arXiv: 1311.1282 (2013)

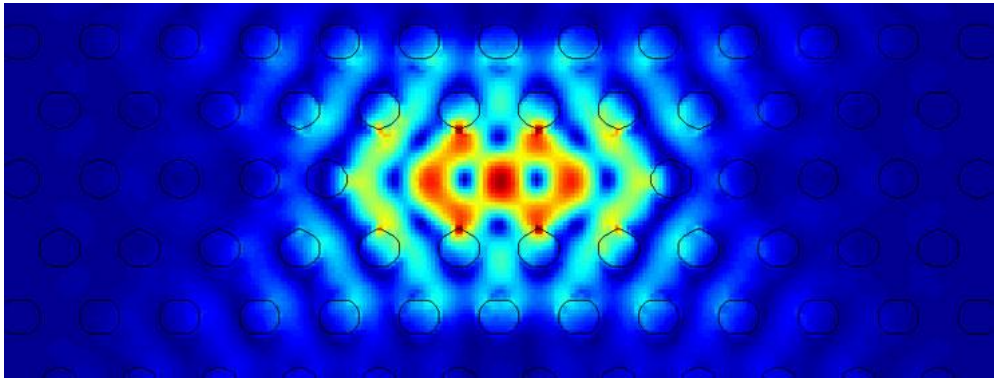
Applications to photonic crystals

- **Application to photonic crystals:**



Lossless materials

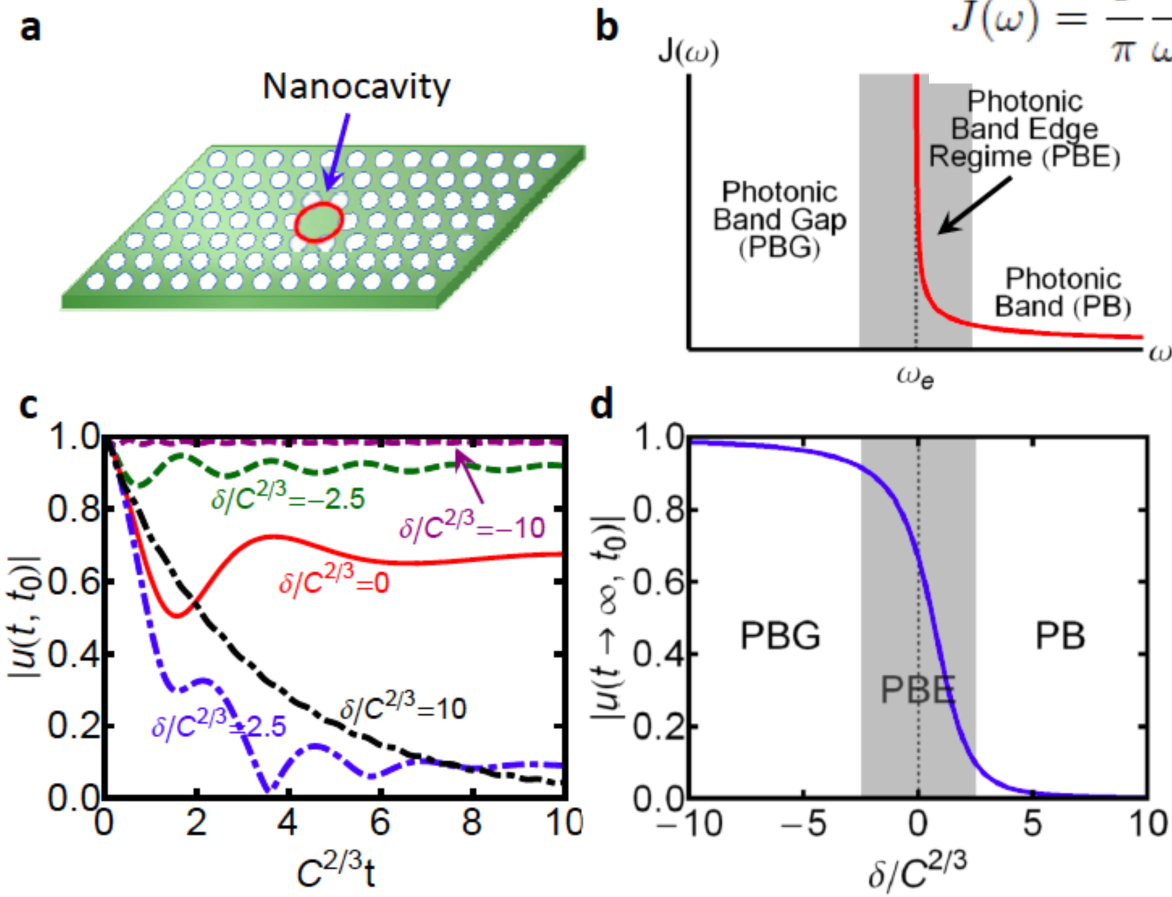
E. Yablonovitch, *Phys. Rev. Lett.* **58**, 2059 (1987)
S. John, *Phys. Rev. Lett.* **58**, 2486 (1987)



Ginzton Lab, Standford

Exact solution of nanocavity in photonic crystals

$$J(\omega) = \frac{C}{\pi} \frac{\sqrt{\omega - \omega_e}}{\omega - \omega_e + \epsilon} \theta(\omega - \omega_e),$$



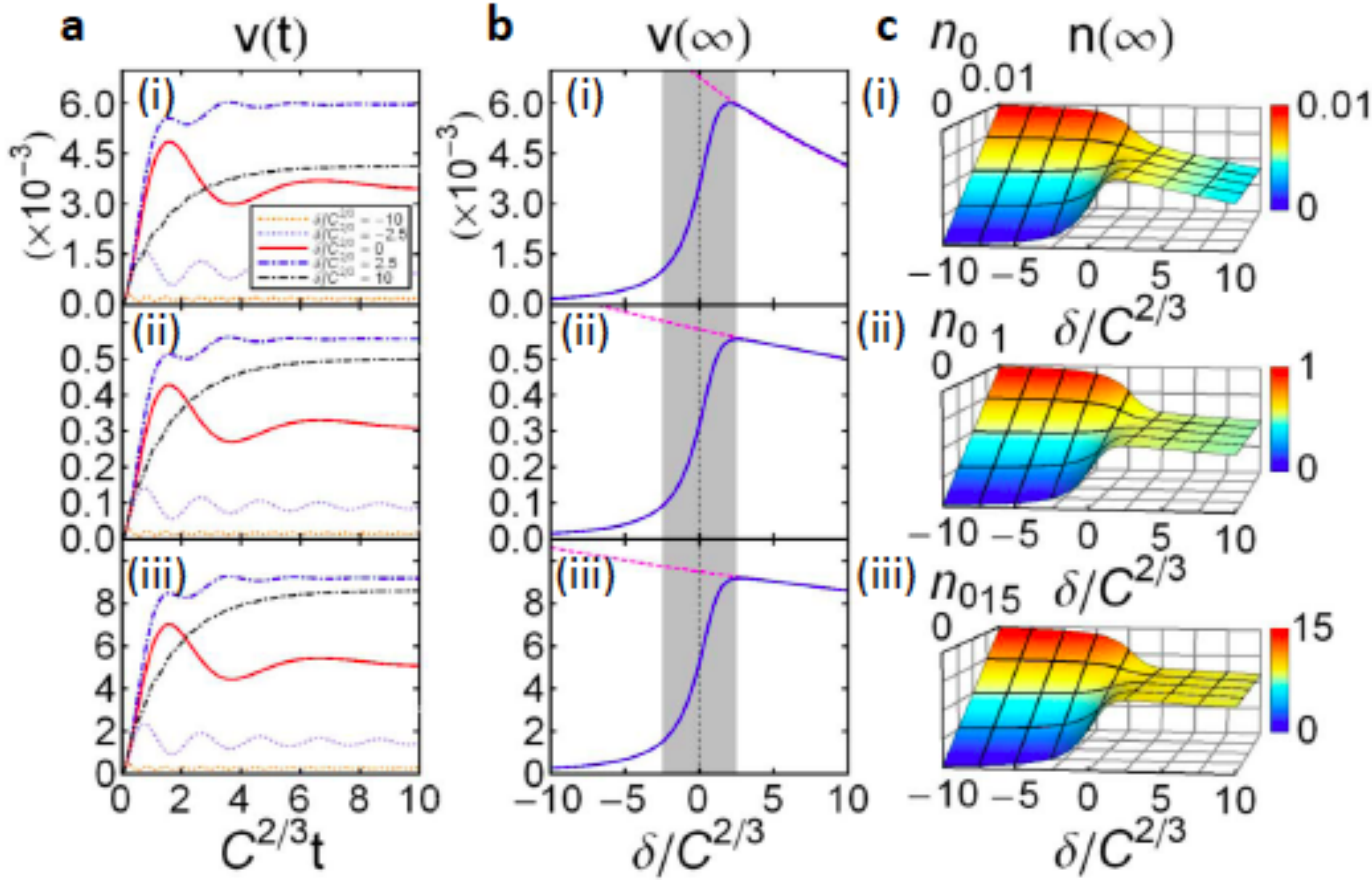
S. John and J. Wang, PRL **64**, 2418 (1990).

$$u(t, t_0) = \frac{2(\omega_e - \omega_b)}{3(\omega_e - \omega_b) + \delta} \cdot e^{-i\omega_b (t - t_0)} + \frac{C}{\pi} \int_{\omega_e}^{\infty} d\omega \frac{\sqrt{\omega - \omega_e} e^{-i\omega(t - t_0)}}{(\omega - \omega_c)^2 (\omega - \omega_e) + C^2}$$

Lo, Xiong & WMZ, arXiv: 1311.5409 (2013)

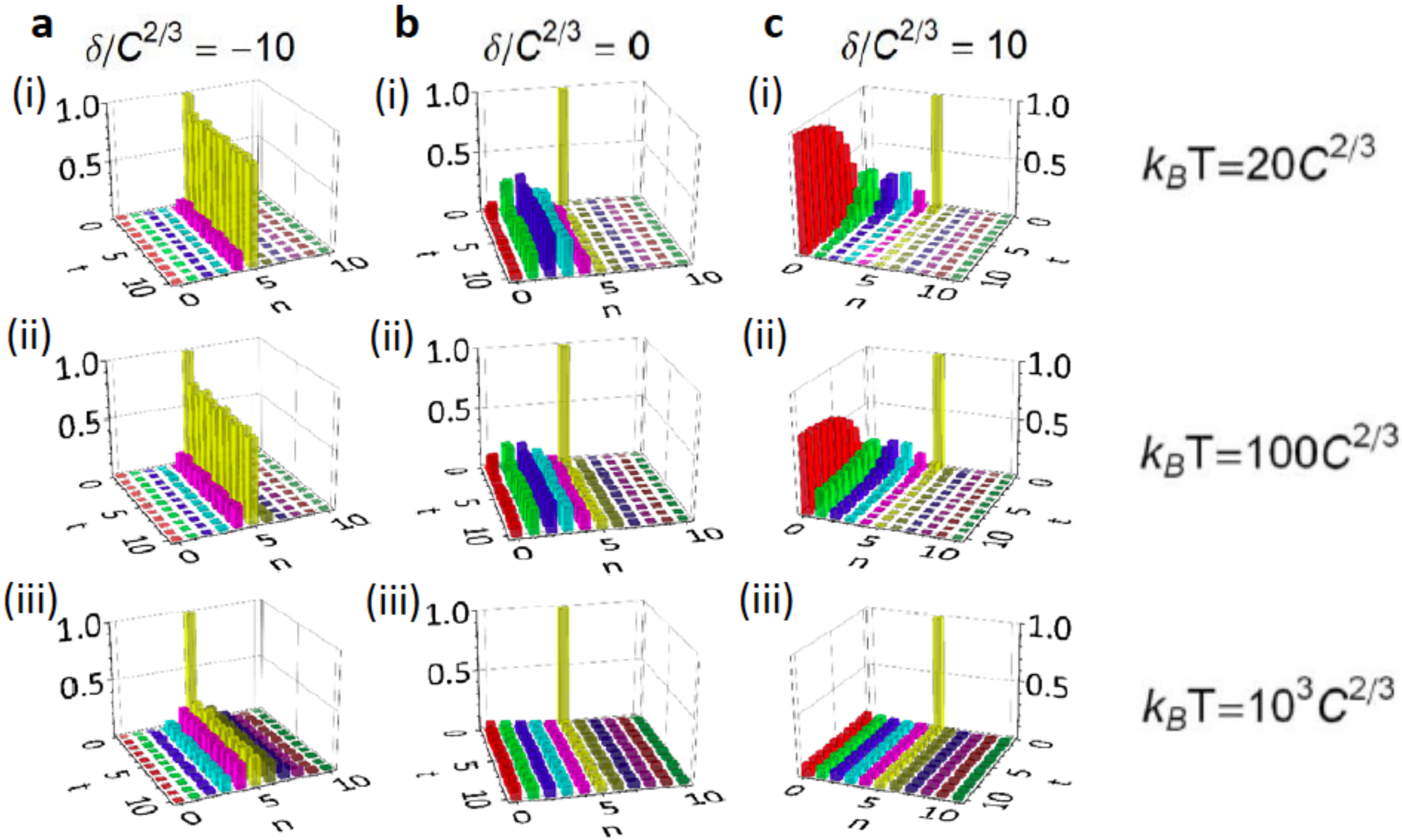
Thermal fluctuation:

Lo, Xiong & WMZ, arXiv: 1311.5409 (2013)



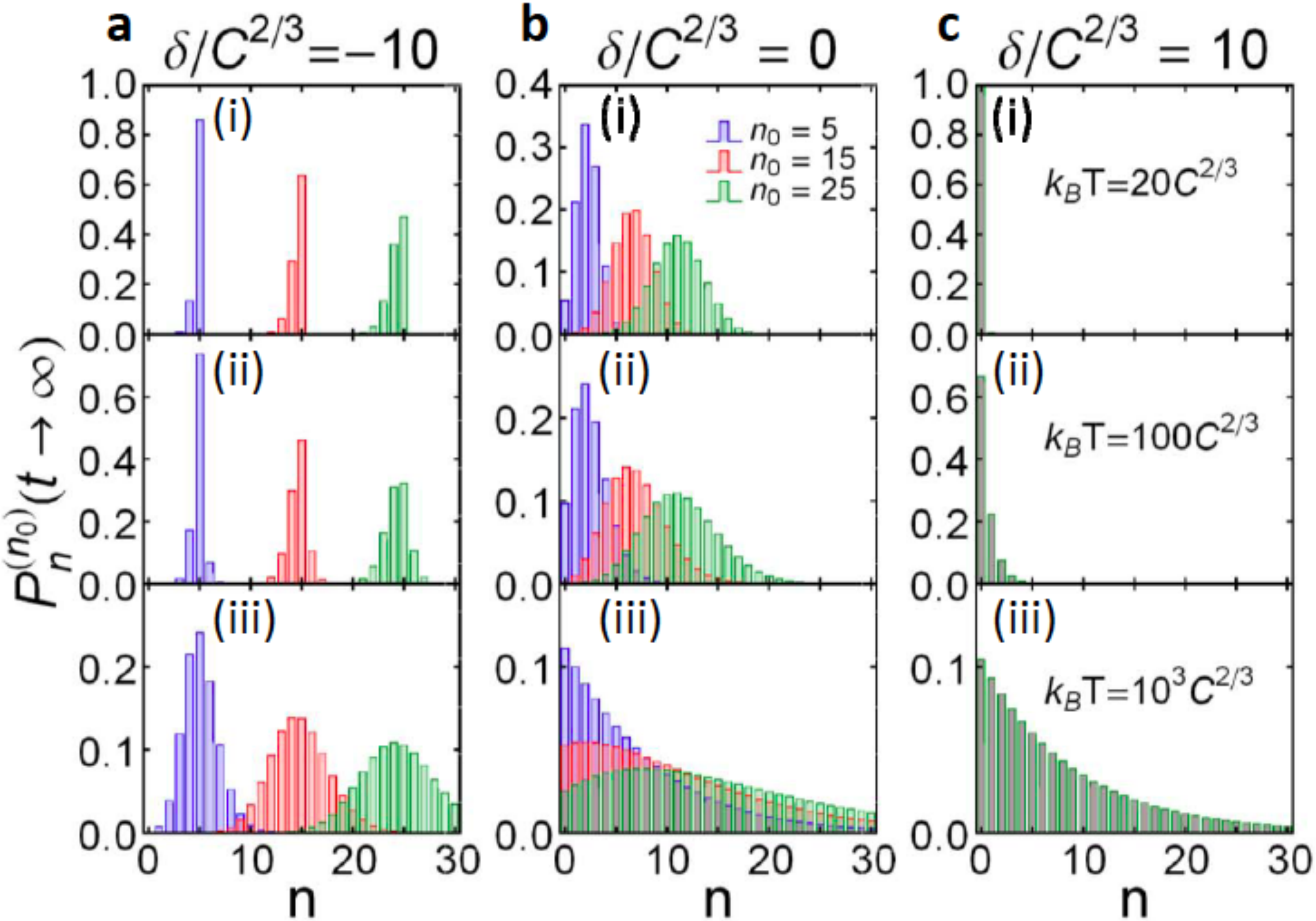
Evolution of an initial Fock state: n_0=5

Lo, Xiong & WMZ, arXiv: 1311.5409 (2013)



Breakdown of Bose-Einstein distribution……

Lo, Xiong & WMZ, arXiv: 1311.5409 (2013)



Summary

- all results presented here are solved exactly from the exact master equation of non-interacting open systems.

- all results are only determined from the spectral density of the environment, so are generic.

- it provides a rigorous proof to the foundations of statistical mechanics.

- new feature (qumemory states) of open quantum systems is discovered, does it imply new physics??

- hopefully it also provides a basis for quantum control??

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