

# Quantum Communication

Serge Massar

Université Libre de Bruxelles

# Plan

- Why Quantum Communication?
- Prepare and Measure schemes
  - QKD
- Using Entanglement
- Teleportation
- Communication Complexity
- And now what?

[Talk: Theoretical Concepts & Illustrative Experiments](#)

# Quantum Communication

Why?

How?

# Quantum Communication

## Why?

- Quantum Crypto
  - Q. Key Distribution
  - Other protocols
    - Coin Tossing, etc...
- Communication Complexity
- Foundations of Physics

## How?

# Quantum Communication

## Why?

- Quantum Crypto
  - Q. Key Distribution
  - Other protocols
    - Coin Tossing, etc...
- Communication Complexity
- Foundations of Physics

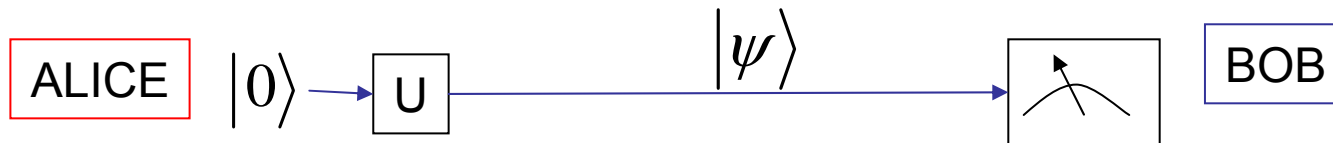
## How?

### Photons

$$\vec{E}(\vec{x}, t) = A\vec{u} \cos[\omega t - \vec{k} \cdot \vec{x} - \varphi]$$

- $\vec{u}$  Polarization
  - $\omega, t$  Frequency/Energy
  - $\vec{k}, \vec{x}$  Momentum/Position
  - $A, \varphi$  Amplitude/Phase
- Wavelength
    - Visible: Free Space
    - Near IR: fiber optics  $\lambda \approx 1.5\mu m$

- Protocols in which a single qubit is
  - Prepared
  - Sent
  - Measured



- Quantum Key Distribution

# Quantum Key Distribution

Alice

Eve

Bob

- Alice and Bob want to share a secret key

$$r_1 r_2 r_3 \dots r_N \in \{0,1\}$$

$$r_1 r_2 r_3 \dots r_N$$

# Quantum Key Distribution

Alice

Eve

Bob

- Alice and Bob want to share a secret key

$$r_1 r_2 r_3 \dots r_N \in \{0,1\}$$

$$r_1 r_2 r_3 \dots r_N$$

- Eve should not learn the key
- If Eve tries to learn the key, she is detected



# Quantum Key Distribution

Alice

Eve

Bob

- Alice and Bob want to share a secret key

$$r_1 r_2 r_3 \dots r_N \in \{0, 1\}$$

$$r_1 r_2 r_3 \dots r_N$$

- Eve should not learn the key
- If Eve tries to learn the key, she is detected

Use quantum communication  
& uncertainty principle / no cloning theorem

# QKD

- If Alice prepares:  
two orthogonal states

- If Bob measures:  
in basis

$$|0\rangle \pm e^{i\varphi} |1\rangle \xrightarrow{\text{Send to Bob}} |0\rangle \pm e^{i\varphi} |1\rangle$$

- Then Bob learns which  
state was prepared by  
Alice

# QKD

- If Alice prepares:  
two orthogonal states

- If Bob measures:  
in basis

$$|0\rangle \pm e^{i\varphi} |1\rangle \xrightarrow{\text{Send to Bob}} |0\rangle \pm e^{i\varphi} |1\rangle$$

- Then Bob learns which  
state was prepared by  
Alice

!!!But Eve can also learn the state by  
Measuring in same basis!!!

# QKD: Trick

- Alice randomly prepares

- Bob randomly measures in

$$\begin{array}{ccc} \text{bit}=0,1 & |0\rangle \pm |1\rangle & \xrightarrow{\text{Send to Bob}} & |0\rangle \pm |1\rangle & 0^\circ \text{ basis} \\ \text{bit}=0,1 & |0\rangle \pm i|1\rangle & & |0\rangle \pm i|1\rangle & 90^\circ \text{ basis} \end{array}$$

Now Eve is stymied.

In which basis to measure?

!!If she learns information, she disturbs the state!!

# QKD: Trick

- Alice randomly prepares

- Bob randomly measures in

$$\begin{array}{l} \text{bit}=0,1 \quad |0\rangle \pm |1\rangle \\ \text{bit}=0,1 \quad |0\rangle \pm i|1\rangle \end{array} \xrightarrow{\text{Send to Bob}} \begin{array}{l} |0\rangle \pm |1\rangle \quad 0^\circ \text{ basis} \\ |0\rangle \pm i|1\rangle \quad 90^\circ \text{ basis} \end{array}$$

Now Eve is stymied.

In which basis to measure?

!!If she learns information, she disturbs the state!!

Alice and Bob can obtain a secret key by revealing publicly at a later stage the basis used. If the basis are the same, the prepared and measured state constitute the secret key.

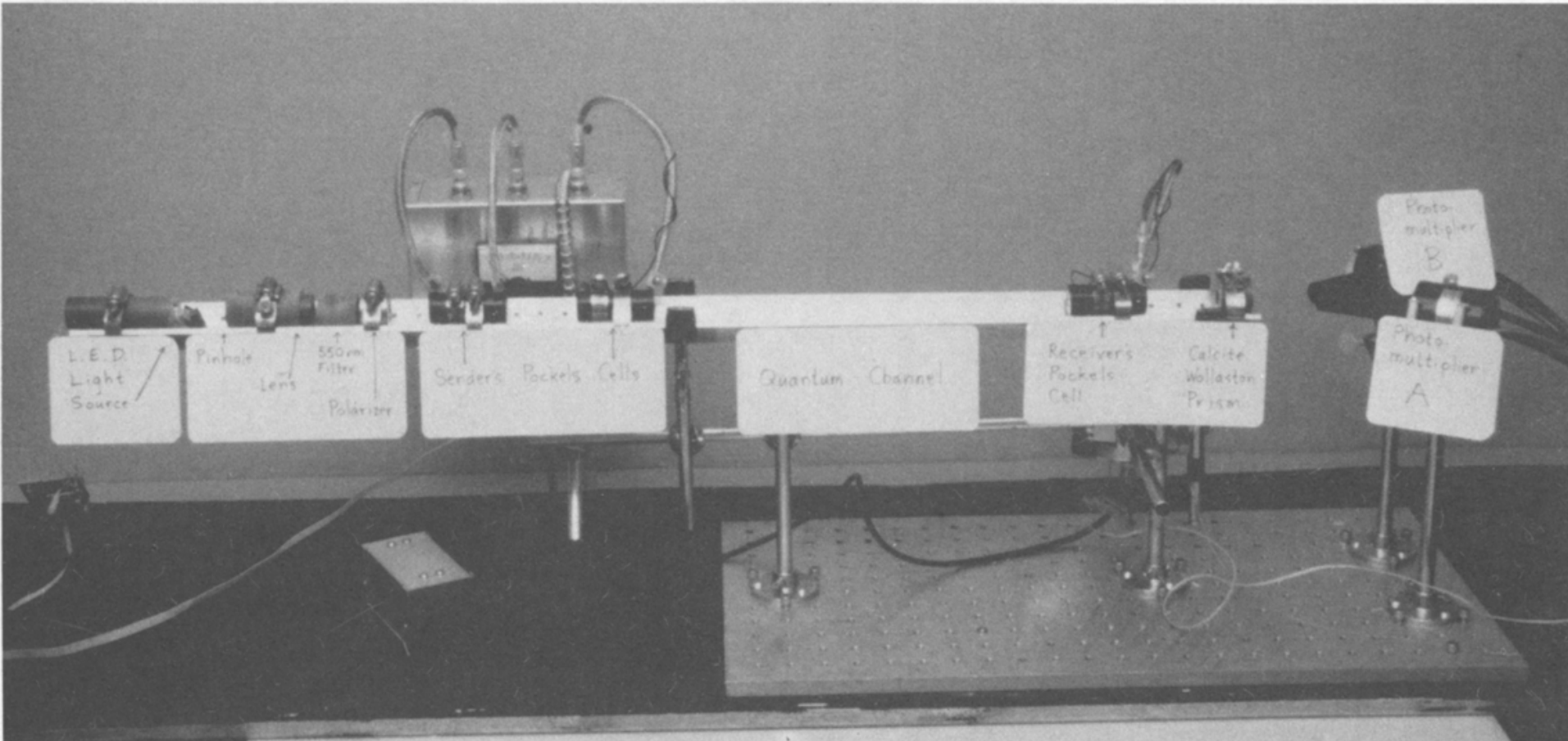
# QKD: Example

Emission $\varphi$	$0^\circ$	$90^\circ$	$180^\circ$	$90^\circ$	$0^\circ$	$270^\circ$	$180^\circ$	$0^\circ$	$90^\circ$
Bit Sent	0	0	1	0	0	1	1	0	0
Mst Basis	$90^\circ$	$90^\circ$	$0^\circ$	$90^\circ$	$0^\circ$	$0^\circ$	$0^\circ$	$90^\circ$	$0^\circ$
Mst Result	1	0	1	0	0	1	1	1	0
Key	X	0	1	0	0	X	1	X	X

- QKD needs single photon states
- In practice: Attenuated coherent states  
« The poor man's single photon source »

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle \approx \left(1 - \frac{|\alpha|^2}{2}\right) |0\rangle + \alpha |1\rangle + \dots$$

# First QKD Experiment



Propagation distance: 30cm

Key rate  $\approx$  1 bit/s

Journal of Cryptology 5, 3-28 (1992)



# Quantum Cryptography Today

- Key distribution over 50km of optical fiber
- Secret key rate: 1Mbit/s
- Continuous operation for 36hours
- Technique used: time bins

A. R. Dixon *et al.*, *Applied Physics Letters*, **96**, 161102 (2010)

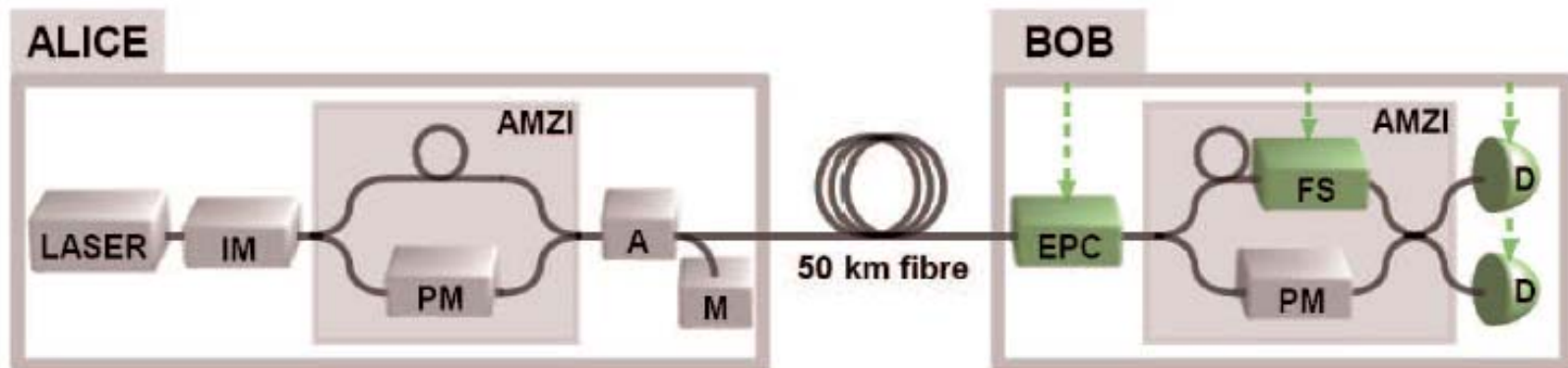
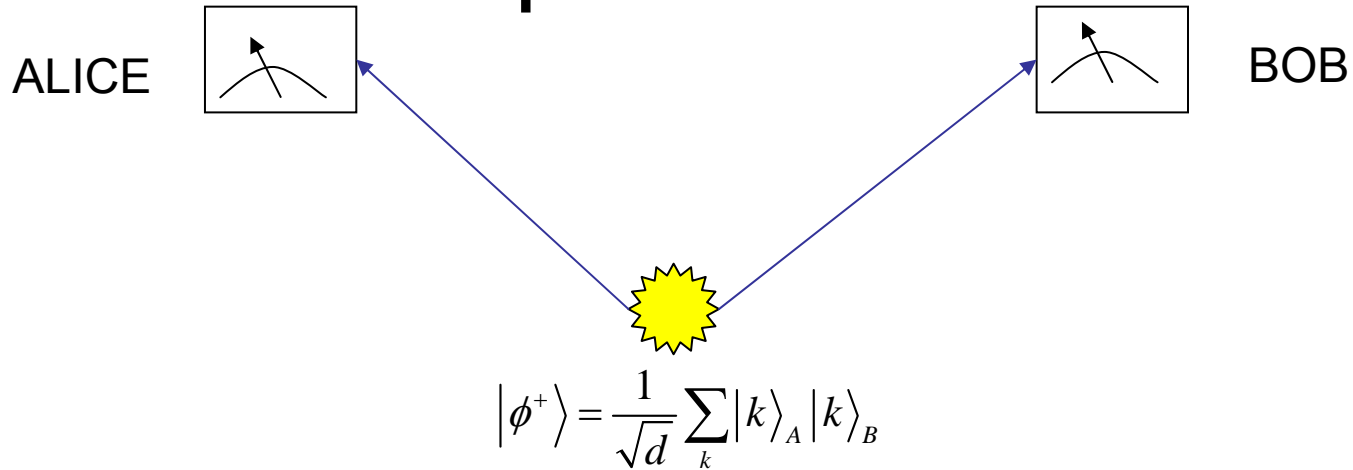
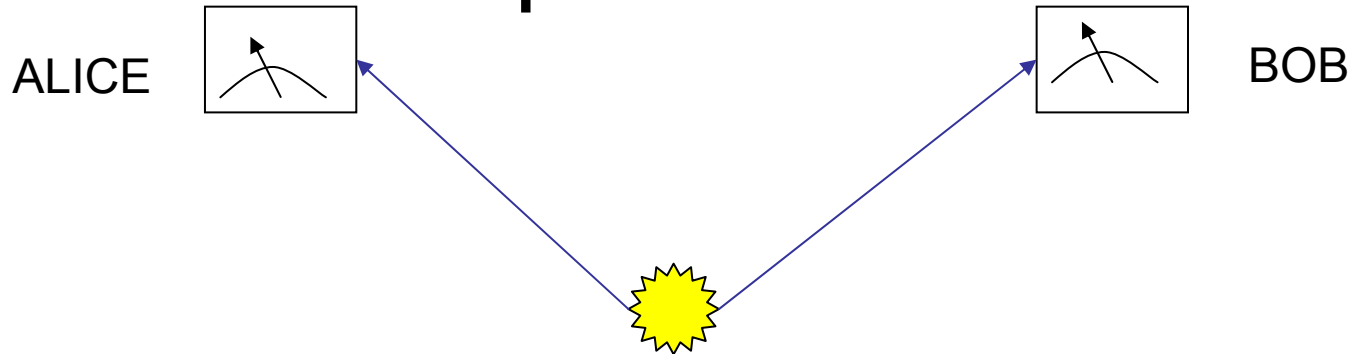


FIG. 1. Schematic of QKD system. IM denotes fiber intensity modulator, PM phase modulator, A attenuator, M optical power meter, EPC electrically-driven polarization controller, FS fiber stretcher, D InGaAs APD detectors. Components in green are feedback-controlled as part of the active stabilization system.

# Experiments with entangled photons



# Experiments with entangled photons

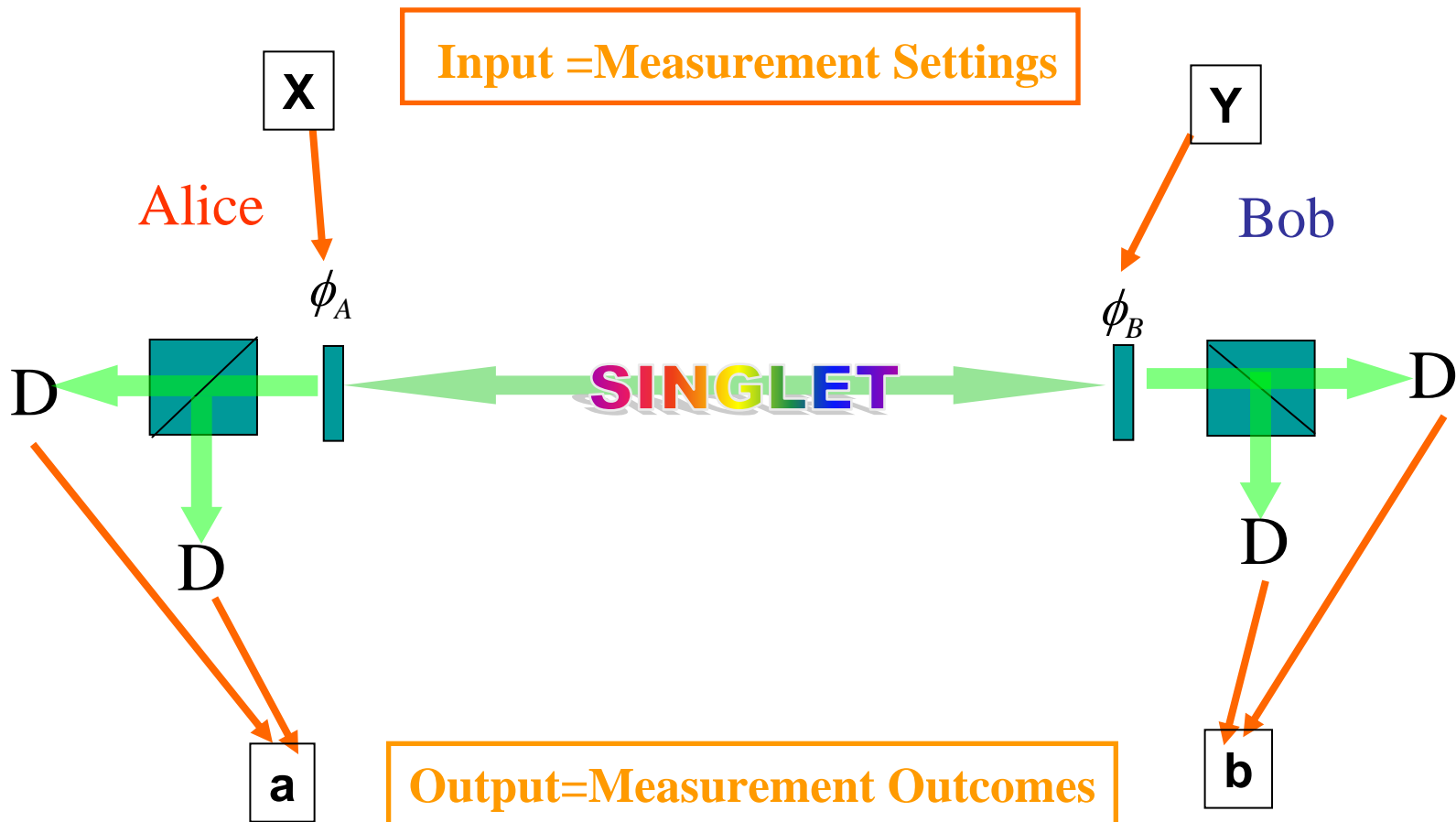


$$|\phi^+\rangle = \frac{1}{\sqrt{d}} \sum_k |k\rangle_A |k\rangle_B$$

## Why?

- Q. Comm. Over longer distances:
  - Slightly further than prepare and measure schemes
  - First step towards quantum repeaters
- Fundamental test of Nature:
  - Quantum Non Locality
  - Device Independent Quantum Cryptography

# Non Locality: Aspect type experiment



$$P(ab|XY) = P(\text{A outcome} \ \& \ \text{B outcome} \ | \ \text{A mst setting} \ \& \ \text{B mst setting})$$

# Implications of Non Locality

## Local Hidden Variable Model

$$P(ab | xy) = \int d\lambda P(\lambda) P(a | x\lambda) P(b | y\lambda)$$

If a lhv description is possible,  $P(ab|xy)$  satisfies all Bell inequalities

- local deterministic description of measurements is possible

If lhv description is impossible: (Quantum) Non Locality

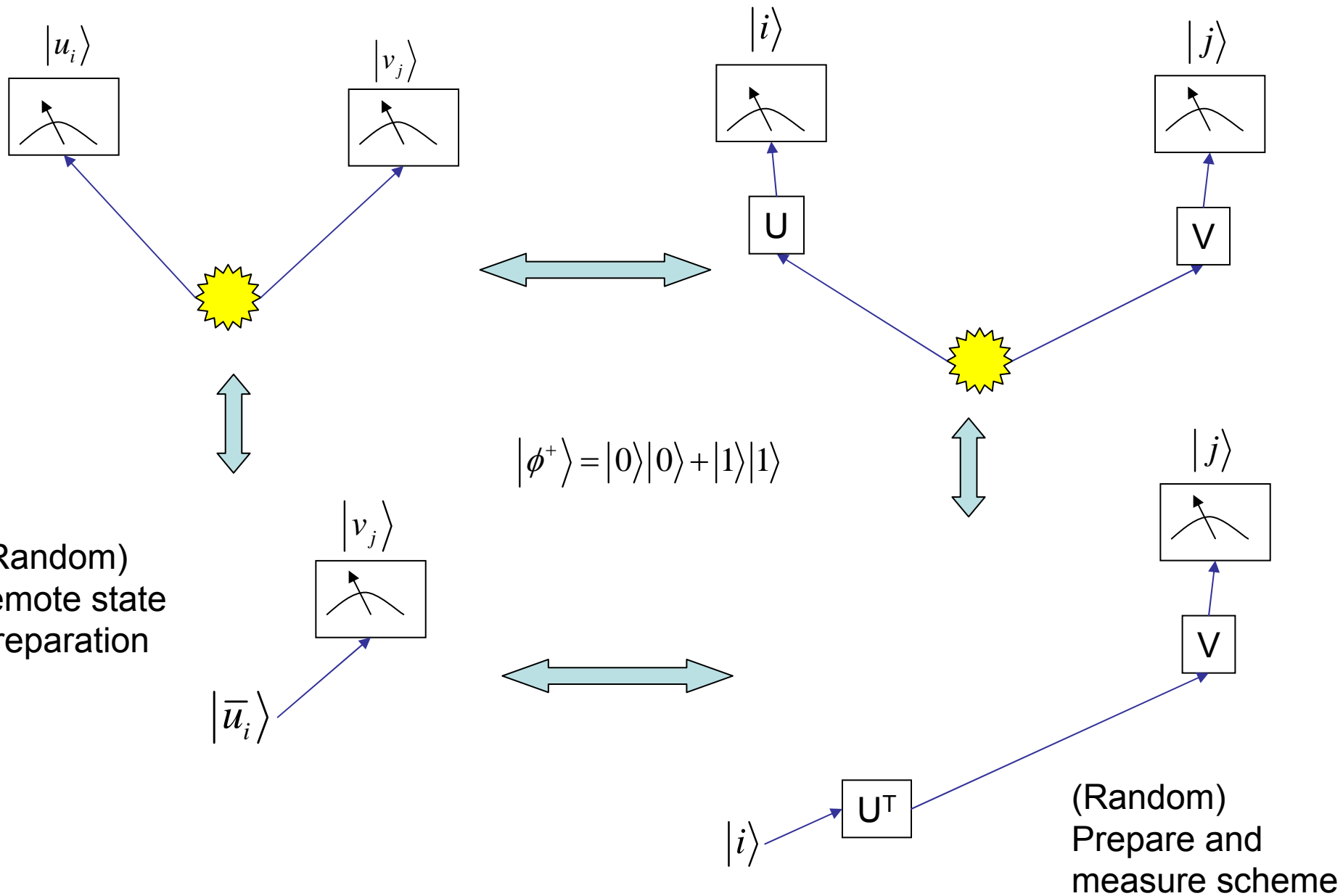
- measurements results are random, must be secret
- detected by Bell inequality violation

# Experiments with entangled particles

Equivalence with remote state preparation

Equivalence with prepare and measure

# Equivalences between schemes

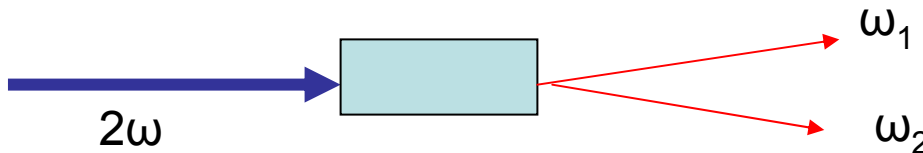


# Entangled Photon source

- Frequency Doubling



- Parametric Down Conversion

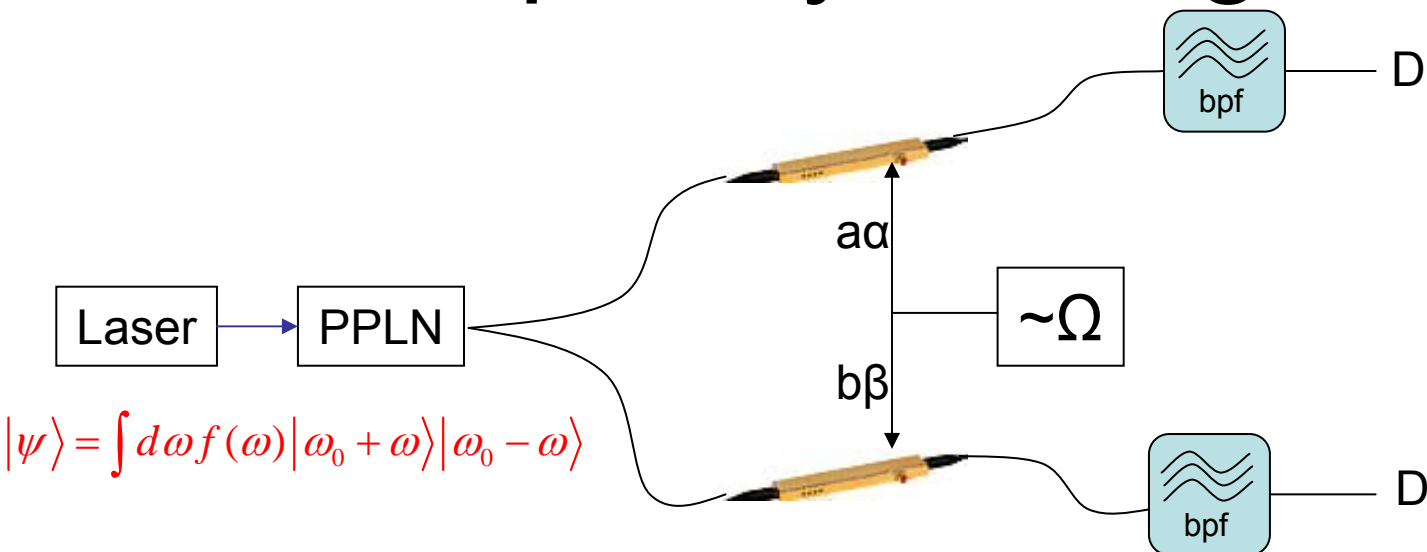


$\omega_1 + \omega_2 = 2\omega$  : Energy Conservation

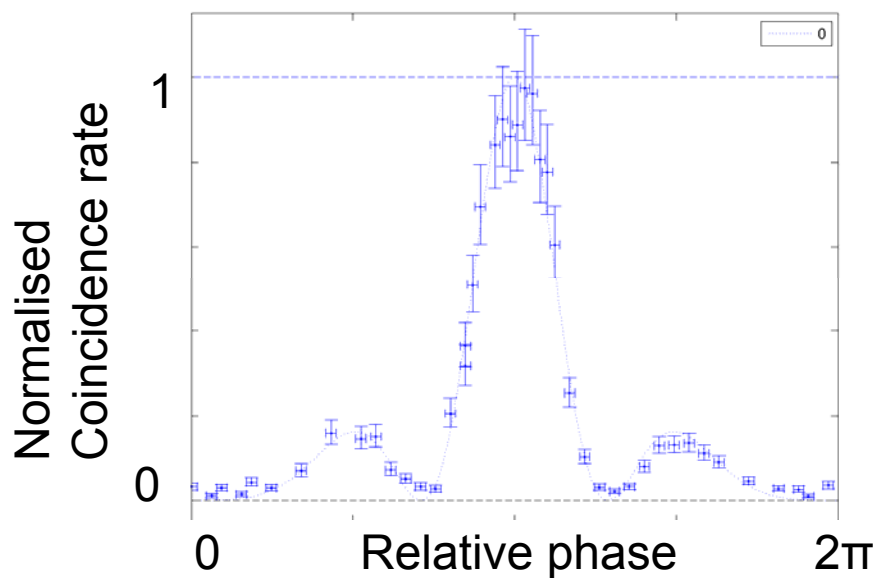
(approximate) Momentum Conservation (Phase Matching Condition)

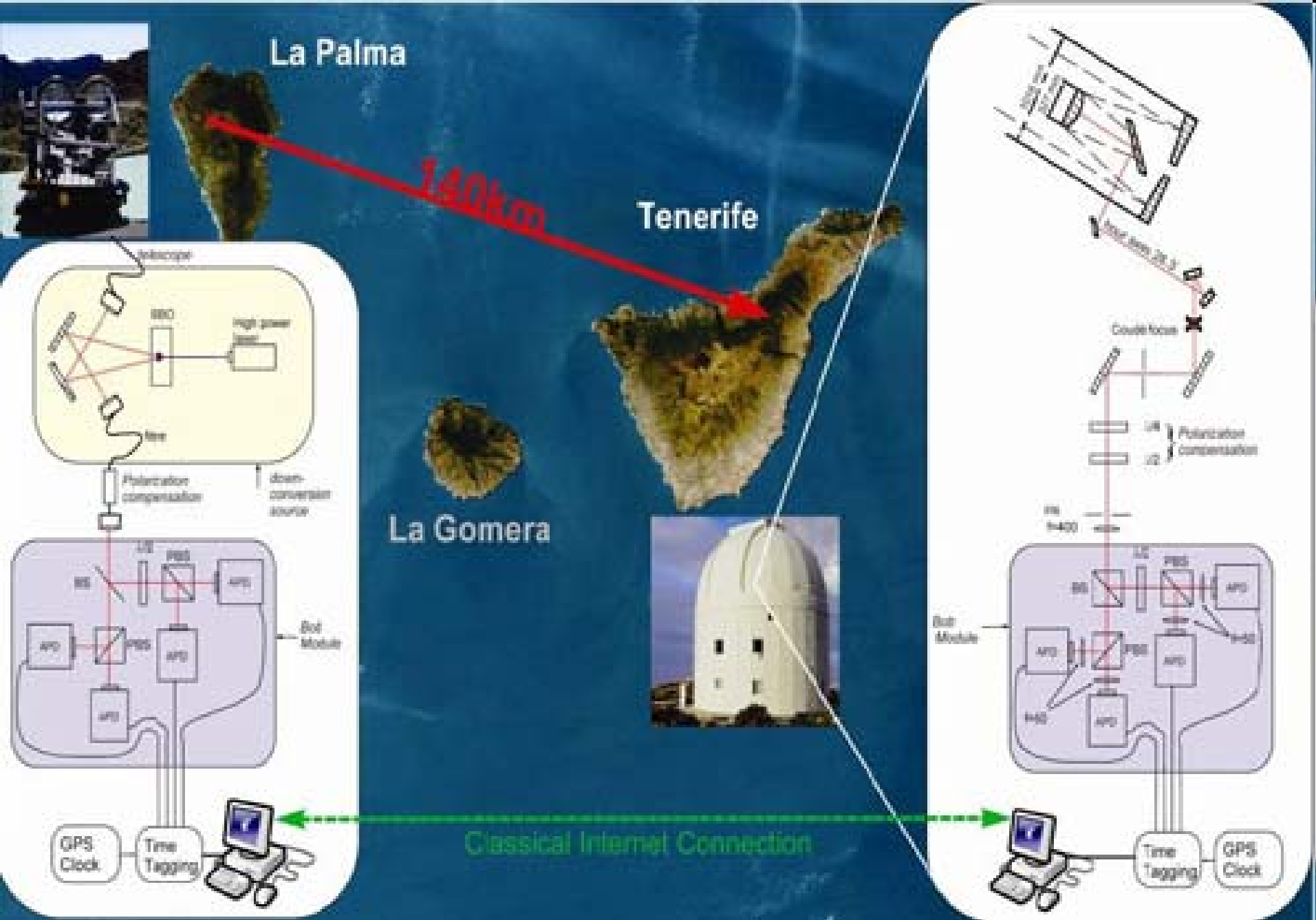


# Frequency Entanglement

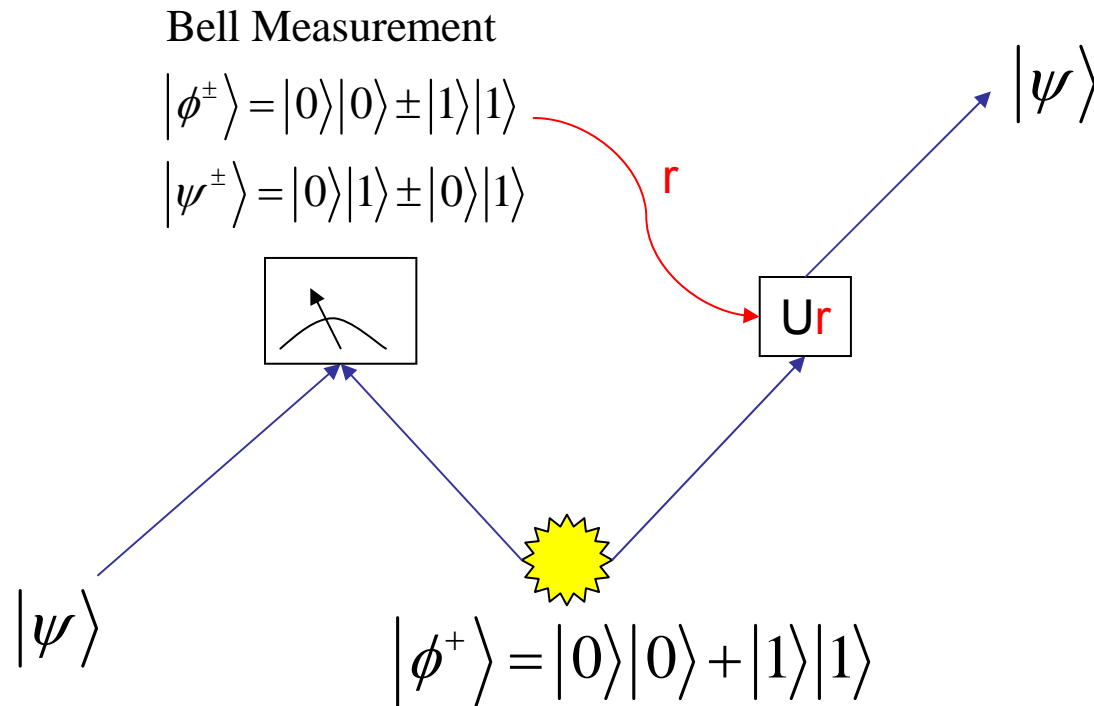


$$|\psi\rangle = \int d\omega f(\omega) |\omega_0 + \omega\rangle |\omega_0 - \omega\rangle$$

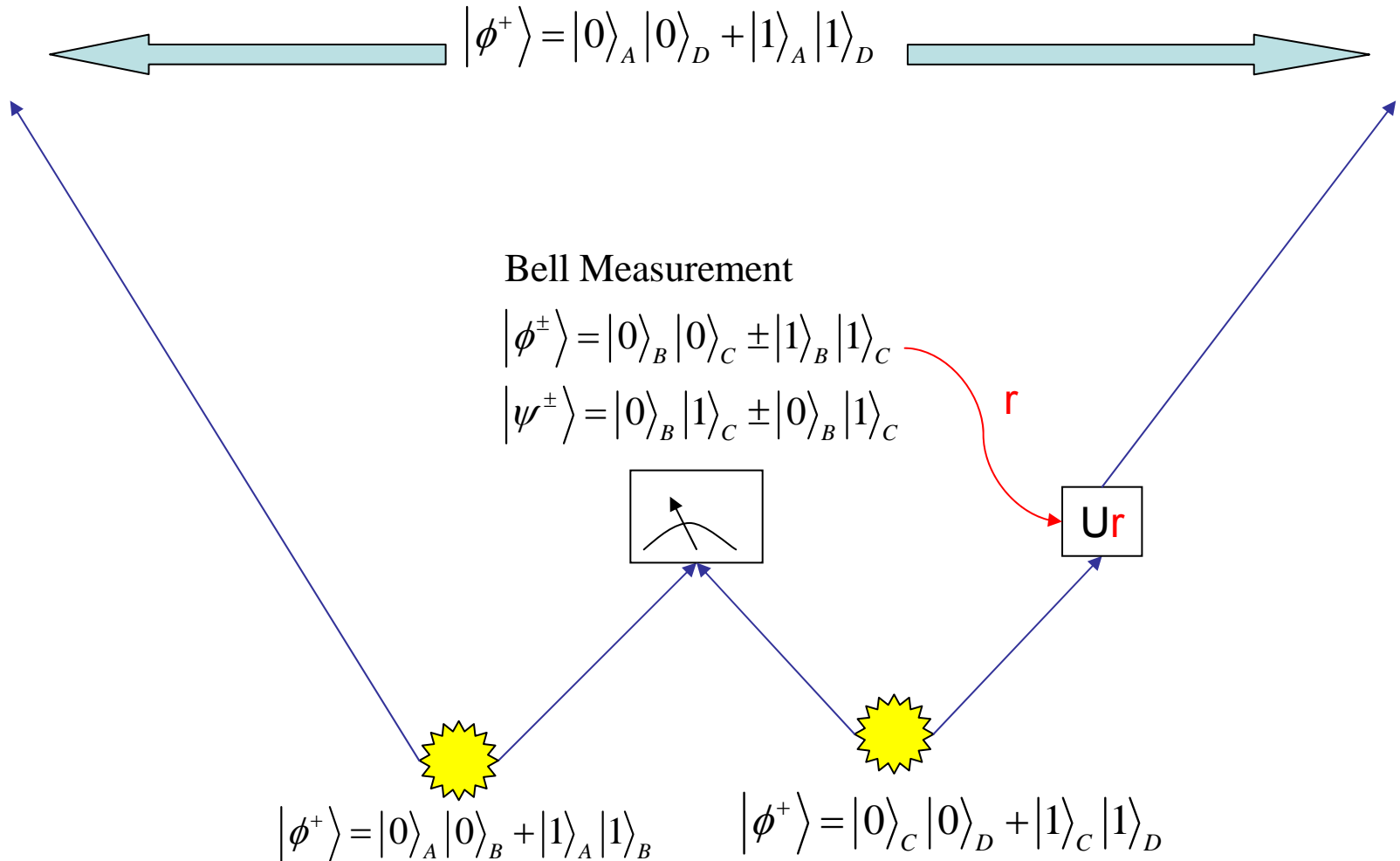




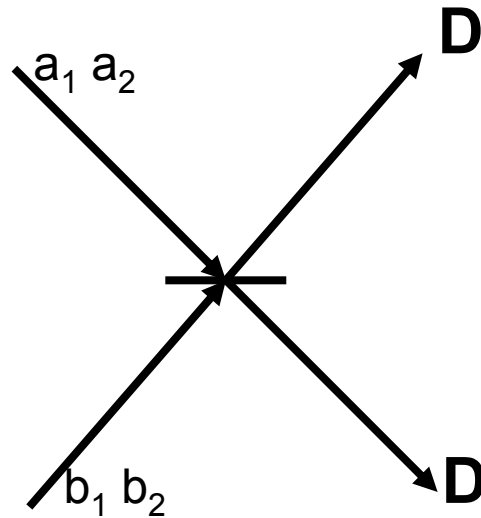
# Quantum Teleportation



# Entanglement Swapping



# Bell State Measurement with Photons



$$|\psi\rangle = (\alpha a_1^\dagger b_1^\dagger + \beta a_2^\dagger b_1^\dagger + \gamma a_1^\dagger b_2^\dagger + \delta a_2^\dagger b_2^\dagger) |0\rangle$$

Two photons

Two modes in beam a

Two modes in beam b

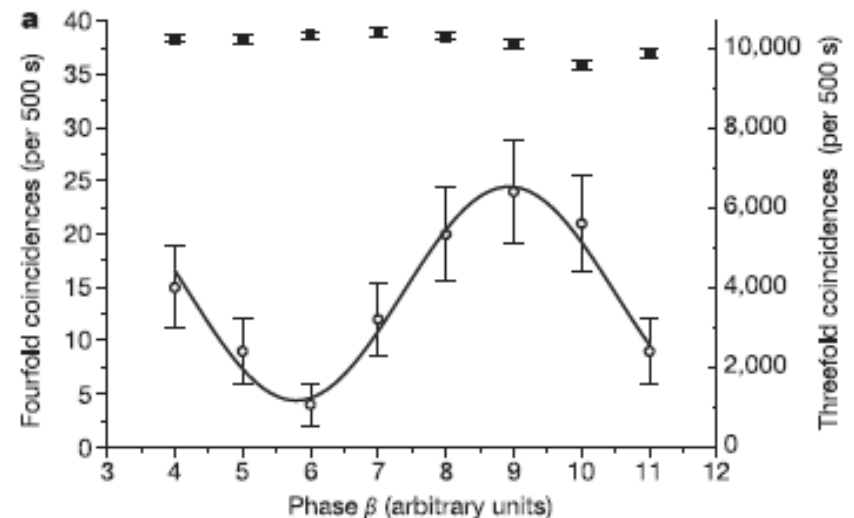
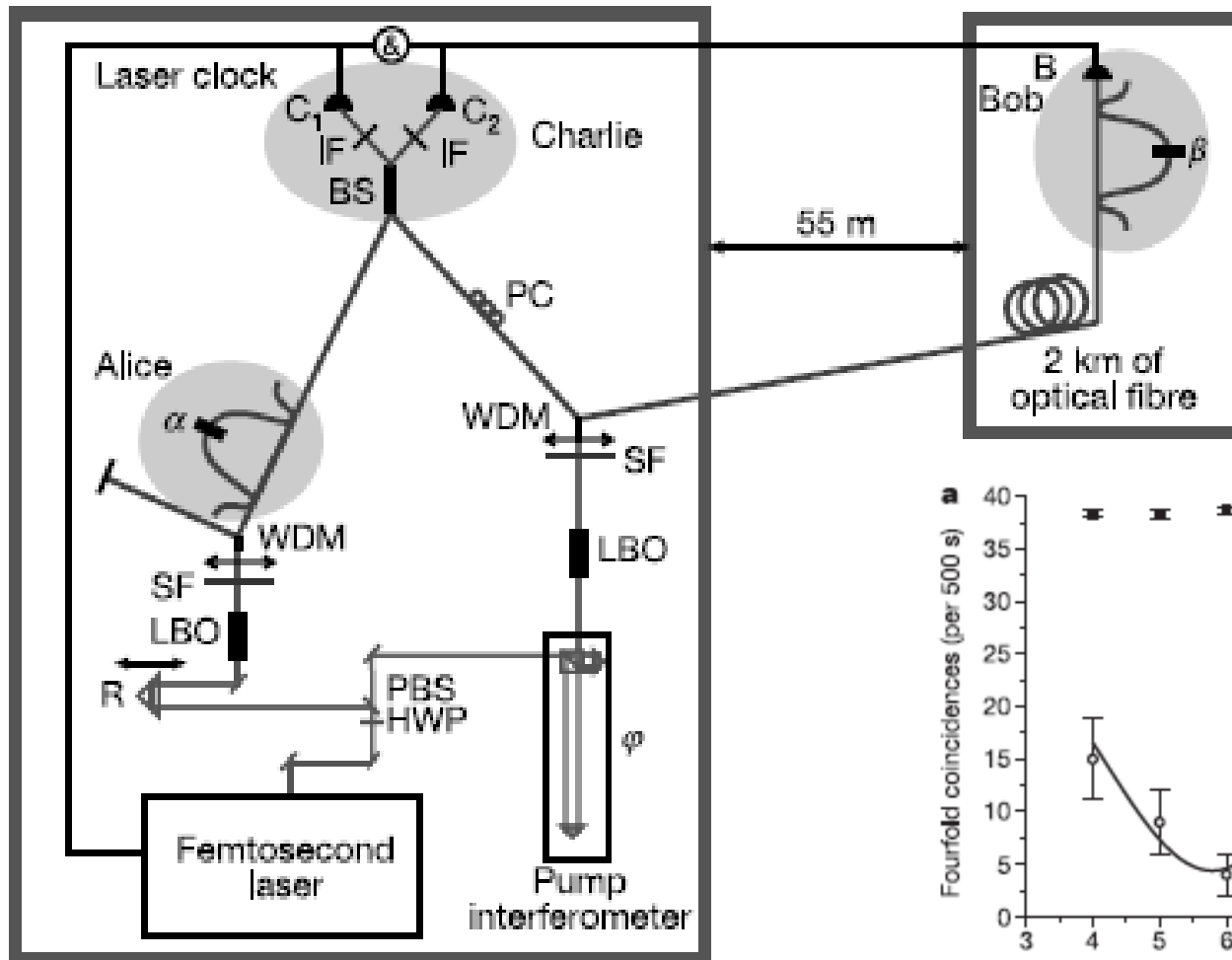
Coincident detection in both detectors implies that initial state was

$$|\psi\rangle = \left( \frac{1}{\sqrt{2}} a_1^\dagger b_2^\dagger - \frac{1}{\sqrt{2}} a_2^\dagger b_1^\dagger \right) |0\rangle$$

With probability 1/4 one measures a Bell state

Experimental Quantum Teleportation.  
 Telecommunication Wavelengths  
 distance 55m, passing through a spool of 2km optical fiber

*Nature* **421**, 509-513 (2003)



# Quantum Communication with atoms and photons.

## Entanglement of two Yb<sup>+</sup> ions

- Situated in 2 separate vacuum chambers separated by 1m
- 1 event every 10 minutes

### Advantages:

- Information can be stored
- Interfacable with quantum computer
- Detection loophole closed.

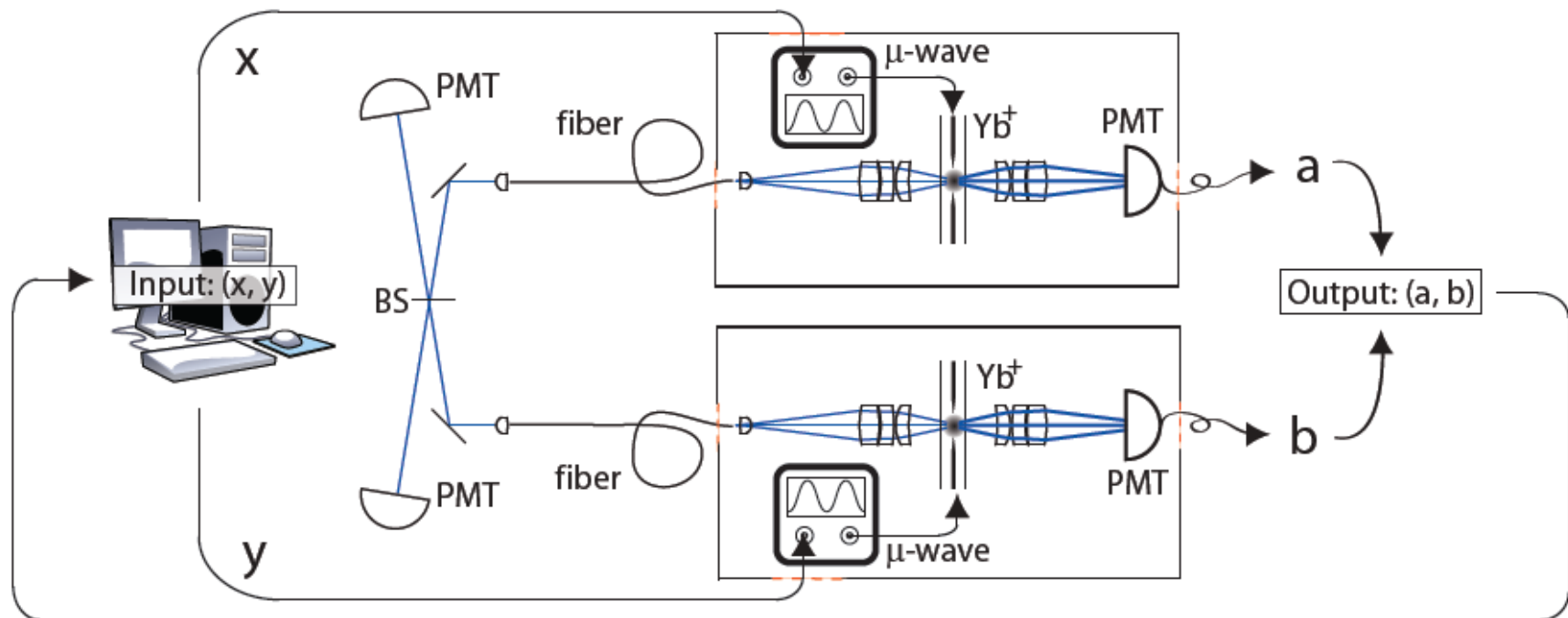
# Quantum Communication with atoms and photons.

## Entanglement of two Yb<sup>+</sup> ions

- Situated in 2 separate vacuum chambers separated by 1m
- 1 event every 10 minutes

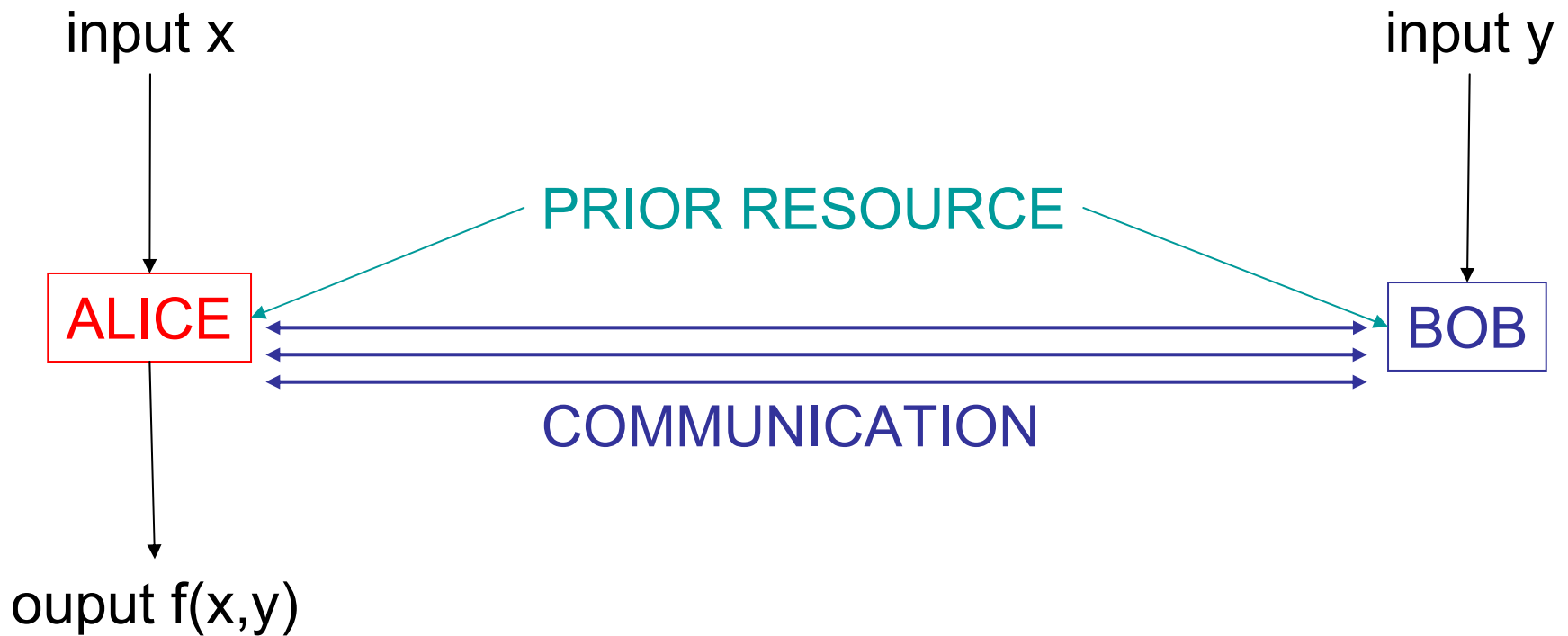
### Advantages:

- Information can be stored
- Interfacable with quantum computer
- Detection loophole closed.



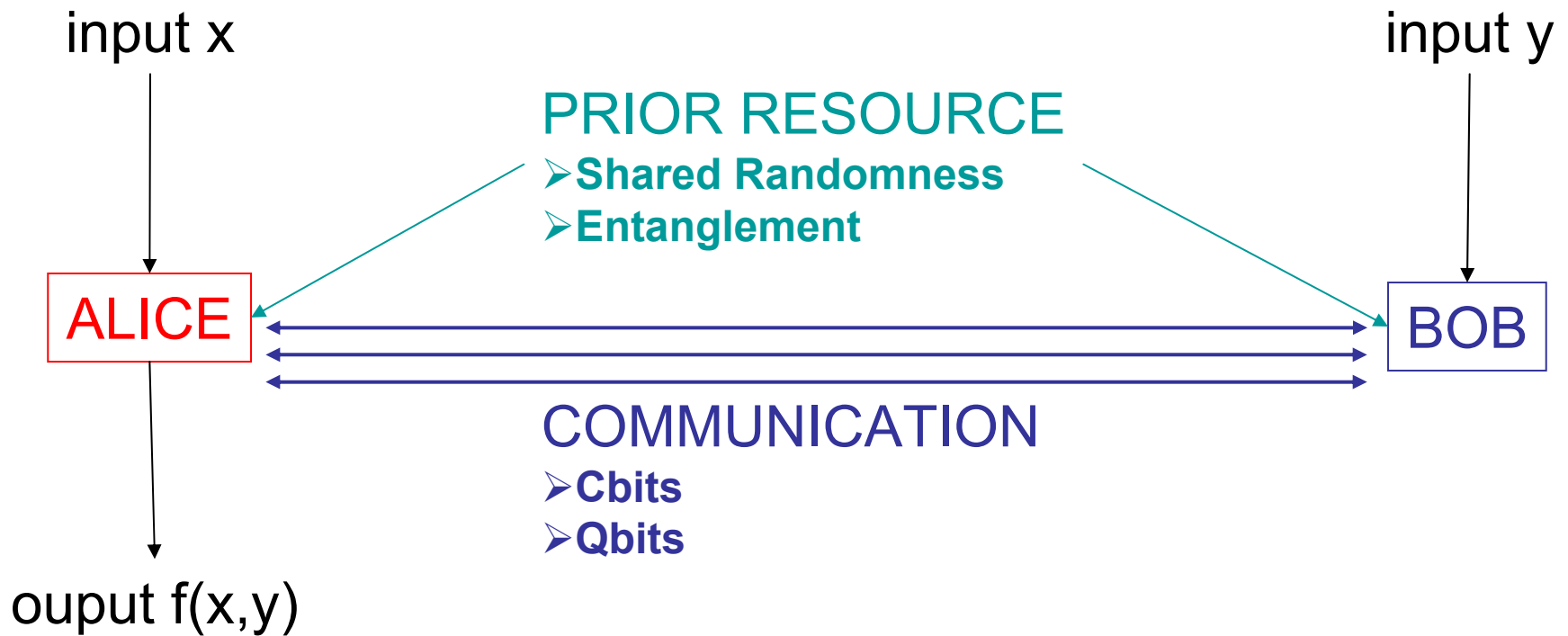


# Quantum Communication Complexity



TASK: Minimum Communication to provide the correct output

# Quantum Communication Complexity



**TASK:** Minimum Communication to provide the correct output

# Example: Equality



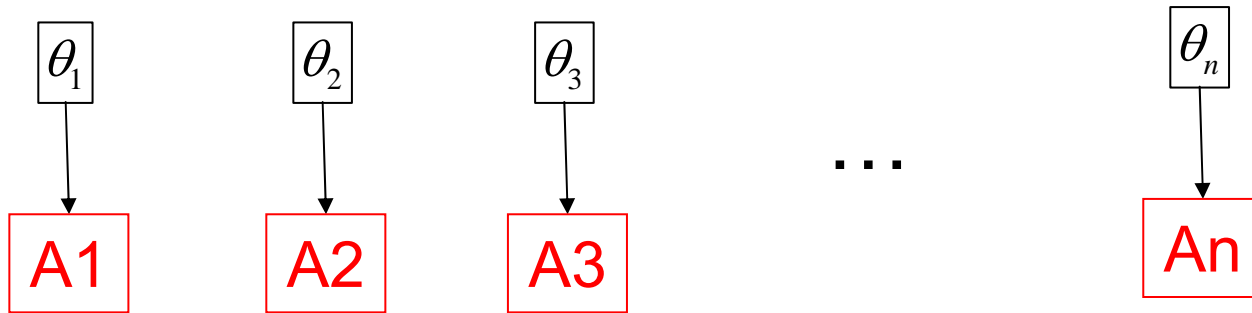
# Example: Equality



- No Error:
  - $n$  cbits of communication required
- Small Error probability & shared randomness
  - $\text{Log}(n)$  cbits of communication required
- Deutsch-Jozsa setting: either  $x=y$  or  $x$  differs from  $y$  in exactly  $n/2$  positions
  - $O(0.007n)$  cbits required
  - $\text{Log}(n)$  qubits required
  - $\text{Log}(n)$  ebits +  $\text{Log}(n)$  cbits

# Example: Sum mod $2\pi$

$$\text{PROMISE } \sum_i \theta_i = 0 \text{ or } \pi \pmod{2\pi}$$

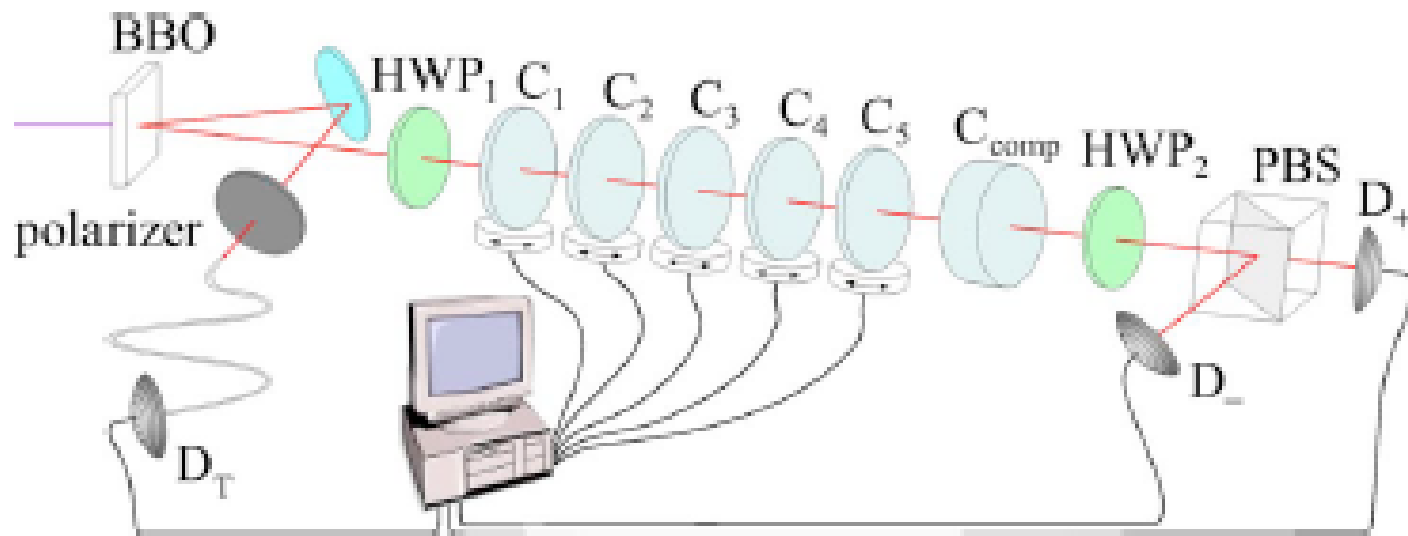


PRIOR RESOURCES  
COMMUNICATION

Question: is  $\sum_i \theta_i = 0 \text{ or } \pi \pmod{2\pi}$

- Bounded Error: requires  $O(n \log(n))$  cbits
- $n$  qubits
- 1GHZ state +  $n$  cbits

# Experimental Realisation of Sum mod $2\pi$



# Conclusion

## The future of Quantum Communication

- Faster
  - Better detectors
- Further
  - Via satellite (?)
  - Repeaters
- Interfacing with stationary qubits
  - Quantum memories for light
  - Error Correction