Quantum control for open quantum systems

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Open quantum systems and reduced density matrices

- In many fields of modern sciences, one has to deal with open quantum systems in contact with their surroundings or environments.
- Most often, one is concerned with only the system dynamics and the key quantity is the reduced system density matrix $\rho(t)$ defined as the partial trace of the total system-plus-reservoir density operator $\rho_T(t)$ over the reservoir degrees of freedom; i.e., $\rho(t) = \text{Tr}_R[\rho_T(t)]$.
- The evolution equation of the reduced density matrix is governed by the reduced Liouville equation or called the quantum master equation that can be Markovian or non-Markovian.

$$\dot{\rho}(t) = \mathcal{L}\rho(t); \quad \dot{\rho}(t) = \mathcal{L}(t)\rho(t); \quad \dot{\rho}(t) = \int_0^t \mathcal{L}(t-\tau)\rho(\tau)d\tau$$
Fighting against decoherence

• Various methods have been proposed to reduce the effect of unwanted interaction with environments, such as the quantum error-correction code, decoherence-free subspaces, dynamical decoupling techniques, quantum Zeno effect, quantum feedback control, quantum optimal control theory ...

• Here, I will describe two approaches to steer the dynamics of open quantum systems against decoherence.
  – closed-loop measurement-guided quantum feedback control
  – open-loop quantum optimal control theory
Outline

• Describe two approaches to steer the dynamics of two open quantum systems against decoherence.
  – The first one is to generate and stabilize entangled qubit states in a superconducting circuit cavity quantum electrodynamics (QED) system through closed-loop measurement-guided quantum feedback control.
  – The second one is to apply open-loop quantum optimal control theory to find the control sequences of high-fidelity quantum gates for qubits embedded in a non-Markovian environment.
Cavity QED

- **Cavity**: optical or microwave resonator constructed from highly reflecting mirrors or material walls.
- **QED (Quantum Electrodynamics)**: interaction of some material system (usually atomic) with the quantized electromagnetic modes (photons) inside the cavity.

\[
H = \hbar \omega_r (a^\dagger a + \frac{1}{2}) + \frac{\hbar \Omega}{2} \sigma_z \\
+ \hbar g (a^\dagger \sigma_- + a \sigma_+) \\
+ H_\kappa + H_\gamma
\]
Circuit QED

- 1D transmission line resonator consists of a full-wave section of superconducting coplanar wave guide.
- A Cooper-pair box qubit (an effective two-level atom) is placed between the superconducting lines and is capacitively coupled to the center trace at a maximum of the voltage standing wave, yielding a strong electric dipole interaction between the qubit and a single photon in the cavity.

A. Blais et al., PRA 69, 062320 (2004)

- Exceptionally small cavity volume (one million times smaller than 3D cavities)
- Large artificial atom size (10000 times larger than an atom)

Leading to strong coupling with $g \gg \kappa, \gamma$
Superconducting Cooper-pair Box

Condition: Two-level charge qubit

\[ \Delta_{gap} > E_c \]
\[ 4E_c \gg E_J \gg kT \]

"SQUID box" to vary \( E_J \)

\[ E_{el} = 4E_c \left( \frac{C_g V_g}{2e} - \frac{1}{2} \right) \]
\[ \hbar \Omega_E = \sqrt{E_{el}^2 + E_J^2} \]
\[ E_{J,eff} = E_J \cos(\pi\phi / \phi_0) \]
Transmon qubit

A very large dipole moment qubit with good coherence

Evolution of entanglement fidelity

\[ F = E \left( \langle \phi^+ | \psi(t) \rangle \langle \psi(t) | \phi^+ \rangle \right) \]

where \( |\phi^+\rangle = (|01\rangle + |10\rangle) / \sqrt{2} \)

300 quantum trajectories are generated all with the initially disentangled state

\[ |\psi_0\rangle = \frac{1}{2} (|00\rangle + |01\rangle + |10\rangle + |11\rangle) \otimes |\alpha\rangle \] with \( \alpha = 3 \)

\( \varepsilon = 100, \ \kappa = 100, \ \chi = 25, \ \lambda = 100, \ p = 3 \)

\( T = 2000 \times dt, \ dt = 0.0001, \ \Gamma = 0.003. \)

- The feedback scheme converge on the maximally entangled state \( |\phi^+\rangle \) in a short time with high probability.
- The measurement record indicates the state of the qubits, and therefore we can tell when we have converged on the entangled state.

Generation and stabilization of a three-qubit entangled W state in circuit QED via quantum feedback control

Shang-Yu Huang,¹,² Hsi-Sheng Goan,¹,², * Xin-Qi Li,³ and Gerard J. Milburn⁴

Circuit cavity quantum electrodynamics (QED) is proving to be a powerful platform to implement quantum feedback control schemes due to the ability to control superconducting qubits and microwaves in a circuit. Here, we present a simple and promising quantum feedback control scheme for deterministic generation and stabilization of a three-qubit W state in the superconducting circuit QED system. The control scheme is based on continuous joint Zeno measurements of multiple qubits in a dispersive regime, which enables us not only to infer the state of the qubits for further
Real-time quantum feedback prepares and stabilizes photon number states

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Generation & Stabilization

The $|W^-\rangle$ state can be generated from the states $|000\rangle, |W^-\rangle, |W^+\rangle, |111\rangle$ and stabilized by the feedback control.

$$F(t) = \text{Tr}\left[\rho(t)\rho_{W^-}\right]$$

$$|W^-\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$$
Quantum optimal control for open quantum systems

Goan, Hsi-Sheng

with Bin Hwang, Yi Chou, Tsun-Yi Yang, Jung-Shen Tai, Shang-Yu Huang, ...

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Quantum optimal control theory

- Quantum optimal control theory (QOCT) is a powerful tool that provides a variational framework for calculating the optimal shaped pulse to maximize a desired physical objective (or minimize a physical cost function) within certain constraints.

- The field of quantum optimal control has developed rapidly since the 1980s. The key aspects of the theory were formulated by Tannor, Rice, Kosloff, Shapiro, Brumer, Rabitz and ...

- (Quantum) optimal control theory:
  - Standard gradient optimization methods
  - A somewhat different approach: Krotov iterative method
Optimal control of the silicon-based donor-electron-spin quantum computing

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Near time-optimal CNOT gate control sequence

The minimum time sequence that meets the required fidelity is the near time-optimal control sequence.
Canonical decomposition of CNOT gate for global control e-spin QC

\[ U\left(\frac{\pi}{8}\right) = \exp\left[i\frac{\pi}{8}\sigma^1 e \cdot \sigma^2 e\right] \]

CNOT gate operation time: 297ns

Quantum control and manipulation

- Most of the control sequences implemented in quantum experiments are developed and designed based on the assumption of having ideal (closed) quantum coherent systems.
- However, almost every quantum system interacts inevitably with its surrounding environment resulting in decoherence and dissipation of the quantum system.
- Thus precisely controlling realistic open quantum systems is one of the most important and timely issues in the field of QIP.
Fighting against decoherence

• Various methods have been proposed to reduce the unwanted interaction with environments, such as the quantum error-correction code, quantum Zeno effect, and decoherence-free subspaces, quantum feedback control, ...

• **Dynamical decoupling techniques**: a succession of short and strong pulses to the system designed to suppress decoherence; (concatenated dynamical decoupling...)

• **Optimal control theory**: a powerful tool that allows us to design and realize accurate quantum gates by selecting optimal pulse shapes (arbitrarily shaped pulses and duration; or continuous dynamical modulation) for the external control within experimental capabilities.
Optimal Control of a Qubit Coupled to a Non-Markovian Environment

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A central challenge for implementing quantum computing in the solid state is decoupling the qubits from the intrinsic noise of the material. We investigate the implementation of quantum gates for a paradigmatic, non-Markovian model: a single-qubit coupled to a two-level system that is exposed to a heat bath. We systematically search for optimal pulses using a generalization of the novel open systems gradient ascent pulse engineering algorithm. Next to the known optimal bias point of this model, there are optimal pulses which lead to high-fidelity quantum operations for a wide range of decoherence parameters.

\[ H_S(t) = E_1(t) \sigma_z + \Delta \sigma_x + E_2 \tau_z + \Lambda \sigma_z \tau_z. \]
Non-Markovian spin-boson model

- Model:
  \[ H_S(t) = -\varepsilon(t)\sigma_z / 2 - \Omega\sigma_x / 2 \]
  \[ H_I = \sum_{\lambda} c_q \sigma_x (b_q + b_q^\dagger), \quad H_B = \sum_{\lambda} \hbar \omega_q b_q^\dagger b_q \]

- The perturbative non-Markovian open quantum system theory may be categorized into two classes:
  - time-nonlocal (time-convolution) method
  - time-local (or time-convolutionless) method.

- To second-order in the system-environment Hamiltonian (in the interaction picture)

\[ \frac{d\tilde{\rho}(t)}{dt} = -\frac{1}{\hbar^2} \text{Tr}_R \int_0^t dt' [\tilde{H}_I(t), [\tilde{H}_I(t'), \tilde{\rho}(t') \otimes R_0]] \]
\[ H_I = \sum_\lambda c_q \sigma_x (b_q + b_q^\dagger), \quad \tilde{H}_I(t) = U_S^\dagger(t) H_I U_S(t) = \tilde{\sigma}_x(t) B(t) \]

where \( \tilde{\sigma}_x(t) = U_S^\dagger(t) \sigma_x U_S(t), \quad U_S(t) = \mathcal{T} e^{(-i/\hbar) \int_0^t H_S(t') dt'} \),

\[ H_S(t) = -\varepsilon(t) \sigma_z / 2 - \Omega \sigma_x / 2, \]

\[ B(t) = \sum_q (c_q b_q e^{-i\omega_q t} + c_q a_q^\dagger e^{i\omega_q t}). \]

In the interaction picture:

\[
\dot{\rho}(t) = -\frac{1}{\hbar^2} \int_0^t \text{Tr}_B[\tilde{\sigma}_x(t) B(t), [\tilde{\sigma}_x(t') B(t'), \tilde{\rho}(t') \otimes R_0]] dt' \\
= -\frac{1}{\hbar^2} \int_0^t dt' \{ [\tilde{\sigma}_x(t) \tilde{\sigma}_x(t') \tilde{\rho}(t') - \tilde{\sigma}_x(t') \tilde{\rho}(t') \tilde{\sigma}_x(t)] \\
\times C(t - t') + [\tilde{\rho}(t') \tilde{\sigma}_x(t') \tilde{\sigma}_x(t) - \tilde{\sigma}_x(t) \tilde{\rho}(t') \tilde{\sigma}_x(t')] \\
\times C(t' - t) \}, \quad (A3)
\]
Bath CF:

\[ C(t - t') \equiv \text{Tr}_B[B(t)B(t')R_0] \]

\[ = \int_0^\infty d\omega J(\omega)\left[ (n(\omega) + 1)e^{-i\omega(t-t')} + n(\omega)e^{i\omega(t-t')} \right] \]

\[ = \int_0^\infty d\omega J(\omega) \cos[\omega(t - t')] \coth \left[ \frac{\hbar \omega}{2k_B T} \right] \]

\[ - i \int_0^\infty d\omega J(\omega) \sin[\omega(t - t')] , \quad (A4) \]

Transferring to the Schrödinger picture:

\[ J(\omega) = \sum_q |c_q|^2 \delta(\omega - \omega_q) \]

\[ \dot{\rho}(t) = -\frac{i}{\hbar} [H_S(t), \rho(t)] - \frac{i}{\hbar} \{[\sigma_x, \mathcal{K}(t)] - [\mathcal{K}^\dagger(t), \sigma_x] \}, \]

where

\[ \mathcal{K}(t) = -\frac{i}{\hbar} U_S(t) \left[ \int_0^t dt' C(t - t')\tilde{\sigma}_x(t')\tilde{\rho}(t') \right] U_S^\dagger(t), \]

control-dissipation correlation
Quantum master equation with control-dissipation correlation

\[
\dot{\rho}(t) = \mathcal{L}_S(t)\rho(t) + \left\{ \mathcal{L}_x \mathcal{K}(t) + [\mathcal{L}_x \mathcal{K}(t)]^\dagger \right\}
\]

where \( \mathcal{L}_S(t)A = (-i/\hbar)[H_S(t), A] \), \( H_S(t) = -\frac{\varepsilon(t)}{2}\sigma_z - \frac{\Omega}{2}\sigma_x \)

\( \mathcal{L}_x A = (-i/\hbar)[\sigma_x, A] \).

dissipator \( \mathcal{K}(t) = -\frac{i}{\hbar}\int_0^t dt' C(t-t') \mathcal{U}_S(t,t')\sigma_x \rho(t') \),

propagator \( \mathcal{U}_S(t,t') = \mathcal{T}_+ \exp \left[ \int_{t'}^t \mathcal{L}_S(\tau)d\tau \right] \),

bath CF \( C(t-t') = \int_0^\infty d\omega J(\omega) \cos[\omega(t-t')]\coth(\frac{\beta\hbar\omega}{2}) \)

\[-i\int_0^\infty d\omega J(\omega) \sin[\omega(t-t')] \].
Optimal control objective

- Performance (cost) function:

\[
J = F_{tr} - \int_{0}^{t_f} dt' \lambda(t')[\varepsilon(t') - \varepsilon_0(t')]^2
\]

\[
F_{tr} = \frac{1}{N} \text{Re} \left[ \text{Tr} \left\{ Q_D^\dagger \hat{G}(T) \right\} \right]
\]

- Penalty or constraint: control energy be minimal in balance with meeting the control objectives.
- \( \lambda(t) \) is a positive weight function that can be adjusted and chosen empirically.
- \( \varepsilon_0(t) \) is the standard control values can be chosen arbitrarily.
- Goal: reach a desired target (super)operation \( Q_D \) by maximizing the performance function (high fidelity and low energy cost) in a given operation time \( t_f = T \).
The Krotov optimization method

• A somewhat different QOCT approach from the standard gradient optimization methods is the Krotov iterative method.

• The Krotov method has several appealing advantages over the gradient methods:
  – monotonic increase of the objective with iteration number,
  – no requirement for a line search, and
  – macrosteps at each iteration.

The Krotov optimal control method algorithm

• **STEP 1:** Guess an initial control sequence

• **STEP 2:** Use equation of motion to find trajectory (forward propagator).

\[ \dot{\rho}(t) = \Lambda(t)\rho(t) \]

\[ \frac{\partial \tilde{G}(t,t')}{\partial t} = \hat{\Lambda}(t)\tilde{G}(t,t') \]

• **STEP 3:** Find an auxiliary functional (backward propagator).

\[ \frac{\partial \hat{B}(t,t')}{\partial t} = -\Lambda^{\dagger}(t)\hat{B}(t,t') \]

\[ \hat{B}(t = T) = \hat{Q}^{\dagger} \]

• **STEP 4:** Find the new control sequence

\[ \varepsilon_{k+1}(t) = \varepsilon_k(t) + \frac{1}{2\lambda(t)}\text{Re}[\text{Tr}\{\hat{B}_k(t) \frac{\partial \Lambda(t)}{\partial \varepsilon(t)} \hat{G}_{k+1}(t)\}] \]

• **STEP 5:** Repeat step 2 to step 4, until a preset fidelity (error) threshold is reached or a given number of iterations has been performed.
Quantum master equation with control-dissipation correlation

\[ \dot{\rho}(t) = \mathcal{L}_S(t)\rho(t) + \left\{ \mathcal{L}_x \mathcal{K}(t) + [\mathcal{L}_x \mathcal{K}(t)]^\dagger \right\} \]

where \( \mathcal{L}_S(t)A = (-i / \hbar)[H_S(t), A], \quad H_S(t) = -\frac{\varepsilon(t)}{2}\sigma_z - \frac{\Omega}{2}\sigma_x \)

\( \mathcal{L}_x A = (-i / \hbar)[\sigma_x, A]. \)

dissipator \( \mathcal{K}(t) = -\frac{i}{\hbar}\int_0^t dt' C(t - t')\mathcal{U}_S(t, t')\sigma_x \rho(t'), \)

propagator \( \mathcal{U}_S(t, t') = \mathcal{T}_+ \exp\left[\int_{t'}^t \mathcal{L}_S(\tau)d\tau\right], \)

bath CF \( C(t - t') = \int_0^\infty d\omega J(\omega)\cos[\omega(t - t')]\coth\left(\frac{\beta\hbar\omega}{2}\right) \)

\[ -i\int_0^\infty d\omega J(\omega)\sin[\omega(t - t')]. \]
Extended Liouville space and time-local equations of motion

- Expand the bath CF as a sum of exponential functions
  \[ C(t-t') = \sum_j C_j(t-t') = \sum_j C_j(0)e^{\gamma_j(t-t')} \]

- The dissipation operator: \( \mathcal{K}(t) = \sum_j \mathcal{K}_j(t) \)
  \[ \mathcal{K}_j(t) = -i/\hbar \int_0^t dt' C_j(0)e^{\gamma_j(t-t')} \mathcal{U}_s(t,t')\sigma_x\rho(t'), \]

- Replace dissipator \( \mathcal{K}(t) \) with auxiliary density matrix \( \mathcal{K}_j(t) \)
  \[ \dot{\rho}(t) = \mathcal{L}_S(t)\rho(t) + \left\{ \mathcal{L}_x\mathcal{K}(t) + [\mathcal{L}_x\mathcal{K}(t)]^\dagger \right\} \]
  \[ \Rightarrow \dot{\rho}(t) = \mathcal{L}_S(t)\rho(t) + \sum_j \left\{ \mathcal{L}_x\mathcal{K}_j(t) + [\mathcal{L}_x\mathcal{K}_j(t)]^\dagger \right\} \]

- Find the equation of motion for \( \mathcal{K}_j(t) \)
  \[ \dot{\mathcal{K}}_j(t) = (-i/\hbar)C_j(0)\sigma_x\rho(t) + [\mathcal{L}_S(t) + \gamma_j]\mathcal{K}_j(t) \]
  Form coupled linear local-in-time diff. eqs. for \( \{\rho, \mathcal{K}_j, \mathcal{K}_j^\dagger\} \)
Fitting the bath correlation function

\[ J(\omega) = \sum_{q} |c_q|^2 \delta(\omega - \omega_q) = \alpha \omega e^{-\omega/\omega_c} \]

\[ \omega_c = 7.5\Omega, \ k_B T = 0.2\Omega, \ \alpha = 0.1 \]

- Need only three or four exponential functions to directly expand the bath CF:

\[ C(t-t') = \int_{0}^{\infty} d\omega J(\omega) \cos[\omega(t-t')] \coth(\frac{\beta \hbar \omega}{2}) \]

\[ -i \int_{0}^{\infty} d\omega J(\omega) \sin[\omega(t-t')] \]
Extremely efficient to deal with the time-nonlocal non-Markovian equation of motion.

The required information one needs is the knowledge of the bath or noise spectral density which is experimentally accessible [e.g., see, J. Bylander et al., Nat. Phys. 7, 565 (2011)].
Error vs. time for an ideal Z-gate

Restriction: \( \varepsilon(t) \leq 30\Omega \)

- The inset shows the optimal control pulse for any \( \Omega t_f > 0.3 \).
Z-gate error and control pulse

P. Rebentrost et al.,

The static $\Delta \sigma_x$ induces at least a full loop around x axis.
State evolution of the ideal Z-gate with an initial $|+\rangle$ state

Our optimal control pulse strategy does not require a full loop around $x$-axis and thus can achieve a Z-gate in a much shorter operation time.


$$H_s(t) = -\frac{\varepsilon(t)}{2} \sigma_z - \frac{\Omega}{2} \sigma_x$$
**Z-gate in an Ohmic bath with** $\omega_c = 20\Omega$

$$J(\omega) = \sum_{q} |c_q|^2 \delta(\omega - \omega_q) = \alpha \omega e^{-\omega/\omega_c}$$

The error \( \| Q_D - \hat{G}(T) \|_F^2 / 2 \mathcal{N} = \text{Tr} \left[ (Q_D - \hat{G}(T))^\dagger (Q_D - \hat{G}(T)) \right] / 2 \mathcal{N} \) (Frobenius norm) increases when the operation time become longer.

It is possible to achieve high-fidelity Z-gates with errors < 10^{-5} at large times.

The longer bath correlation time allows the optimal control sequence to counteract and suppress the contribution from the bath.
The error increases as cutoff frequency $\omega_c$ becomes bigger.

Memory effect plays an important role in determining the gate error.

The optimal control sequence that suppresses the decoherence induced by bath is totally different from that of the ideal unitary case.
Error of identity-gate

- Achieving a high-fidelity identity gate at long times implies having the capability for arbitrary state preservation, i.e., restoring arbitrary state robustly against the bath.
- The gate errors are expected to be much lower if one has independent control over both $\sigma_z$ and $\sigma_x$ terms in the qubit Hamiltonian and if there is no restriction on the control parameter strengths.

$\omega_c = \Omega$
More works on quantum optimal control

• Two-qubit gates taking into account more realistic bath spectral densities.

• One-qubit gates with an exact master equation.

• A small number of noise qubits interacting with one or more system qubits (full Hamiltonian approach; also including influence from baths).

Refs:


Quantum Noise in the Josephson Charge Qubit

O. Astafiev,¹,* Yu. A. Pashkin,¹,† Y. Nakamura,¹,² T. Yamamoto,¹,² and J. S. Tsai¹,²

\[ J(\omega) = \sum_q |c_q|^2 \delta(\omega - \omega_q) \]

\[ S(\omega) = J(\omega) \coth\left(\frac{\hbar \omega}{2kT}\right) \]
An open quantum system model with an exact master equation

\[ H_{\text{tot}} = H_s + H_B + H_{SB} \]

\[ H_s = \frac{\omega + \varepsilon(t)}{2} \sigma_z \equiv \frac{\omega(t)}{2} \sigma_z \]

\[ H_B = \sum_{\lambda} \omega_{\lambda} a_{\lambda}^+ a_{\lambda} \]

\[ H_{SB} = \sum_{\lambda} (g_{\lambda}^* \sigma_- a_{\lambda}^+ + g_{\lambda} \sigma_+ a_{\lambda}) \]

\[ \alpha(t-s) \equiv \sum_{\lambda} |g_{\lambda}|^2 e^{-i\omega_{\lambda}(t-s)} \]

\[ \dot{\rho}_t = -\frac{i\omega}{2} [\sigma_z, \rho_t] + (F(t) + F^*(t)) \left( \sigma_- \rho_t \sigma_+ - \frac{1}{2} \{\sigma_+ \sigma_-, \rho_t\} \right) + \left( \frac{F(t) - F^*(t)}{2} \right) [\sigma_+ \sigma_-, \rho_t] \]

\[ F(t) \equiv \int_0^t \alpha(t-s)f(t,s)ds \]

\[ \partial_t f(t,s) = [i\omega + F(t)] f(t,s) \]

\[ f(s,s) = 1. \]
Model

- \( H = H_s + H_c(t) + H_I + H_B \)
- System:
  \[
  H_s = \sum_{i=1}^{2} \left[ -\left( \frac{E_m}{4} + \frac{E_c^i}{4} \right) \sigma_z^i - \frac{E_j^i}{2} \sigma_x^i \right] + \frac{E_m}{4} \sigma_z^1 \sigma_z^2
  \]
- Control:
  \[
  H_c(t) = \left( \frac{E_m}{2} n^2(t) + \frac{E_c^1}{2} n^1(t) \right) \sigma_z^1 + \left( \frac{E_m}{2} n^1(t) + \frac{E_c^2}{2} n^2(t) \right) \sigma_z^2
  \]
  \[
  n^1(t), n^2(t) \in [0,1]
  \]
  Since \( n^i = \frac{V_g^i c_g^i}{2e} \) is the number of the tunneling cooper pair.
Implementation of CNOT gate in NV center

Blue #1: NV electron spin
Blue #2: $^{13}\text{C}$ nuclear spin
Red: $^{15}\text{N}$ nuclear spin
Bath: from other spins

$$G = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$
How to generate these optimal control pulses experimentally?

• Accurately determination of the parameters in Hamiltonian.
• Calibration of the system response with amplifiers and control fields.
• The optimal pulse sequences look experimentally challenging, but not impossible.
• Commercial devices (e.g., Tektronix AWG70001A) for generating arbitrary wave forms with 10 bits of vertical resolution at a sample rate of 50 GSa/s, a bit rate of 12.5 Gb/s and a rise/fall time smaller than 27 ps are now available.
• Such a device should enable generation of complex signals in a time scale of sub-nanoseconds to nanoseconds.
• In principle the pulses could also be improved using closed-loop optimization where measurement data is immediately fed back to the optimizer to improve the pulses without full knowledge of the system.
Summary I

• A universal and efficient QOCT based on the Krotov method for a time-nonlocal non-Markovian open quantum system has been presented and applied to obtain control sequences and gate errors for Z and identity gates.

• The control-dissipation correlation and the memory effect of the bath are crucial in achieving high-fidelity gates.

• Recent experiments on measuring noise spectral density, [e.g., J. Bylander et al., Nat. Phys. 7, 565 (2011)], engineering external environment, simulating open quantum system, and observing non-Markovian dynamics could facilitate the experimental realization of the QOCT in non-Markovian open quantum systems in the near future.
Summary II

- The 3-qubit entangled $|W\rangle$ state can be generalized and stabilized with high fidelity in the circuit QED system in which the cooper pair box qubits are fixed in the transmission line resonator.

- This quantum feedback control scheme is simple, and promising. The scheme is robust against measurement inefficiency and differences in qubit decay rates.