Oxford Seminar

Saturday, March 28, 2009
6:45 PM

Title: TOP?

Topological Phases of Matter : Why they are Interesting and Useful

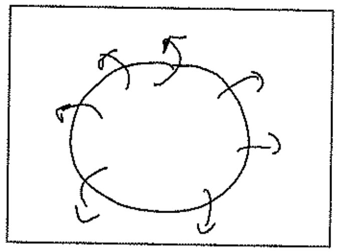
Ref: Rev Mod Phys Sept 2008 Nayak, Simon, Stern, Freedman, DasSarma) SIMON PHYSICS WORLD 2010

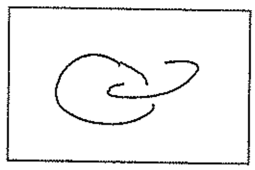
- Some have heard stories sound similar -- new perspective
- Success if everyone learns something new

- Story starts with Lord Kelvin around 1867
- Kelvin had a friend Tait
  - [comments on tate -- golf -- success]
- Tait had built a machine that created smoke rings
  ... and this caught Kelvin's attention:

They are stable (kelvin theorem)

FAR LEFT



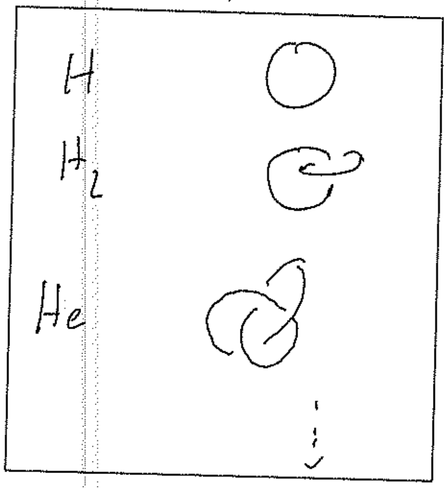


- Also at the time everyone believed that we are surrounded by Aether
- At the time one of the big mysteries in science was the fact that elements were discrete
- Kelvin had a brilliant idea: Elements corresponded to Knots of Vortices in the Aether

Saturday, March 28, 2009
7:16 PM

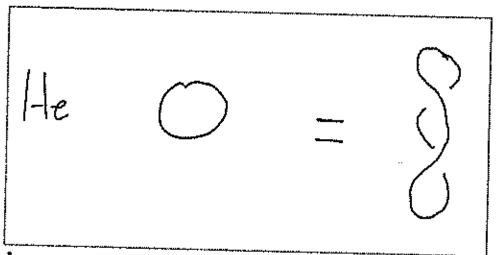
- Being sloppy about knot vs link
- This is a nice idea - it explained discreteness - And was popular for a while, but eventually most physicists had decided it was no good.
- But for a few decades it had some strong proponents - perhaps the strongest being Tait
- Tait was so enamoured of this idea that he spent much of the rest of his life trying to compile a "periodic table of knots" - i.e, list all possible knots ergo all elements.

Add to prior



- Tait is often considered the father of the mathematical theory of knots (which has been a very rich branch of mathematics)
- During exploration, Tait hit on what is now the fundamental question of knot theory. "can one be smoothly deformed into another without cutting"

For example



add to prior

- Sounds easy , but is not.
- Over the last 150 years, mathematicians have been studying this question and it is still considered a hard problem, but one tool has been useful in partially solving the problem - the idea of a "knot invariant"
- About to take a short detour into knot theory

Saturday, March 28, 2009
7:45 PM

KNOT (or picture)

$$\text{KNOT} \xrightarrow[\text{KNOT INVARIANT}]{\text{ALGORITHM}} \text{OUTPUT}$$

Middle board
(keep for a bit)

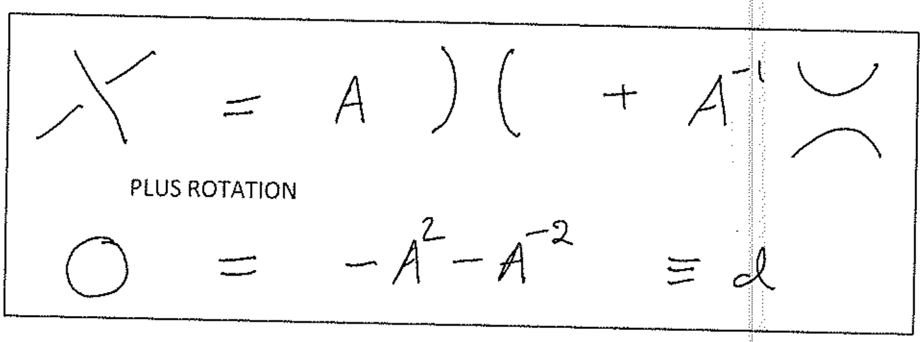
- Topologically equivalent knots must give the same output
- So if the output for two knots comes out different, they must be topologically different knots
- However, sometimes two inequivalent knots give the same output
- Most important question in knot theory is if you can find a knot invariant that can distinguish all knots from each other

FAR RIGHT (KEEP)

$\boxed{\text{KAUFFMAN (JONES)}}$

- Example of a knot invariant: Kauffman Invariant
  (closely related to Jones polynomial.. I'll explain the difference later)

- Two Rules



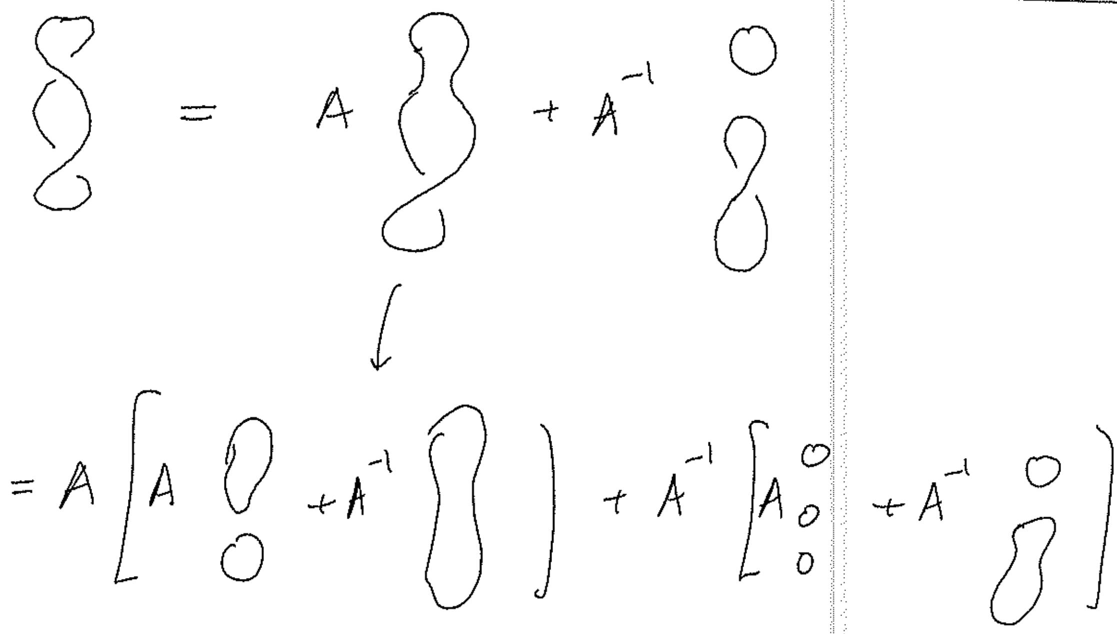
KEEP!

Leave!
Room!
Below
this

Saturday, March 28, 2009
7:59 PM

- An example

Add to prior left



$$= A^2 d^2 + d + d^3 + A^{-2} d^2 = d$$

- We get d, which is not surprising since the open loop is d, and this is just the open loop in disguise and the result depends only on topology
- To the mathematician, the Kauffman invariant is an invariant of regular isotopy.

computationally hard for complex knots.

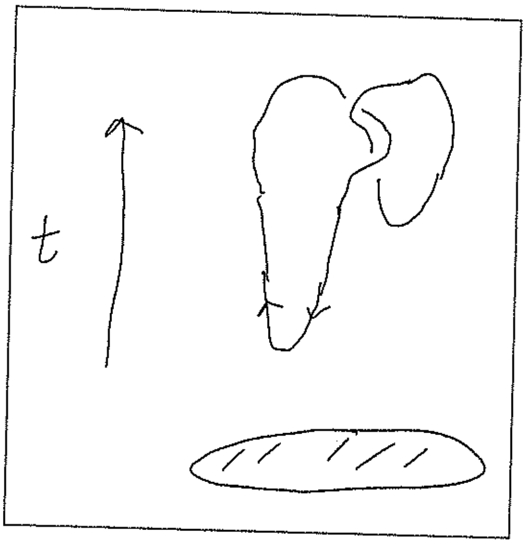
CAN ERASE VORTEX ATOM AND PRIOR LONG CALC

- Return to physics now, to something that sounds different but is actually related

$$\boxed{\text{TOPOLOGICAL QUANTUM FIELD THEORY}}$$

- This is a quantum field theory, or quantum mechanical system where amplitudes depend only on the topology of a process.
- Think about quantum mechanics as Feynman taught us

$$\boxed{Z = \sum_{\text{paths}} e^{iS(\text{path})}}$$

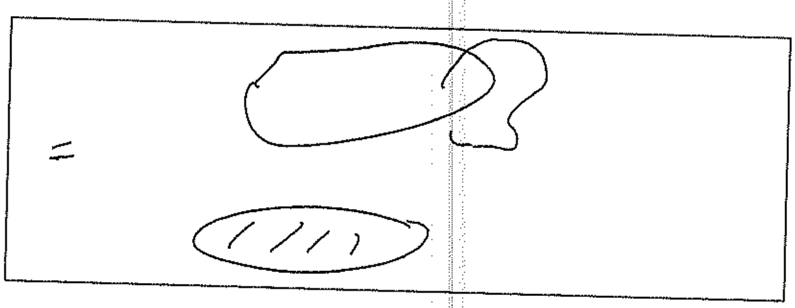


DRAW TOP VIEW OF PROCESS

This is some 2D system (always think in 2+1)

I've drawn some space-time process
That contributes to the sum

For a theory to be topological, the amplitude for this must be =



Since they are topologically the same.

Saturday, March 28, 2009
8:27 PM

- What should be clear is that for topological theory e^{iS} is a knot invariant

- Famously Witten showed made the relationship between certain known TQFTs of a certain known knot invariants. For the Kauffman invariant the corresponding TQFT is known as

Where

$$\boxed{SU(2)_k \Longleftrightarrow A = i e^{\frac{i\pi}{2(k+2)}} \qquad d = 2\cos\left(\frac{\pi}{k+2}\right)}$$

Put this under kauffman rules (KEEP)

- This was the work that won Witten the Field's medal, along with Jones

- Finally, explain the title of my talk:

A Topological Phase of Matter is a Phase of Matter whose low energy, long wavelength description is a TQFT.

- Rather shockingly we believe these things exist (not just figment)

$SU(2)_2$	$\Rightarrow$	$\nu = 5/2$ FQHE	(moore read)
		Sr_2RuO_4 films	
		$^3He\,A$ films	
		Engineered super structures - - -	
$SU(2)_3$	$\Rightarrow$	$\nu = 12/5$ FQHE	(read-rezayi)

UNDER Kauffman rules (KEEP)

- Plus other possibilities with cold atoms, JJ arrays, spin systems, superconducting Top Ins junctions... but none of these have been realized yet

- Evidence that these are TQFTs is not iron clad yet, but in some cases is getting good.

- Strongest evidence is in FQHE, so I'll describe those in a moment, but first

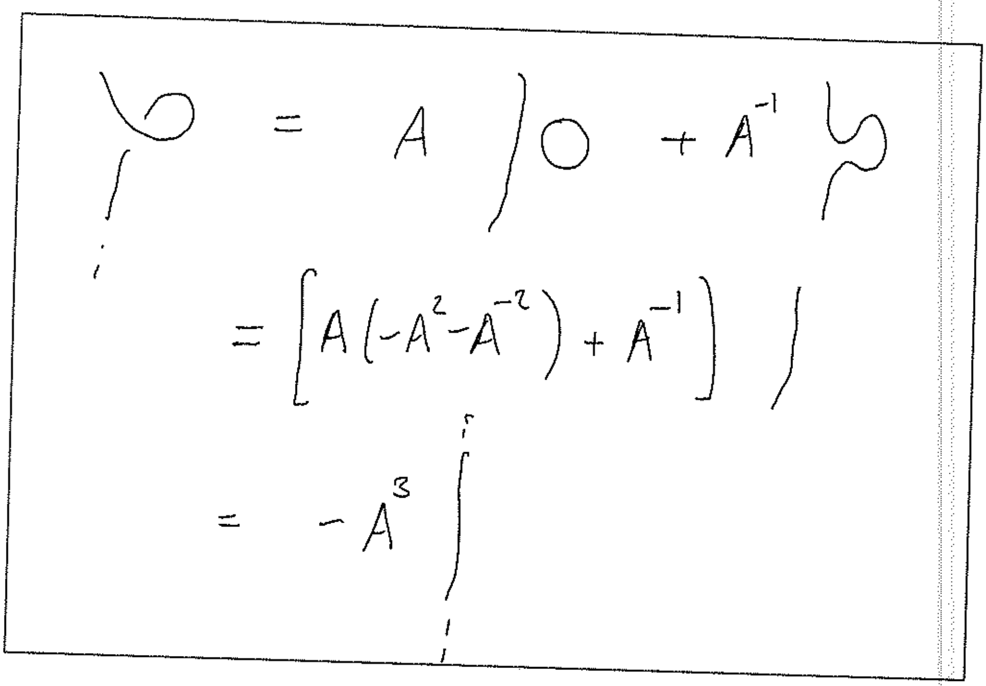
WHY ARE THESE SPECIAL

GROUND STATE (VACUUM) COMPLEX ENTANGLED STATE

Saturday, March 28, 2009
8:46 PM

- (For honesty in advertising).. I should explain the difference between Kauffman and Jones.

  Consider the following (dots mean closed up in a knot somewhere else)

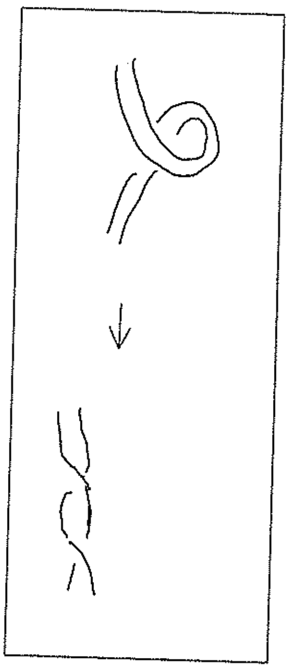


MIDDLE

SAVE THIS.

- is it not a topological invariant?
- Need to realize that particles have finite width

ADD TO PRIOR



- When pulled tight, we introduce a twist. So left and right are not identical
- In Quantum mechanics, rotating a particle with spin gives a phase, and that is exactly what we find ! SPIN STATISTICS
- So Kauffman correctly accounts for the spin of these particles
- Jones is cooked up to remove the phase of the self-rotation And give an invariant that will not care about introducing Self twists.

Saturday, March 28, 2009
9:00 PM

CAN ERASE SPIN STATISTICS PART

Comments on FQHE

(MIDDLE)

2D e's, INTERACTING, + B

- Make electrons 2d by trapping in a thin semiconductor heterostructure
- Then cool down

2D e's, INTERACTING, + B

$\downarrow$

FQHE ($\nu = \ldots$)

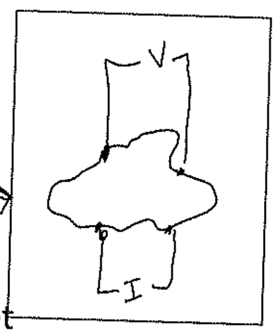
- Nu is ratio of density of electrons to magentic flux density
- Near certain values of nu you see FQHE - a dissipationless state
- All of these are believed to be TQFTs although most are trivial and uninteresting (I'll explain in what sense)

2D e's, INTERACTING, + B

$\downarrow$

FQHE ($\nu = \ldots$) = TQFT

Add last / dotted



- (a) This is symmetry emergence, not symmetry breaking
- (b) would like to integrate out high energy degrees of freedom to get TQFT - but must take roundabout route

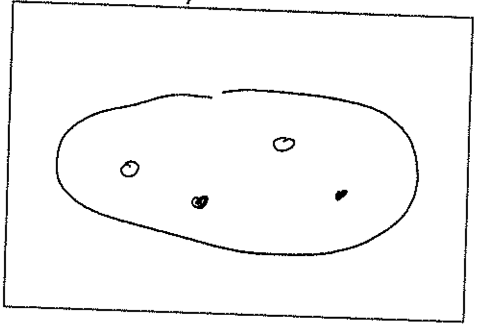
Saturday, March 28, 2009
9:15 PM

CAN ERASE EXCEPT ALL THE KEEP STUFF ON LEFT

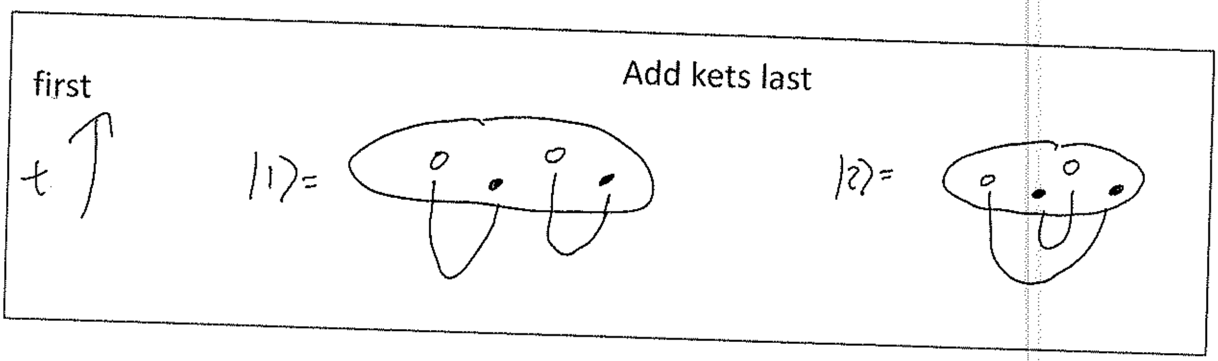
- Return to general properties of TQFTs :

- One important property is that when there are quasiparticles in the system, there becomes a degeneracy of states with the same energy.

Leave Space Above and Below (START LEFT)
(and a bit left)

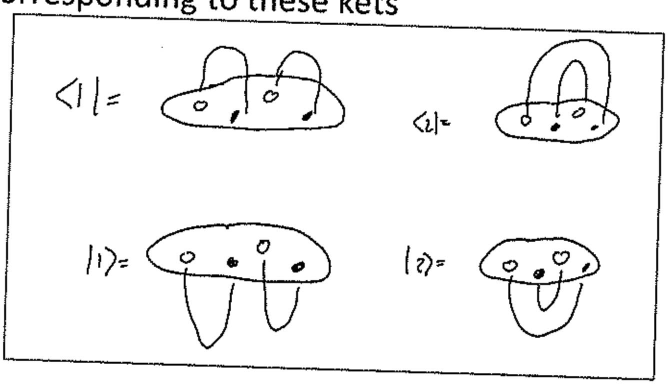


- System with 2 qps and 2 qhs.
- The energy is determined by how far apart they are.
- I claim there is more than one state with the particles in the same position.
- Look at the spacetime history



- The particles could have come from the vacuum in (at least) two different ways Different space-time histories to get to the same positions.

- To show these are distinct, we must also construct the time reversed states, which are the bras corresponding to these kets



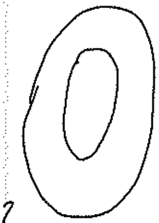
Add top row

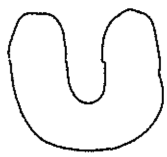
(Supposed to be mirror images)

Saturday, March 28, 2009
9:33 PM

- To find matrix elements, just connect together the bras and kets

$\langle 1|1\rangle =$  $= d^2$

$\langle 2|2\rangle =$  $= d^2$

$\langle 1|2\rangle =$  $= d$

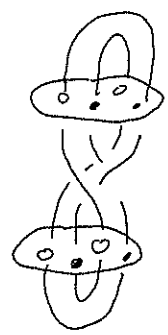
NORMALIZATION

INSERT

- This tells you that 1 and 2 are not the same, and must be linearly independent
- The exception is k=1 which unfortunately describes most FQHE states
- Only these special FQHE states are nontrivial

Note: If in between bottom and top I inserted some arbitrary braid, I would get a different result. This means that braiding particles around each other changes the state of the system. I.e, makes transitions within the hilbert space.

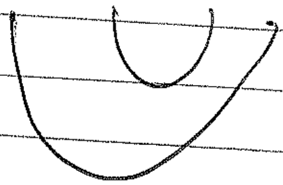
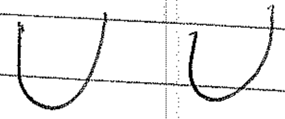
$\langle 2| =$

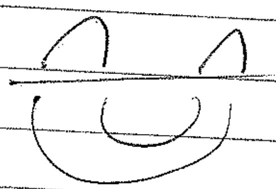
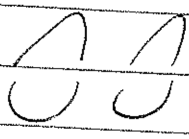


$|2\rangle =$

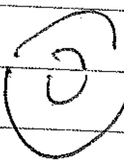
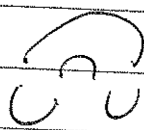
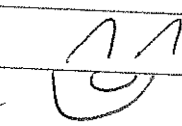
$\langle 2|\text{BRAID}|2\rangle = \ldots$

*

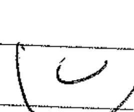
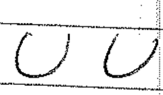
 $-\frac{1}{d}$  $= |1\rangle$

 $-\frac{1}{d}$  $= \langle 0|1\rangle$

$$d - \frac{1}{d} d^2 = 0$$

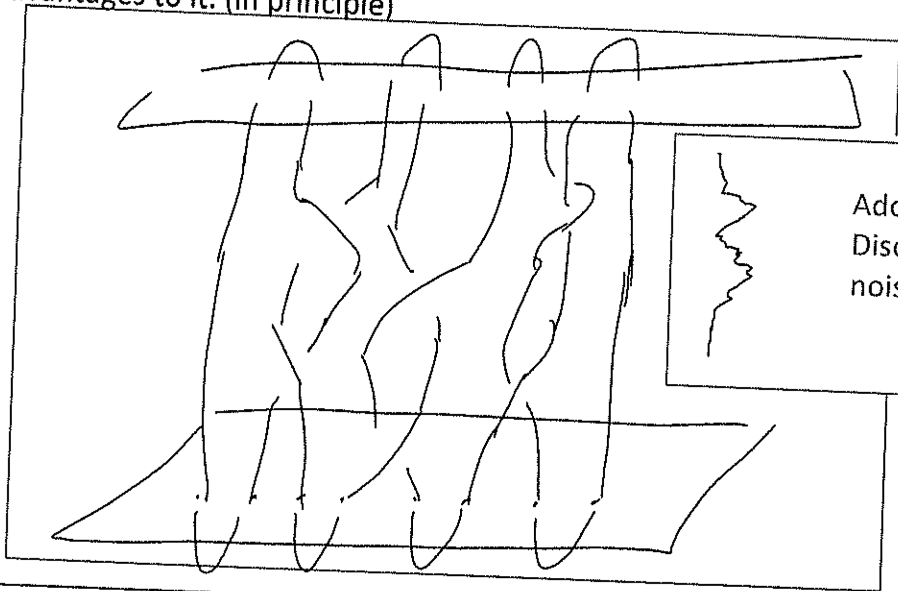
 $-\frac{1}{d}$  $-\frac{1}{d}$  $+\frac{1}{d^2}$ 

$$= d^2 - 1 - 1 + 1 = d^2 - 1$$

$|1\rangle = \frac{1}{\sqrt{d^2-1}}\Big[$  $-\frac{1}{d}$  $\Big]$

Saturday, March 28, 2009
9:49 PM

- When you see a two state system you should immediately think of a qubit.
- And in fact, this idea, known as Topological Quantum Computation, idea credited to Freedman, Kitaev, and collaborators. Actually has some real advantages to it. (in principle)



Add this last in the Discussion about noise

- Start with a bunch of groups of quasiparticles, each group representing a qubit.
- Do a quantum computation (which is just some unitary operation on the hilbert space) by braiding the particles around each other
- Then measure the particles - one way being trying to return the particles to the vacuum. This is the readout.

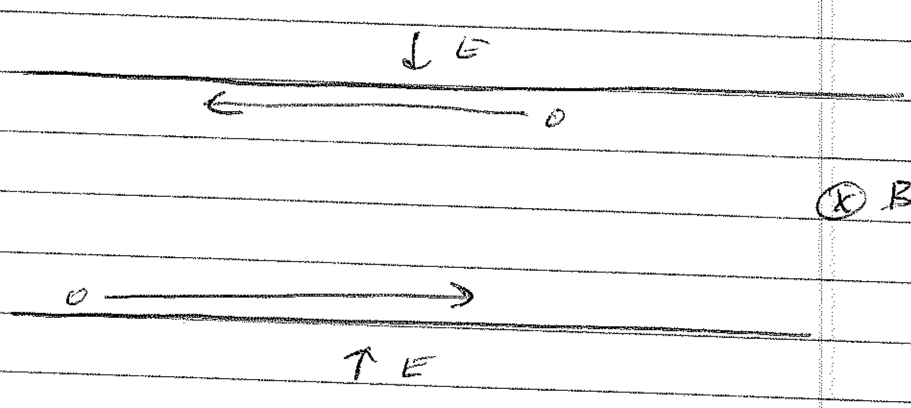
- If you choose the right TQFT, this is just as powerful as any other quantum computer
- The advantage is resistance to noise and coupling to the environment. Senisitivity to noise (or coupling to the environment) is the bugaboo of all quantum computing, but here... it is robust. To see why imagine one of the particles gets hit by noise...

The possible application is why this is potentially useful - and why microsoft is interested

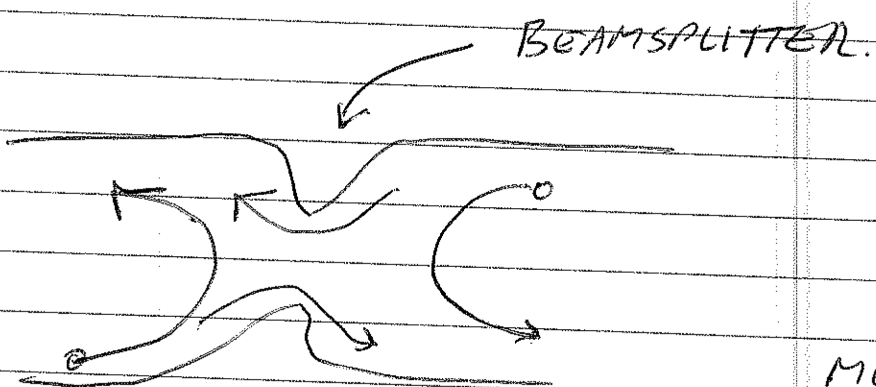
THE FIRST EXPERIMENT TO TEST THESE IDEAS
(and describes a measurement scheme)

FACT ABOUT FQHE

CONFINEMENT GIVES $E \times B$ DRIFT

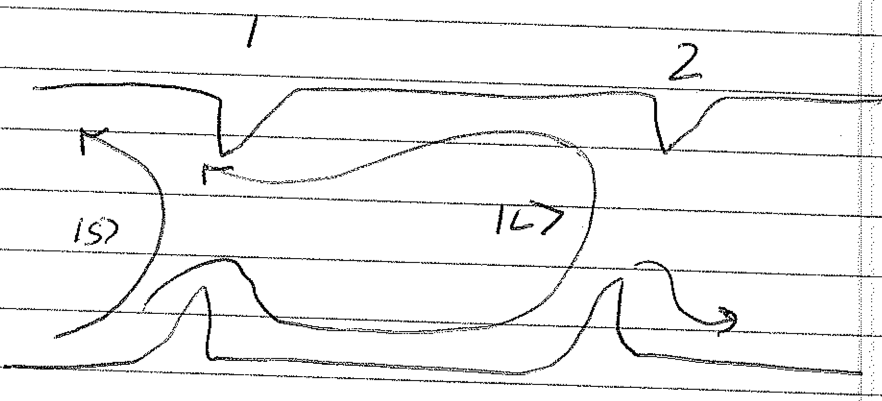


MODIFY



MEASURE BACKSCATTER

=



FABRY-PEROT INTERFEROMETER

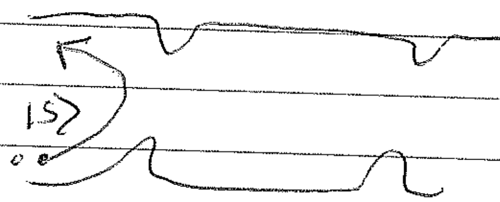
$$|OUT\rangle = t_1 |S\rangle + e^{i\phi} t_2 |L\rangle$$

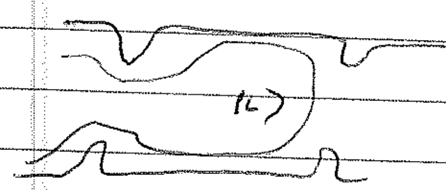
BACKSCATTERED CURRENT =

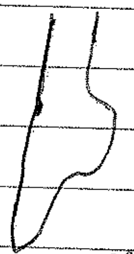
$$\langle OUT | OUT \rangle \sim |t_1|^2 + |t_2|^2 +$$

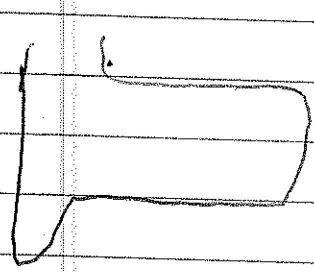
$$2\,\mathrm{Re}\left[e^{i\phi} t_1^* t_2 \langle S | L \rangle \right]$$

CAN VARY ϕ TO MEASURE THIS PIECE.



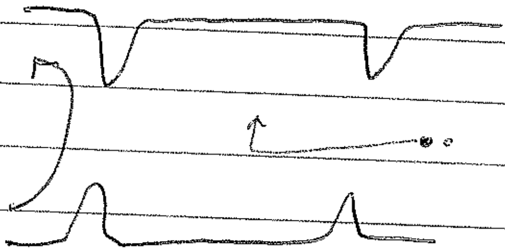


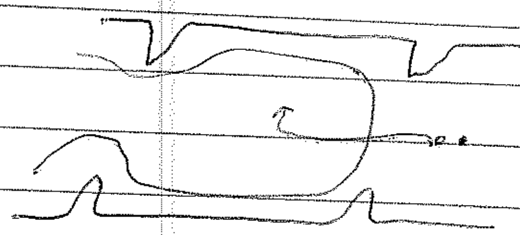
t  $= |S\rangle$

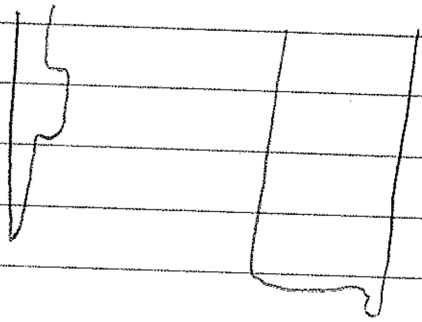
 $= |L\rangle$

↑ PREPARE PARTICLE

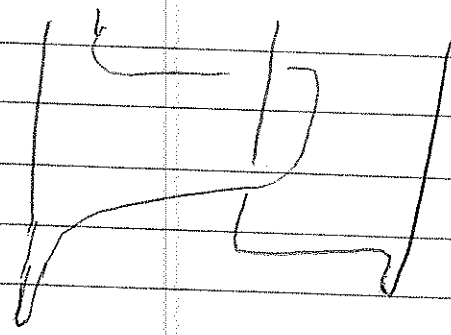
$\langle S | L \rangle =$  $= d$



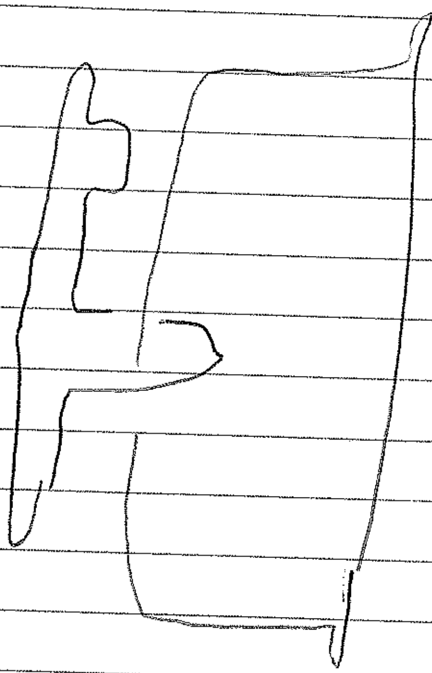




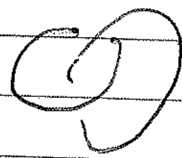
$= |s\rangle$

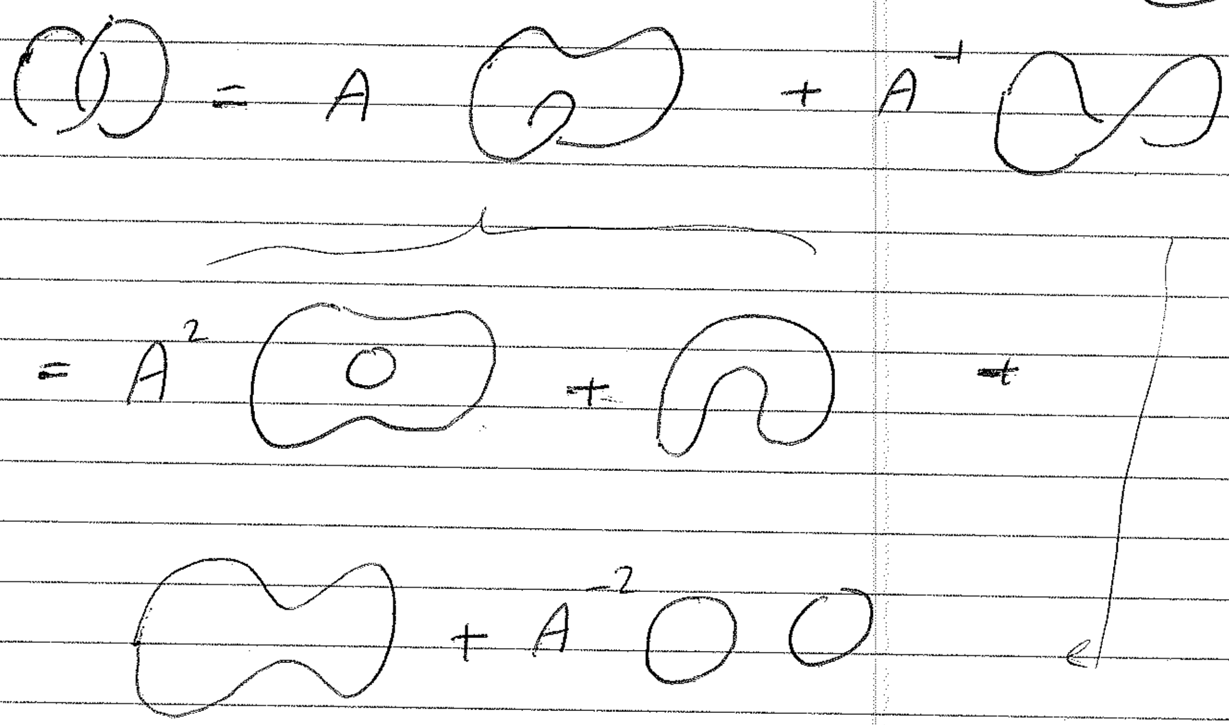


$\langle s|L\rangle =$



$=$





$$= (A^2 + A^{-2})\, d^2 + 2d = -d^3 + 2d$$

$$= 0 \quad \text{for} \quad d = \sqrt{2}$$