Quantum control for open quantum systems

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Open quantum systems and reduced density matrices

- In many fields of modern sciences, one has to deal with open quantum systems in contact with their surroundings or environments.
- Most often, one is concerned with only the system dynamics and the key quantity is the reduced system density matrix $\rho(t)$ defined as the partial trace of the total system-plus-reservoir density operator $\rho_T(t)$ over the reservoir degrees of freedom; i.e., $\rho(t)=\mathrm{Tr}_R[\rho_T(t)]$.
- The evolution equation of the reduced density matrix is governed by the reduced Liouville equation or called the quantum master equation that can be Markovian or non-Markovian.

$$\dot{\rho}(t)=\mathcal{L}\rho(t);\ \ \dot{\rho}(t)=\mathcal{L}(t)\rho(t);\ \ \dot{\rho}(t)=\int_0^t \mathcal{L}(t-\tau)\rho(\tau)d\tau$$

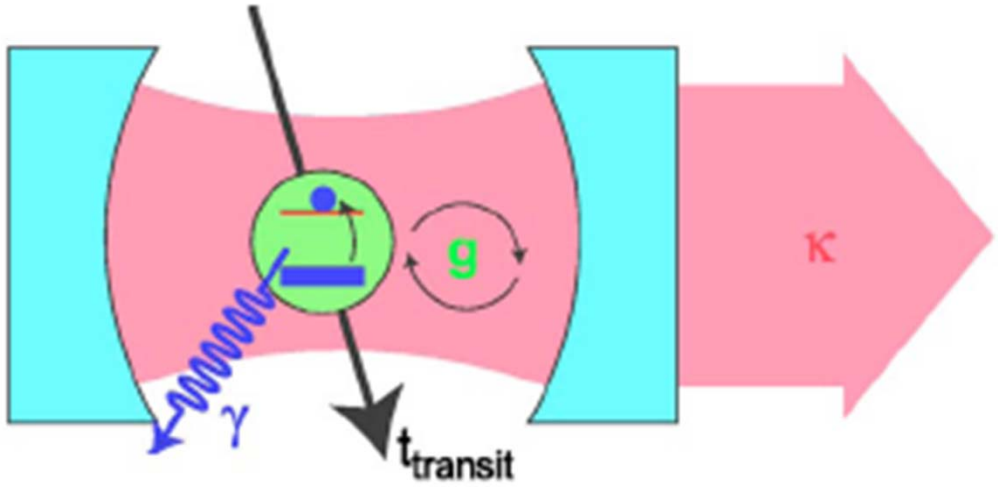
Fighting against decoherence

- Various methods have been proposed to reduce the effect of unwanted interaction with environments, such as the quantum error-correction code, decoherence-free subspaces, dynamical decoupling techniques, quantum Zeno effect, quantum feedback control, quantum optimal control theory …
- Here, I will describe two approaches to steer the dynamics of open quantum systems against decoherence.
  - closed-loop measurement-guided quantum feedback control
  - open-loop quantum optimal control theory

Outline

- Describe two approaches to steer the dynamics of two open quantum systems against decoherence.
  - The first one is to generate and stabilize entangled qubit states in a superconducting circuit cavity quantum electrodynamics (QED) system through closed-loop measurement-guided quantum feedback control.
  - The second one is to apply open-loop quantum optimal control theory to find the control sequences of high-fidelity quantum gates for qubits embedded in a non-Markovian environment.

Cavity QED



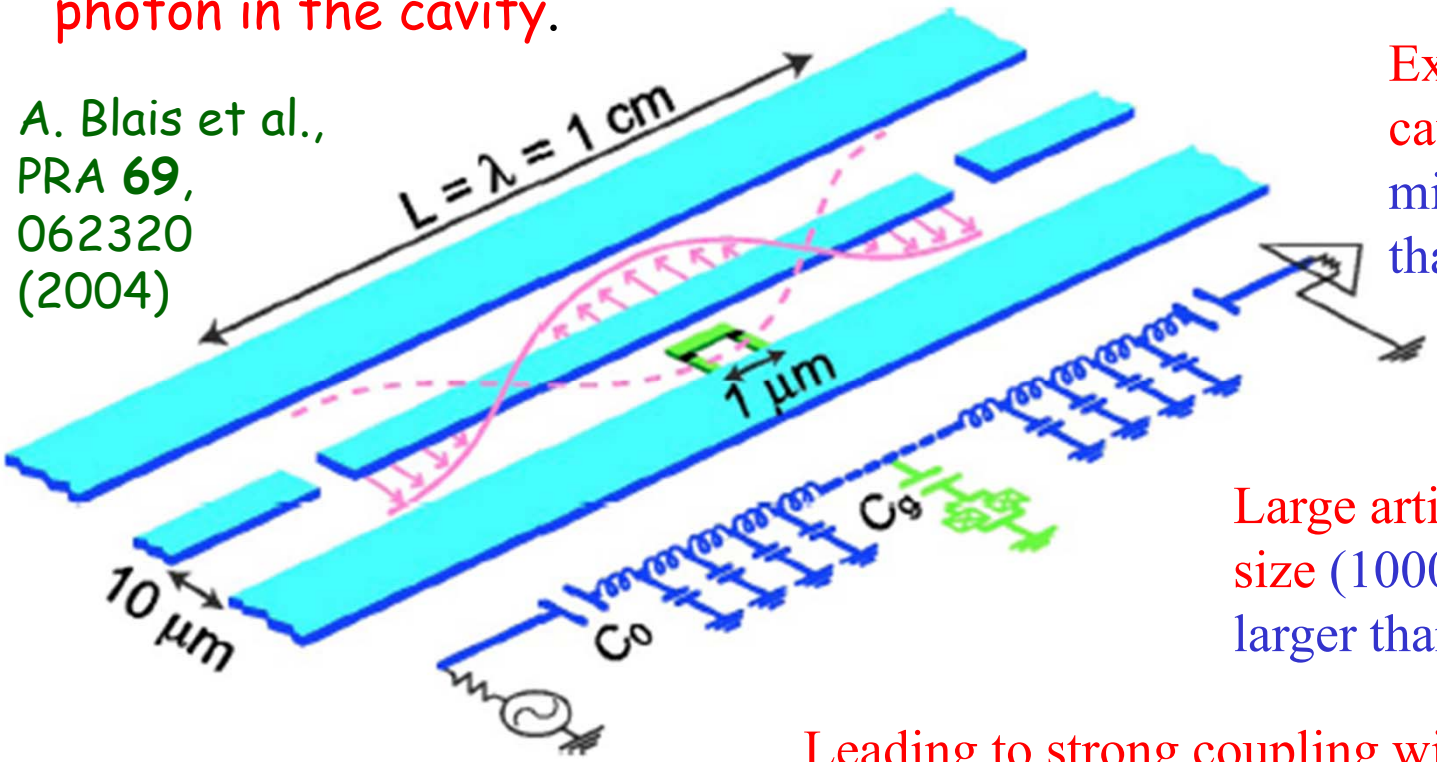
$$H = \hbar\omega_r(a^\dagger a + \frac{1}{2}) + \frac{\hbar\Omega}{2}\sigma_z$$
$$+ \hbar g(a^\dagger \sigma_- + a\sigma_+)$$
$$+ H_\kappa + H_\gamma$$

- Cavity: optical or microwave resonator constructed from highly reflecting mirrors or material walls.
- QED (Quantum Electrodynamics): interaction of some material system (usually atomic) with the quantized electromagnetic modes (photons) inside the cavity.

Circuit QED

- 1D trasmission line resonator consists of a full-wave section of superconducting coplanar wave quide.
- A Cooper-pair box qubit (an effective two-level atom) is placed between the superconducting lines and is capacitively coupled to the center trace at a maximum of the voltage standing wave, yielding a strong electric dipole interaction between the qubit and a single photon in the cavity.

A. Blais et al., PRA **69**, 062320 (2004)

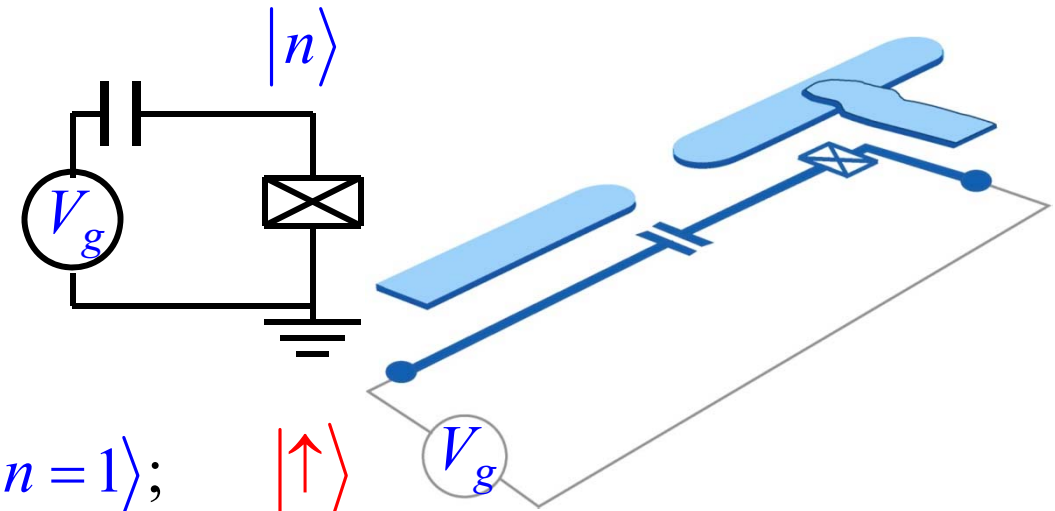


Exceptionally small cavity volume (one million times smaller than 3D cavities)

Large artificial atom size (10000 times larger than an atom)

Leading to strong coupling with $g \gg \kappa, \gamma$

Superconducting Cooper-pair Box



Condition: Two-level charge qubit

$$\Delta_{gap} > E_c$$

$$4E_c \gg E_J \gg kT$$

$|n=1\rangle$; $|\uparrow\rangle_z$

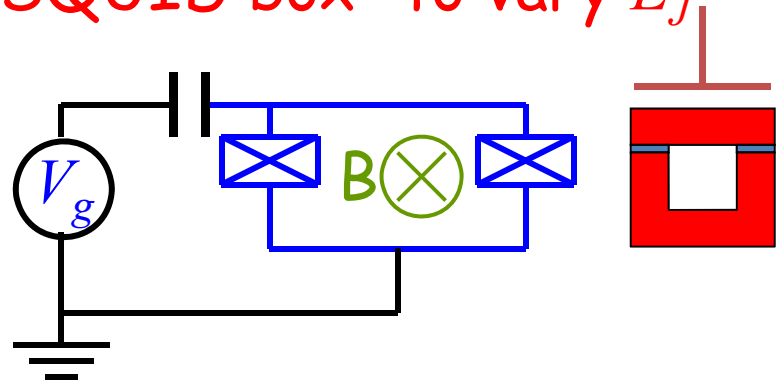
$|n=0\rangle$; $|\downarrow\rangle_z$

$$\hat{H} = \frac{E_{el}}{2}\hat{\sigma}_z + \frac{E_J}{2}\hat{\sigma}_x = \frac{\hbar\Omega_E}{2}\sigma_z^e$$

$$E_{el} = 4E_c\left(\frac{C_g V_g}{2e} - \frac{1}{2}\right)$$

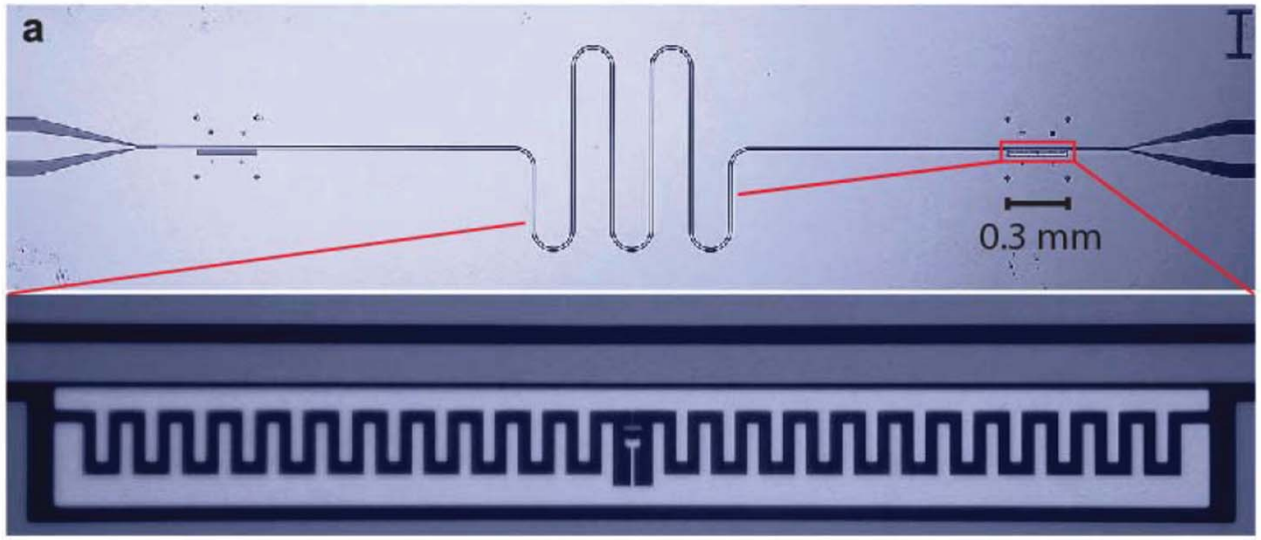
$$\hbar\Omega_E = \sqrt{E_{el}^2 + E_J^2}$$

"SQUID box" to vary E_J

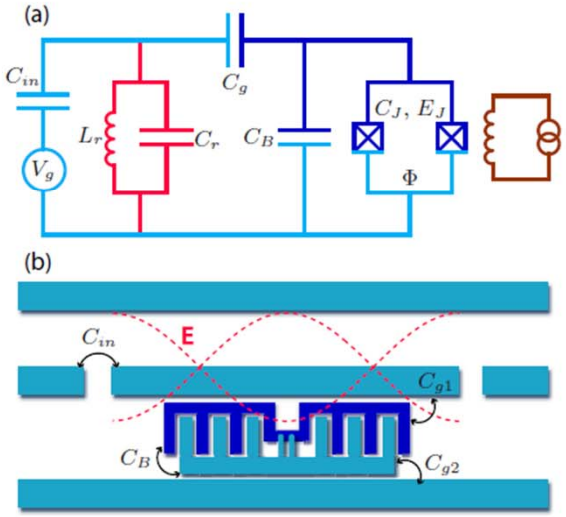


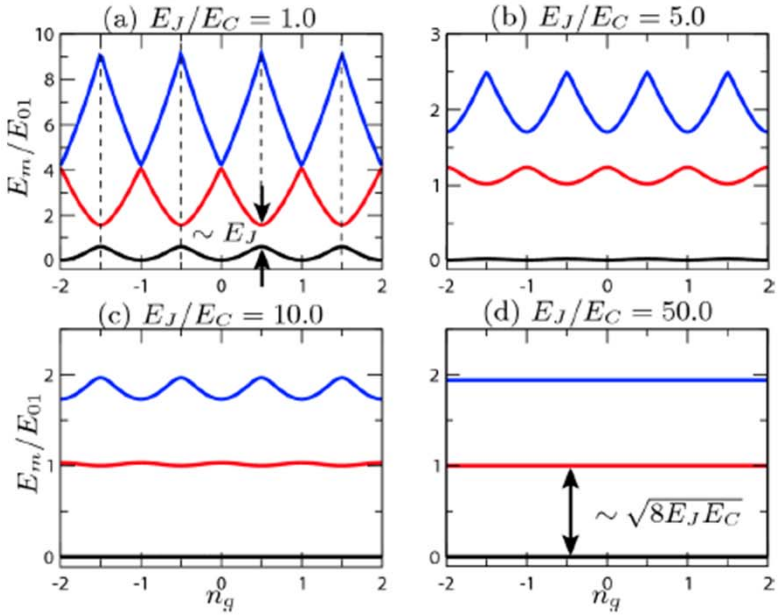
$$E_J^{eff} = E_J \cos(\pi\phi/\phi_0)$$

Transmon qubit



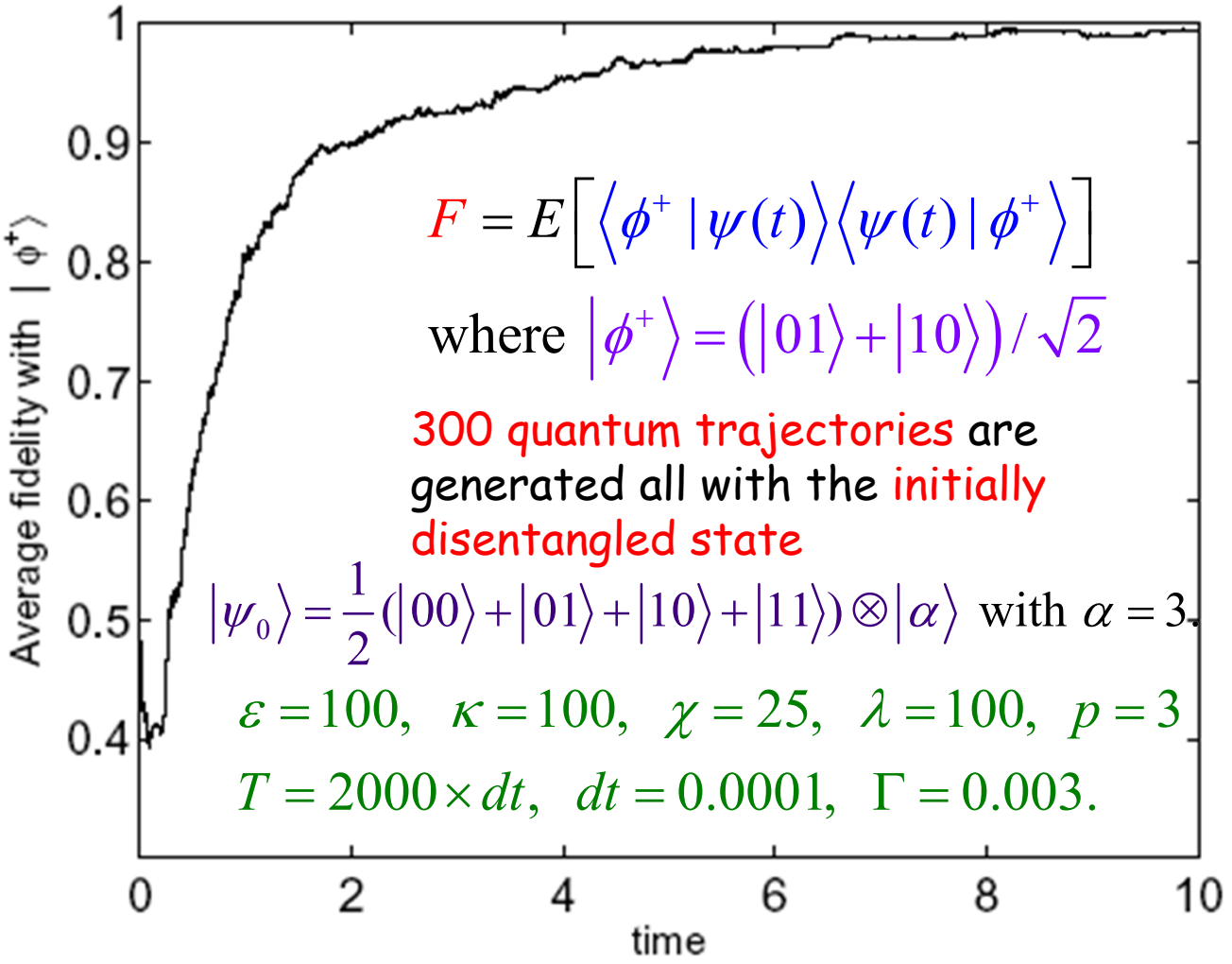
A very large dipole moment qubit with good coherence





J. Koch *et al.*, Phys. Rev. A **76**, 042319 (2007)

Evolution of entanglement fidelity



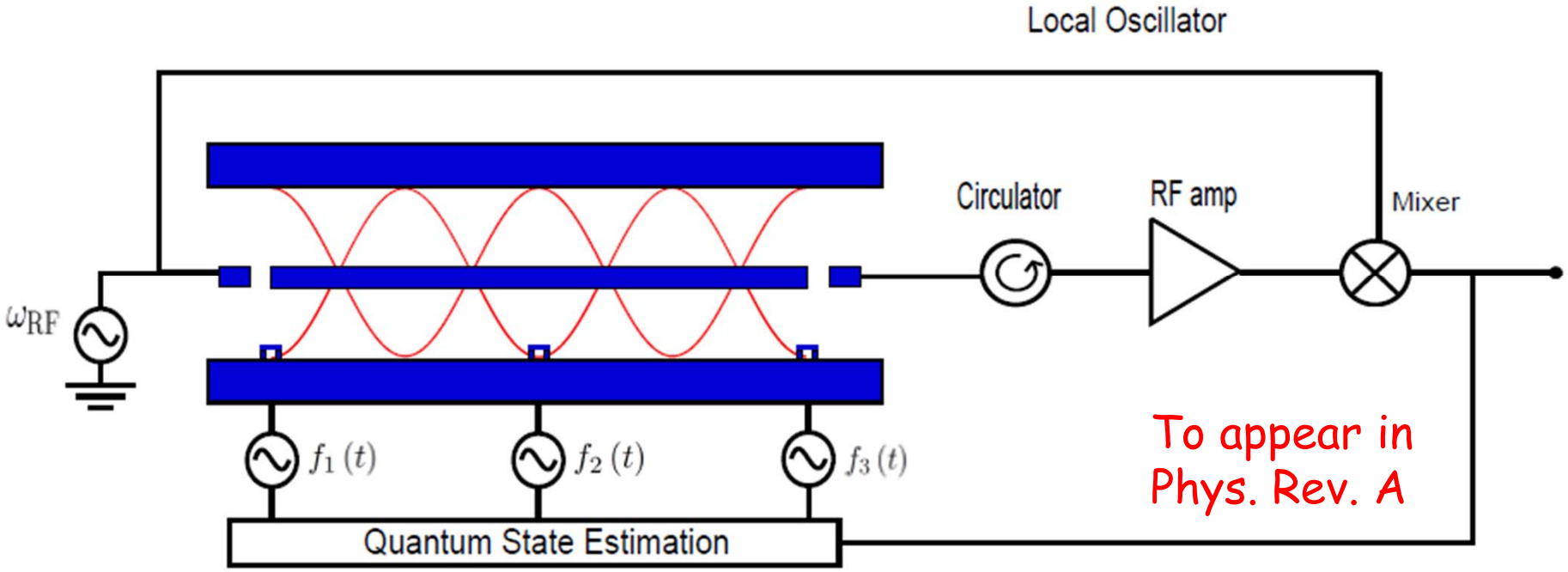
- The feedback scheme converge on the maximally entangled state $|\phi^+\rangle$ in a short time with high probability.
- The measurement record indicates the state of the qubits, and therefore we can tell when we have converged on the entangled state.

M. Sarovar, H.-S. Goan, T. P. Spiller, and G. J. Milburn, *"High fidelity measurement and quantum feedback control in circuit QED"* , Phys. Rev. A **72**, 062327 (2005)

Generation and stabilization of a three-qubit entangled W state in circuit QED via quantum feedback control

Shang-Yu Huang,[1,2] Hsi-Sheng Goan,[1,2,*] Xin-Qi Li,[3] and Gerard J. Milburn[4]

Circuit cavity quantum electrodynamics (QED) is proving to be a powerful platform to implement quantum feedback control schemes due to the ability to control superconducting qubits and microwaves in a circuit. Here, we present a simple and promising quantum feedback control scheme for deterministic generation and stabilization of a three-qubit W state in the superconducting circuit QED system. The control scheme is based on continuous joint Zeno measurements of multiple qubits in a dispersive regime, which enables us not only to infer the state of the qubits for further



To appear in Phys. Rev. A

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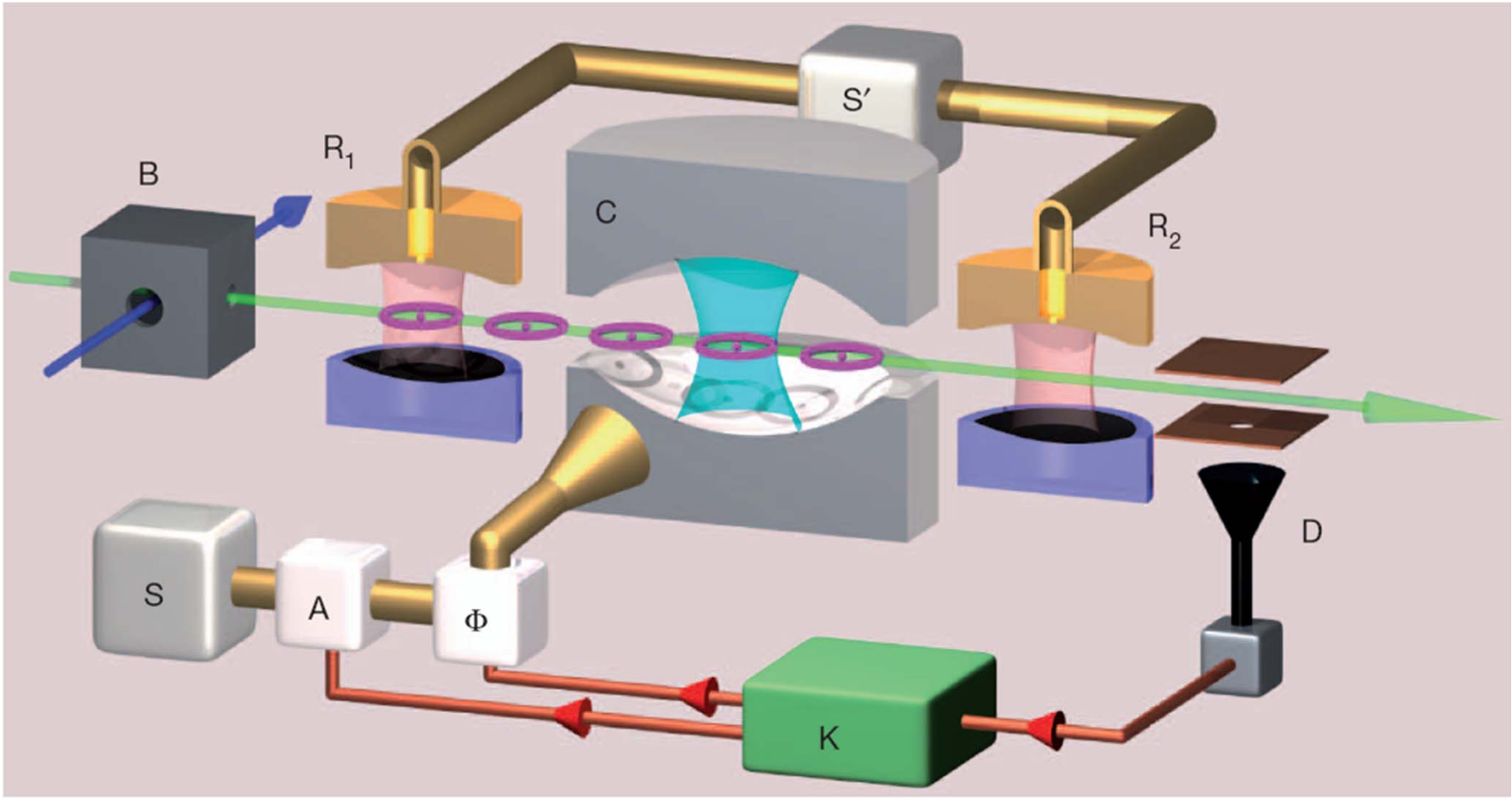
Real-time quantum feedback prepares and stabilizes photon number states

Clément Sayrin[1], Igor Dotsenko[1], Xingxing Zhou[1], Bruno Peaudecerf[1], Théo Rybarczyk[1], Sébastien Gleyzes[1], Pierre Rouchon[2], Mazyar Mirrahimi[3], Hadis Amini[2], Michel Brune[1], Jean-Michel Raimond[1] & Serge Haroche[1,4]

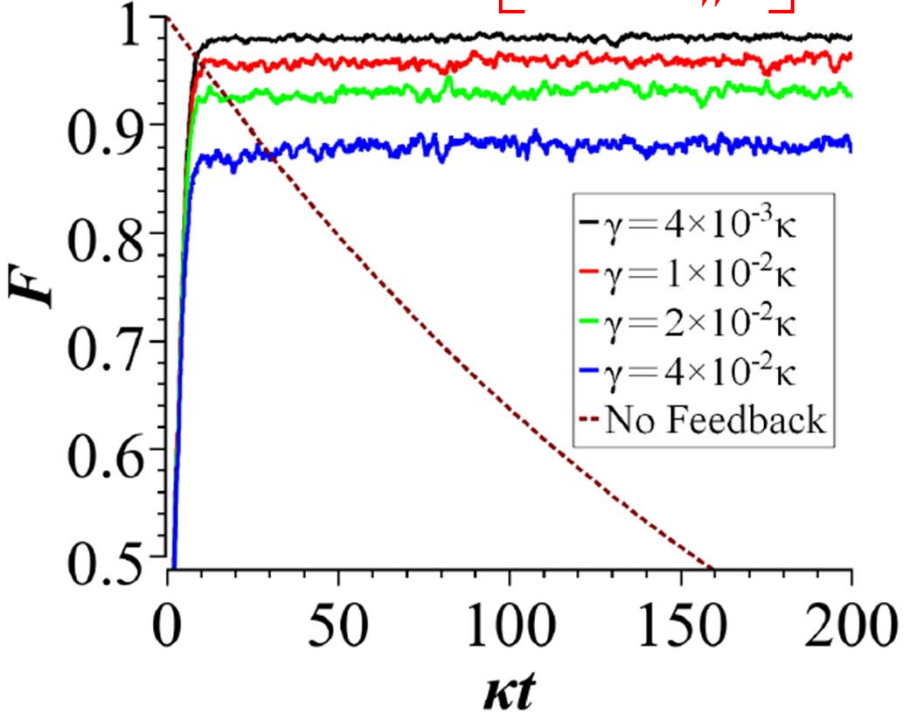


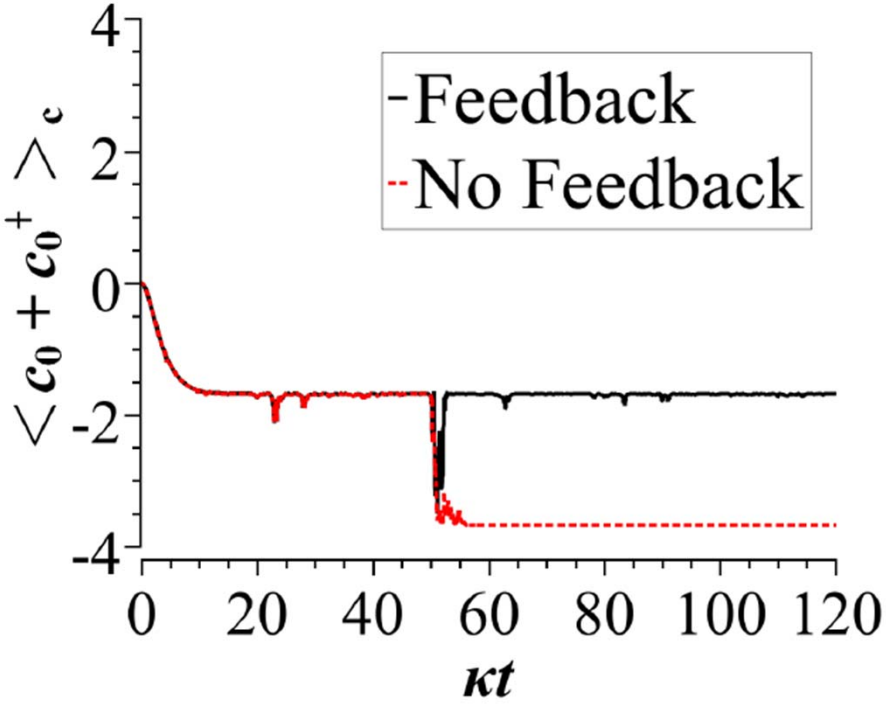
Generation & Stabilization

The $|W^-\rangle$ state can be generated from the states $|000\rangle, |W^-\rangle, |W^+\rangle, |111\rangle$ and stabilized by the feedback control.

$$F(t) = \mathrm{Tr}\left[\rho(t)\rho_{W^-}\right]$$

$$|W^-\rangle = \frac{1}{\sqrt{3}}\left(|100\rangle + |010\rangle + |001\rangle\right)$$





Quantum optimal control for open quantum systems

Goan, Hsi-Sheng

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with Bin Hwang, Yi Chou, Tsun-Yi Yang, Jung-Shen Tai, Shang-Yu Huang, …

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Quantum optimal control theory

- Quantum optimal control theory (QOCT) is a powerful tool that provides a variational framework for calculating the optimal shaped pulse to maximize a desired physical objective (or minimize a physical cost function) within certain constraints.
- The field of quantum optimal control has developed rapidly since the 1980s. The key aspects of the theory were formulated by Tannor, Rice, Kosloff, Shapiro, Brumer, Rabitz and …
- (Quantum) optimal control theory :
  - Standard gradient optimization methods
  - A somewhat different approach: Krotov iterative method

PHYSICAL REVIEW A **79**, 060306(R) (2009) **RAPID COMMUNICATIONS**

Optimal control of the silicon-based donor-electron-spin quantum computing

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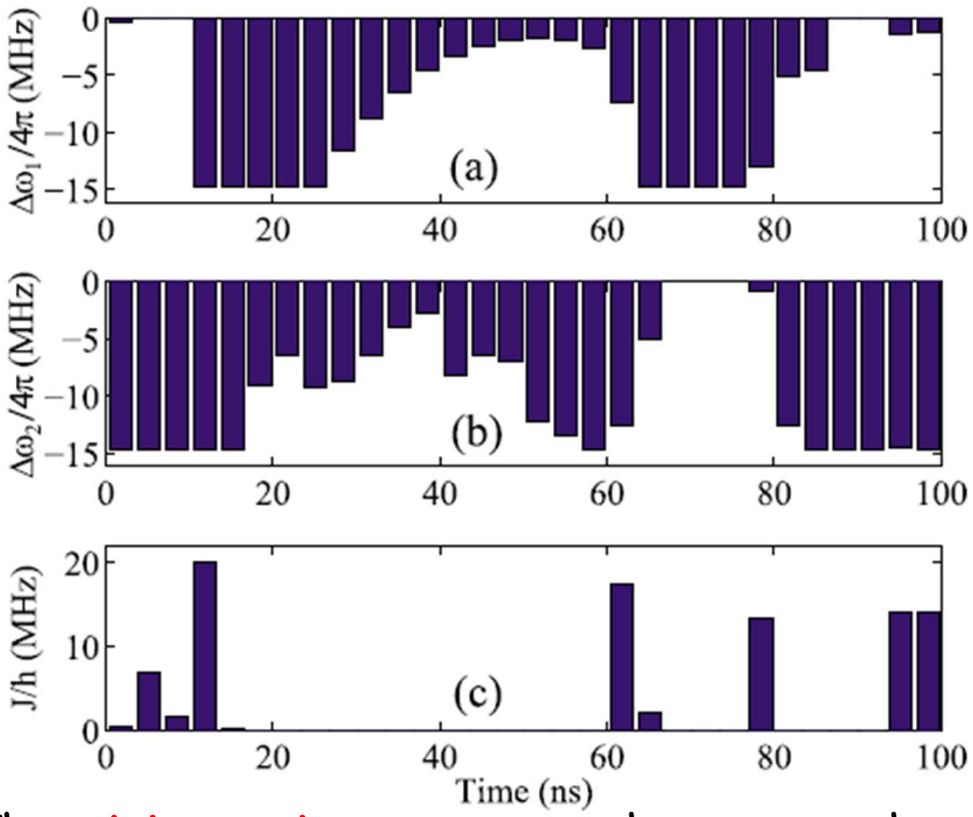
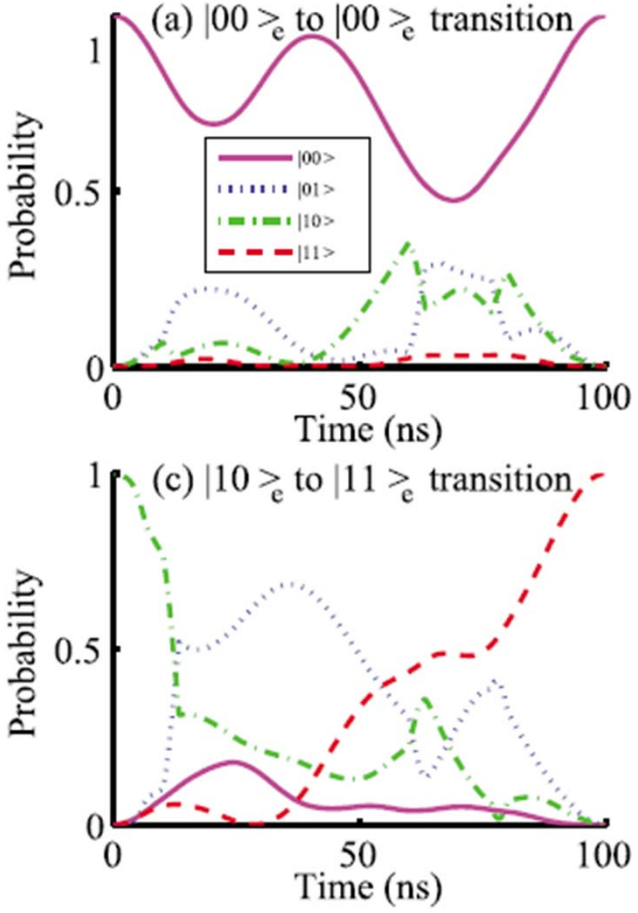
and Center for Quantum Science and Engineering, National Taiwan University, Taipei 10617, Taiwan

Near time-optimal CNOT gate control sequence

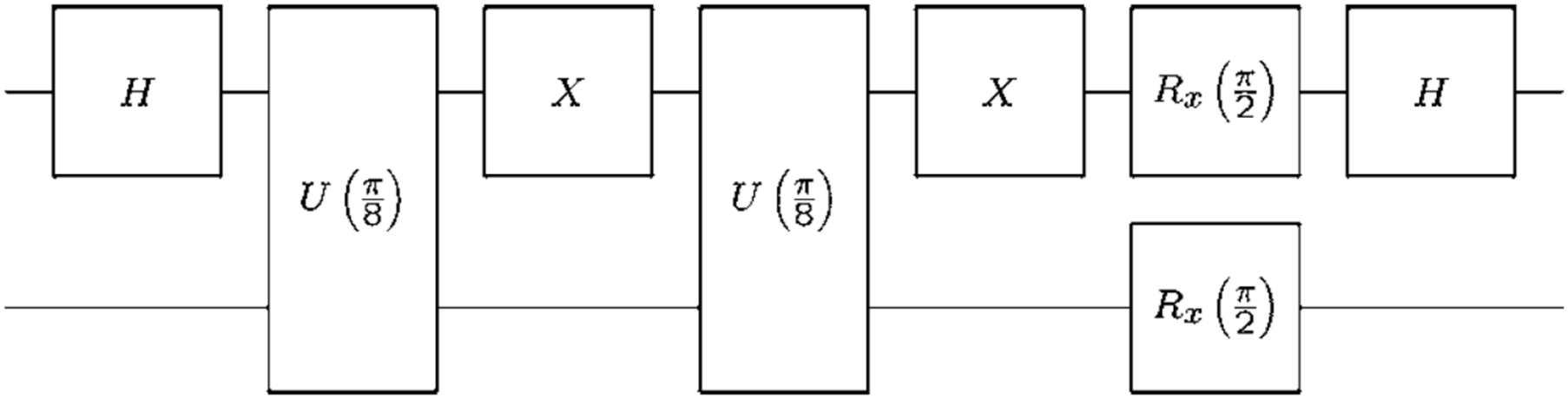


Time evolution of the states



The minimum time sequence that meets the required fidelity is the **near** time-optimal control sequence.

Canonical decomposition of CNOT gate for global control e-spin QC



$$U\left(\frac{\pi}{8}\right) = \exp\left[i\frac{\pi}{8}\boldsymbol{\sigma}^{1e}\cdot\boldsymbol{\sigma}^{2e}\right]$$

CNOT gate operation time: 297ns

C. D. Hill, L. C. L. Hollenberg, A. G. Fowler, C. J. Wellard, A. D. Greentree, and H.-S. Goan, *"Global control and fast solid-state donor electron spin quantum computing"*, Phys. Rev. B **72**, 045350 (2005).

Quantum control and manipulation

- Most of the control sequences implemented in quantum experiments are developed and designed based on the assumption of having ideal (closed) quantum coherent systems.
- However, almost every quantum system interacts inevitably with its surrounding environment resulting in decoherence and dissipation of the quantum system.
- Thus precisely controlling realistic open quantum systems is one of the most important and timely issues in the field of QIP.

Fighting against decoherence

- Various methods have been proposed to reduce the unwanted interaction with environments, such as the quantum error-correction code, quantum Zeno effect, and decoherence-free subspaces, quantum feedback control, …
- Dynamical decoupling techniques: a succession of short and strong pulses to the system designed to suppress decoherence; (concatenated dynamical decoupling…)
- Optimal control theory: a powerful tool that allows us to design and realize accurate quantum gates by selecting optimal pulse shapes (arbitrarily shaped pulses and duration; or continuous dynamical modulation) for the external control within experimental capabilities.

Optimal Control of a Qubit Coupled to a Non-Markovian Environment

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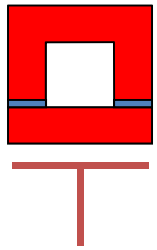
A central challenge for implementing quantum computing in the solid state is decoupling the qubits from the intrinsic noise of the material. We investigate the implementation of quantum gates for a paradigmatic, non-Markovian model: a single-qubit coupled to a two-level system that is exposed to a heat bath. We systematically search for optimal pulses using a generalization of the novel open systems gradient ascent pulse engineering algorithm. Next to the known optimal bias point of this model, there are optimal pulses which lead to high-fidelity quantum operations for a wide range of decoherence parameters.

$$H_S(t) = E_1(t)\sigma_z + \Delta\sigma_x + E_2\tau_z + \Lambda\sigma_z\tau_z.$$

Non-Markovian spin-boson model

- Model: $H_S(t) = -\varepsilon(t)\sigma_z/2 - \Omega\sigma_x/2$



$$H_I = \sum_\lambda c_q \sigma_x (b_q + b_q^\dagger), \quad H_B = \sum_\lambda \hbar\omega_q b_q^\dagger b_q$$

- The perturbative non-Markovian open quantum system theory may be categorized into two classes:
  - **time-nonlocal (time-convolution) method**
  - **time-local (or time-convolutionless) method.**
- To second-order in the system-environment Hamiltonian (in the interaction picture)

$$\frac{d\tilde{\rho}(t)}{dt} = -\frac{1}{\hbar^2}\mathrm{Tr}_R \int_0^t dt' [\tilde{H}_I(t), [\tilde{H}_I(t'), \tilde{\rho}(t') \otimes R_0]]$$

$$H_I = \sum_\lambda c_q \sigma_x (b_q + b_q^\dagger),\ \tilde{H}_I(t) = U_S^\dagger(t) H_I U_S(t) = \tilde{\sigma}_x(t) B(t)$$

where $\tilde{\sigma}_x(t) = U_S^\dagger(t)\sigma_x U_S(t),\ U_S(t) = \mathcal{T}_+ e^{(-i/\hbar)\int_0^t H_S(t')dt'}$,

$$H_S(t) = -\varepsilon(t)\sigma_z/2 - \Omega\sigma_x/2,$$

$$B(t) = \sum_q (c_q b_q e^{-i\omega_q t} + c_q a_q^\dagger e^{i\omega_q t}).$$

In the interaction picture:

$$\begin{aligned}
\dot{\tilde{\rho}}(t) &= -\frac{1}{\hbar^2}\int_0^t \mathrm{Tr}_B[\tilde{\sigma}_x(t)B(t),[\tilde{\sigma}_x(t')B(t'),\tilde{\rho}(t')\otimes R_0]]dt' \\
&= -\frac{1}{\hbar^2}\int_0^t dt'\{[\tilde{\sigma}_x(t)\tilde{\sigma}_x(t')\tilde{\rho}(t') - \tilde{\sigma}_x(t')\tilde{\rho}(t')\tilde{\sigma}_x(t)] \\
&\quad \times C(t-t') + [\tilde{\rho}(t')\tilde{\sigma}_x(t')\tilde{\sigma}_x(t) - \tilde{\sigma}_x(t)\tilde{\rho}(t')\tilde{\sigma}_x(t')] \\
&\quad \times C(t'-t)\}, \qquad \text{(A3)}
\end{aligned}$$

Bath CF:

$$
\begin{aligned}
C(t-t') &\equiv \mathrm{Tr}_B[B(t)B(t')R_0] \\
&= \int_0^\infty d\omega J(\omega)\{[n(\omega)+1]e^{-i\omega(t-t')} + n(\omega)e^{i\omega(t-t')}\} \\
&= \int_0^\infty d\omega J(\omega)\cos[\omega(t-t')]\coth\left[\frac{\hbar\omega}{2k_BT}\right] \\
&\quad - i\int_0^\infty d\omega J(\omega)\sin[\omega(t-t')], \qquad \text{(A4)}
\end{aligned}
$$

$$J(\omega) = \sum_q |c_q|^2 \,\delta(\omega-\omega_q)$$

Transferring to the Schrödinger picture:

$$\dot{\rho}(t) = -\frac{i}{\hbar}[H_S(t),\rho(t)] - \frac{i}{\hbar}\{[\sigma_x,\mathcal{K}(t)] - [\mathcal{K}^\dagger(t),\sigma_x]\},$$

where

control-dissipation correlation

$$\mathcal{K}(t) = -\frac{i}{\hbar}U_S(t)\left[\int_0^t dt' C(t-t')\tilde{\sigma}_x(t')\tilde{\rho}(t')\right]U_S^\dagger(t),$$

Quantum master equation with control-dissipation correlation

$$\dot{\rho}(t) = \mathcal{L}_S(t)\rho(t) + \left\{\mathcal{L}_x \mathcal{K}(t) + [\mathcal{L}_x \mathcal{K}(t)]^\dagger\right\}$$

where $\mathcal{L}_S(t)A = (-i/\hbar)[H_S(t), A]$, $\quad H_S(t) = -\frac{\varepsilon(t)}{2}\sigma_z - \frac{\Omega}{2}\sigma_x$

$$\mathcal{L}_x A = (-i/\hbar)[\sigma_x, A].$$

dissipator $\mathcal{K}(t) = -\frac{i}{\hbar}\int_0^t dt' C(t-t')\mathcal{U}_S(t,t')\sigma_x\rho(t'),$

propagator $\mathcal{U}_S(t,t') = \mathcal{T}_+ \exp\left[\int_{t'}^t \mathcal{L}_S(\tau)d\tau\right],$

bath CF $C(t-t') = \int_0^\infty d\omega J(\omega)\cos[\omega(t-t')]\coth(\frac{\beta\hbar\omega}{2})$

$$-i\int_0^\infty d\omega J(\omega)\sin[\omega(t-t')].$$

Optimal control objective

- Performance (cost) function:

$$J = F_{tr} - \int_0^{t_f} dt' \, \lambda(t')[\varepsilon(t') - \varepsilon_0(t')]^2$$

$$\text{Trace fidelity } F_{tr} = \frac{1}{N} \mathrm{Re}\left[\mathrm{Tr}\left\{ \mathcal{Q}_D^{\dagger} \hat{\mathcal{G}}(T) \right\} \right]$$

- Penalty or constraint : control energy be minimal in balance with meeting the control objectives.
- $\lambda(t)$ is a positive weight function that can be adjusted and chosen empirically.
- $\varepsilon_0(t)$ is the standard control values can be chosen arbitrarily.
- **Goal**: reach a desired target (super)operation Q_D by maximizing the performance function (high fidelity and low energy cost) in a given operation time $t_f = T$.

The Krotov optimization method

- A somewhat different QOCT approach from the standard gradient optimization methods is the Krotov iterative method.
- The Krotov method has several appealing advantages over the gradient methods:
  - monotonic increase of the objective with iteration number,
  - no requirement for a line search, and
  - macrosteps at each iteration.

See, e.g., R. Eitan, M. Mundt and D. J. Tannor, Phys. Rev. A **83**, 053426 (2011)

The Krotov optimal control method algorithm

- STEP 1: Guess an initial control sequence
- STEP 2: Use equation of motion to find trajectory (forward propagator). $\dot{\rho}(t) = \Lambda(t)\rho(t)$ $\frac{\partial \hat{\mathcal{G}}(t,t')}{\partial t} = \hat{\Lambda}(t)\hat{\mathcal{G}}(t,t')$
- STEP 3: Find an auxiliary functional (backward propagator). $\frac{\partial \hat{\mathcal{B}}(t,t')}{\partial t} = -\Lambda^{\dagger}(t)\hat{\mathcal{B}}(t,t')$ $\hat{\mathcal{B}}(t=T) = \hat{\mathcal{Q}}^{\dagger}$
- STEP 4: Find the new control sequence

$$\varepsilon_{k+1}(t) = \varepsilon_k(t) + \frac{1}{2\lambda(t)}\mathrm{Re}[\mathrm{Tr}\{\hat{\mathcal{B}}_k(t)\frac{\partial \Lambda(t)}{\partial \varepsilon(t)}\hat{\mathcal{G}}_{k+1}(t)\}]$$

- STEP 5: Repeat step 2 to step 4, until a preset fidelity (error) threshold is reached or a given number of iterations has been performed.

Quantum master equation with control-dissipation correlation

$$\dot{\rho}(t) = \mathcal{L}_S(t)\rho(t) + \left\{ \mathcal{L}_x \mathcal{K}(t) + [\mathcal{L}_x \mathcal{K}(t)]^\dagger \right\}$$

where $\mathcal{L}_S(t)A = (-i/\hbar)[H_S(t), A]$, $\quad H_S(t) = -\frac{\varepsilon(t)}{2}\sigma_z - \frac{\Omega}{2}\sigma_x$

$$\mathcal{L}_x A = (-i/\hbar)[\sigma_x, A].$$

dissipator $\mathcal{K}(t) = -\frac{i}{\hbar}\int_0^t dt' C(t-t')\mathcal{U}_S(t,t')\sigma_x \rho(t'),$

propagator $\mathcal{U}_S(t,t') = \mathcal{T}_+ \exp\left[\int_{t'}^t \mathcal{L}_S(\tau)d\tau\right],$

bath CF $C(t-t') = \int_0^\infty d\omega J(\omega)\cos[\omega(t-t')]\coth(\frac{\beta\hbar\omega}{2})$

$$-i\int_0^\infty d\omega J(\omega)\sin[\omega(t-t')].$$

Extended Liouville space and time-local equations of motion

- Expand the bath CF as a sum of exponential functions

$$C(t-t') = \sum_j C_j(t-t') = \sum_j C_j(0) e^{\gamma_j (t-t')}$$

- The dissipation operator: $\mathcal{K}(t) = \sum_j \mathcal{K}_j(t)$

$$\mathcal{K}_j(t) = -\frac{i}{\hbar}\int_0^t dt' C_j(0) e^{\gamma_j (t-t')} \mathcal{U}_S(t,t')\sigma_x \rho(t'),$$

- Replace dissipator $\mathcal{K}(t)$ with auxiliary density matrix $\mathcal{K}_j(t)$

$$\dot{\rho}(t) = \mathcal{L}_S(t)\rho(t) + \left\{\mathcal{L}_x \mathcal{K}(t) + [\mathcal{L}_x \mathcal{K}(t)]^\dagger\right\}$$

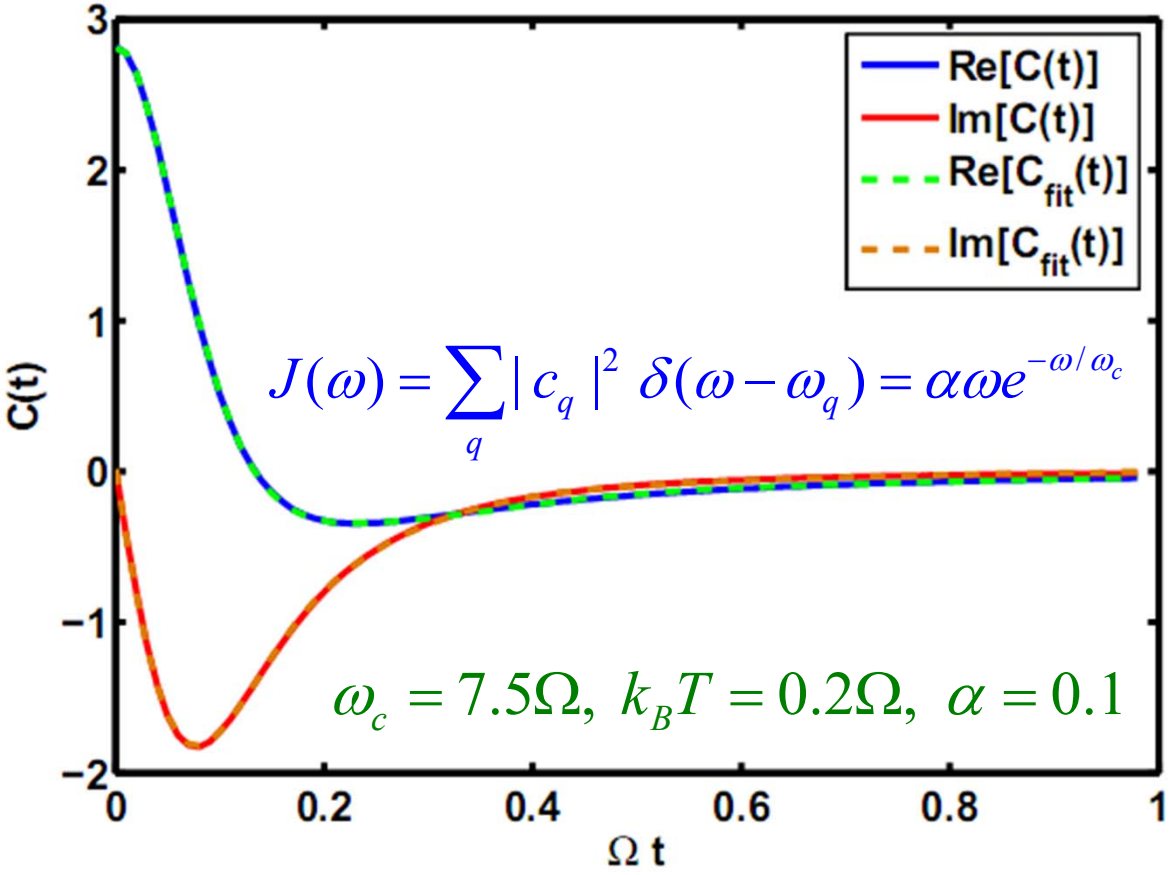
$$\Rightarrow \dot{\rho}(t) = \mathcal{L}_S(t)\rho(t) + \sum_j \left\{\mathcal{L}_x \mathcal{K}_j(t) + [\mathcal{L}_x \mathcal{K}_j(t)]^\dagger\right\}$$

- Find the equation of motion for $\mathcal{K}_j(t)$

$$\dot{\mathcal{K}}_j(t) = (-i/\hbar) C_j(0)\sigma_x \rho(t) + [\mathcal{L}_S(t) + \gamma_j]\mathcal{K}_j(t)$$

Form coupled linear local-in-time diff. eqs. for $\{\rho, \mathcal{K}_j, \mathcal{K}_j^\dagger\}$

Fitting the bath correlation function



Directly expand the bath CF in a sum of exponential functions instead of through the spectral density parameterization.

- Need only three or four exponential functions to directly expand the bath CF:

$$C(t-t')=\int_0^\infty d\omega J(\omega)\cos[\omega(t-t')]\coth(\frac{\beta\hbar\omega}{2})$$

$$-i\int_0^\infty d\omega J(\omega)\sin[\omega(t-t')].$$

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Optimal control for non-Markovian open quantum systems

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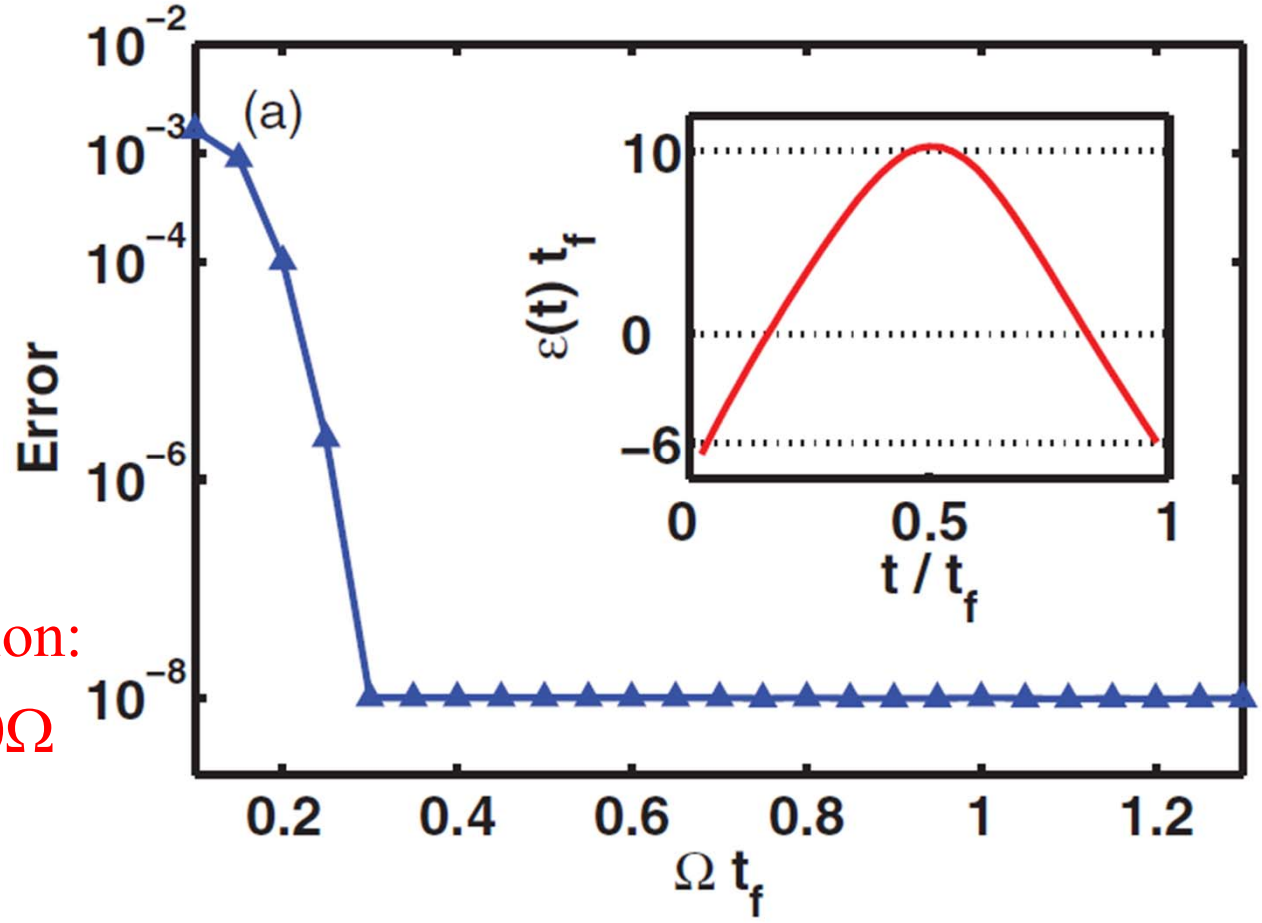
An efficient optimal-control theory based on the Krotov method is introduced for a non-Markovian open quantum system with a time-nonlocal master equation in which the control parameter and the bath correlation function are correlated. This optimal-control method is developed via a quantum dissipation formulation that transforms the time-nonlocal master equation to a set of coupled linear time-local equations of motion in an extended auxiliary Liouville space. As an illustration, the optimal-control method is applied to find the control sequences for high-fidelity Z gates and identity gates of a qubit embedded in a non-Markovian bath. Z gates and identity gates with errors less than 10^{-5} for a wide range of bath decoherence parameters can be achieved for the non-Markovian open qubit system with control over only the σ_z term. The control-dissipation correlation and the memory effect of the bath are crucial in achieving the high-fidelity gates.

- Extremely efficient to deal with the time-nonlocal non-Markovian equation of motion.
- The required information one needs is the knowledge of the bath or noise spectral density which is experimentally accessible [e.g., see, J. Bylander et al., Nat. Phys. **7**, 565 (2011)].

Error vs. time for an ideal Z-gate

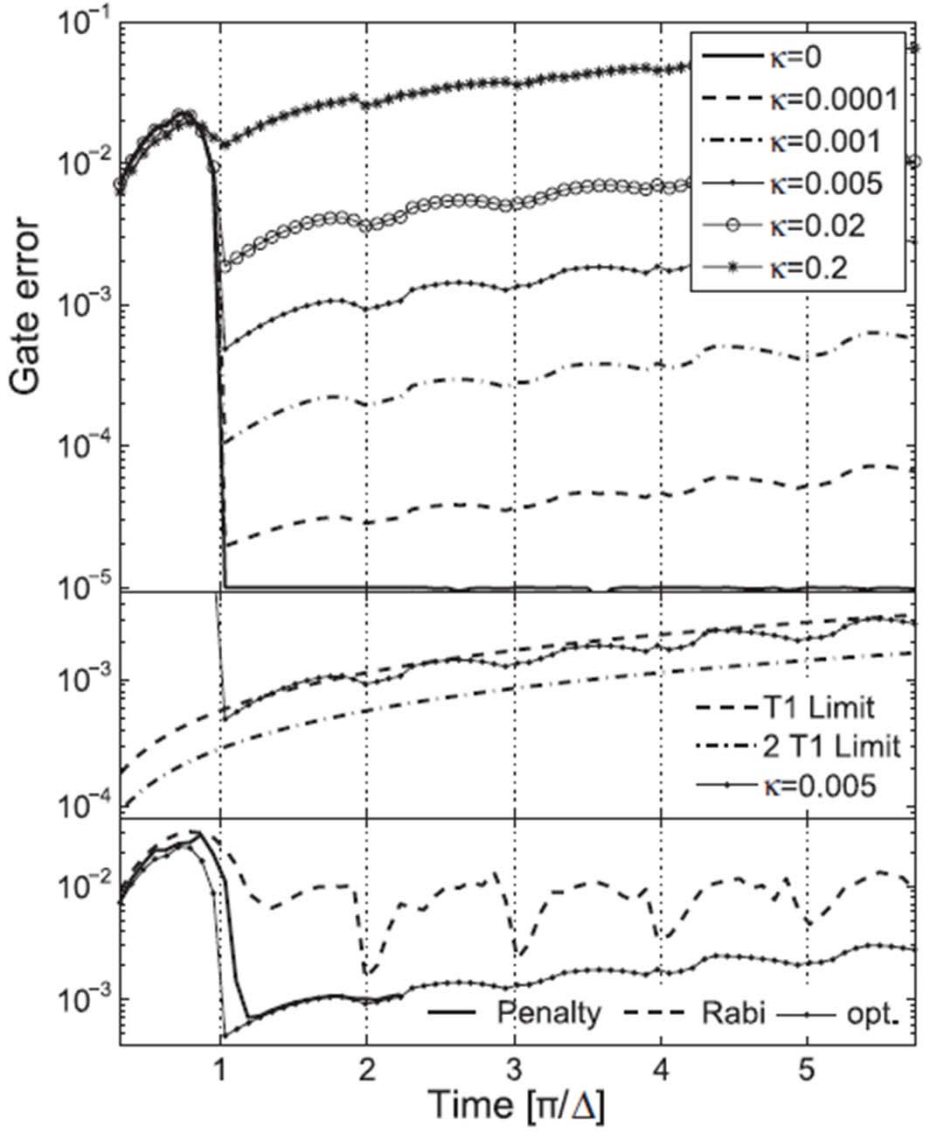


Restriction:

$\varepsilon(t) \leq 30\Omega$

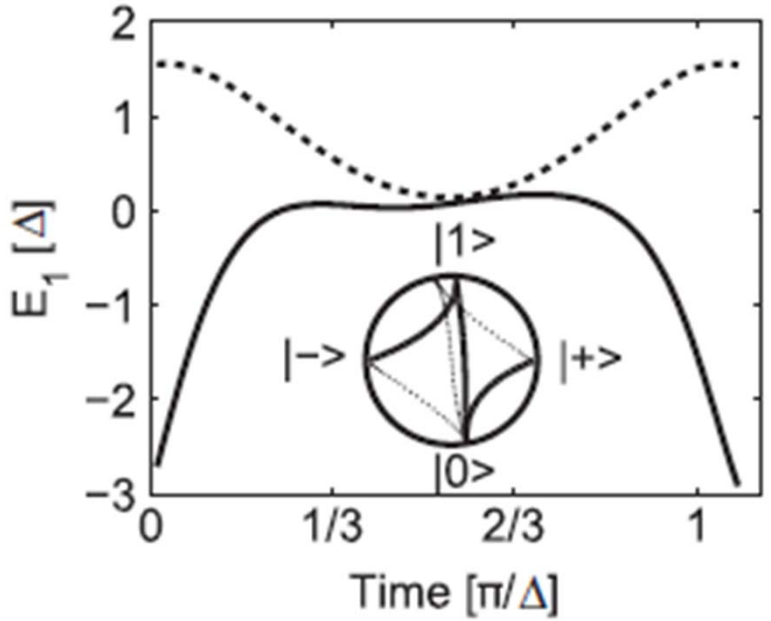
- The inset shows the optimal control pulse for any $\Omega t_f > 0.3$.

Z-gate error and control pulse



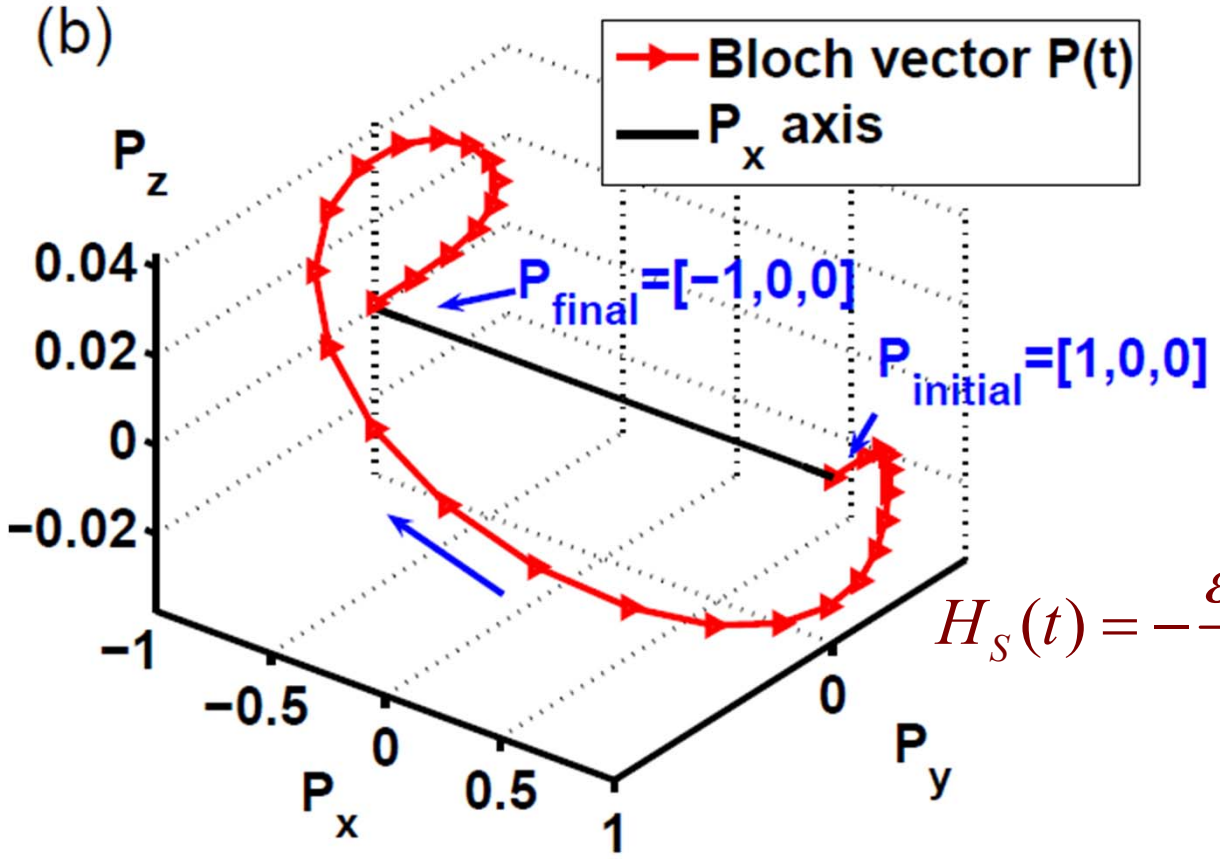
P. Rebentrost et al.,
Phys. Rev. Lett. **102**, 090401 (2009).

$$H_S(t) = E_1(t)\sigma_z + \Delta\sigma_x + E_2\tau_z + \Lambda\sigma_z\tau_z.$$



The static $\Delta\sigma_x$ induces at least a full loop around x axis.

State evolution of the ideal Z-gate with an initial |+> state



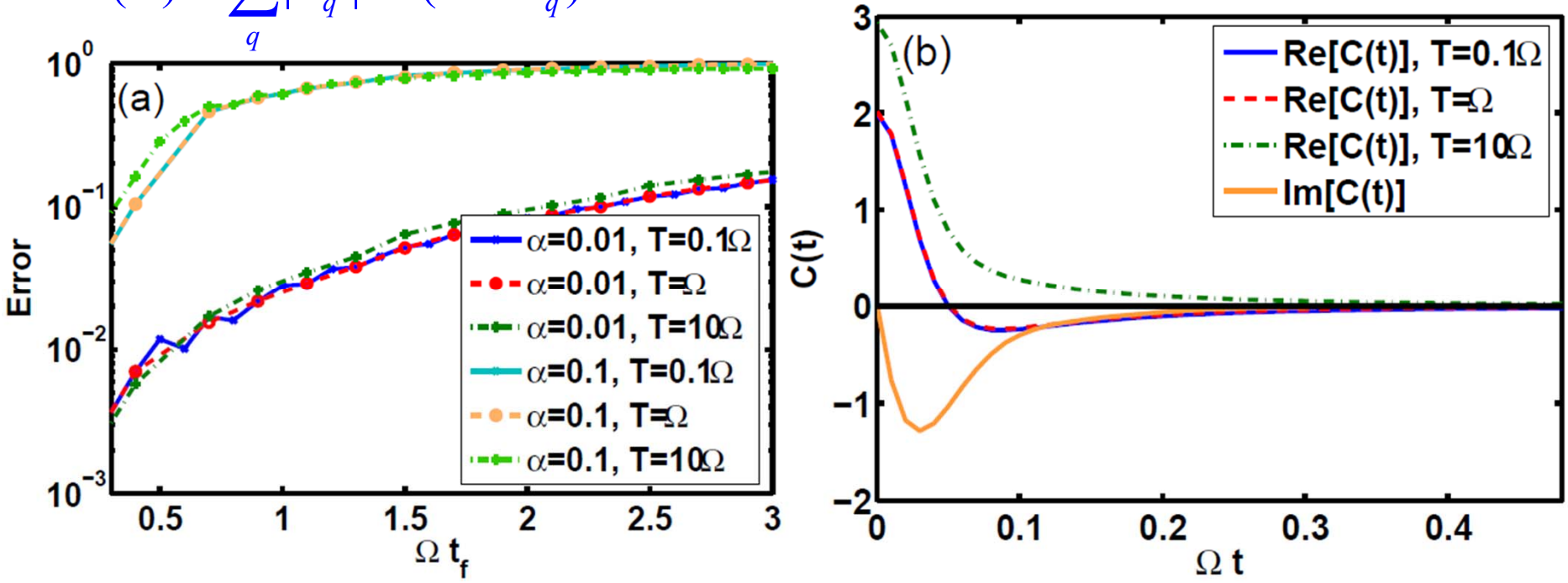
$$H_S(t) = -\frac{\varepsilon(t)}{2}\sigma_z - \frac{\Omega}{2}\sigma_x$$

- Our optimal control pulse strategy does not require a full loop around x-axis and thus can achieve a Z-gate in a much shorter operation time.

Ref: B. Hwang and H.-S. Goan, Phys. Rev. A 85, 032321 (2012).

Z-gate in an Ohmic bath with $\omega_c = 20\Omega$

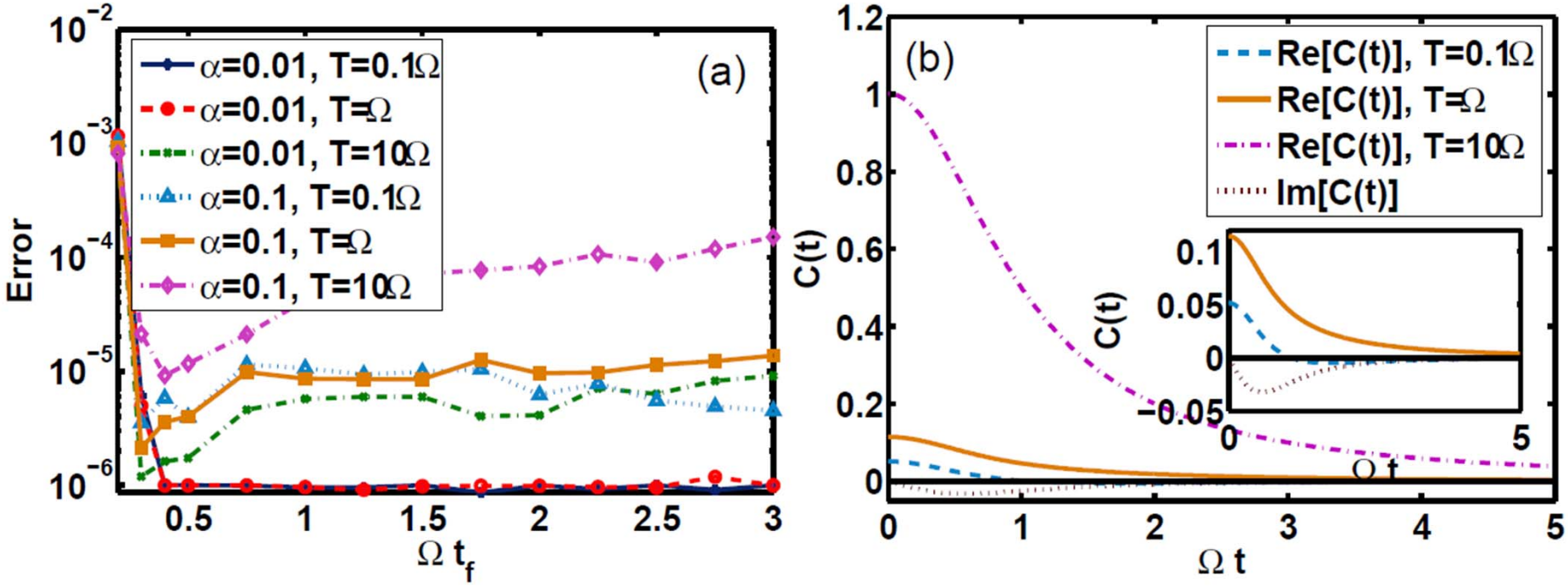
$$J(\omega) = \sum_q |c_q|^2 \, \delta(\omega - \omega_q) = \alpha \omega e^{-\omega/\omega_c}$$



The $\text{error} = \left\| \mathcal{Q}_{\mathcal{D}} - \hat{\mathcal{G}}(T) \right\|_F^2 / 2\mathcal{N} = \text{Tr}\left[(\mathcal{Q}_{\mathcal{D}} - \hat{\mathcal{G}}(T))^\dagger (\mathcal{Q}_{\mathcal{D}} - \hat{\mathcal{G}}(T)) \right] / 2\mathcal{N}$ (Frobenius norm) increases when the operation time become longer.

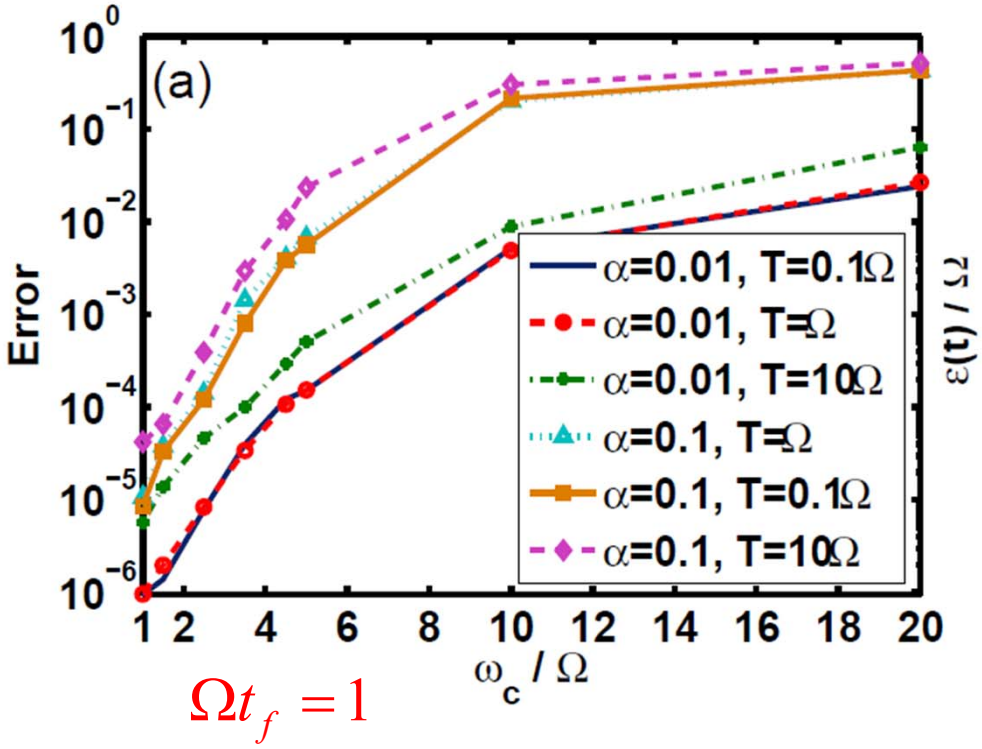
Ref: B. Hwang and H.-S. Goan, Phys. Rev. A 85, 032321 (2012).

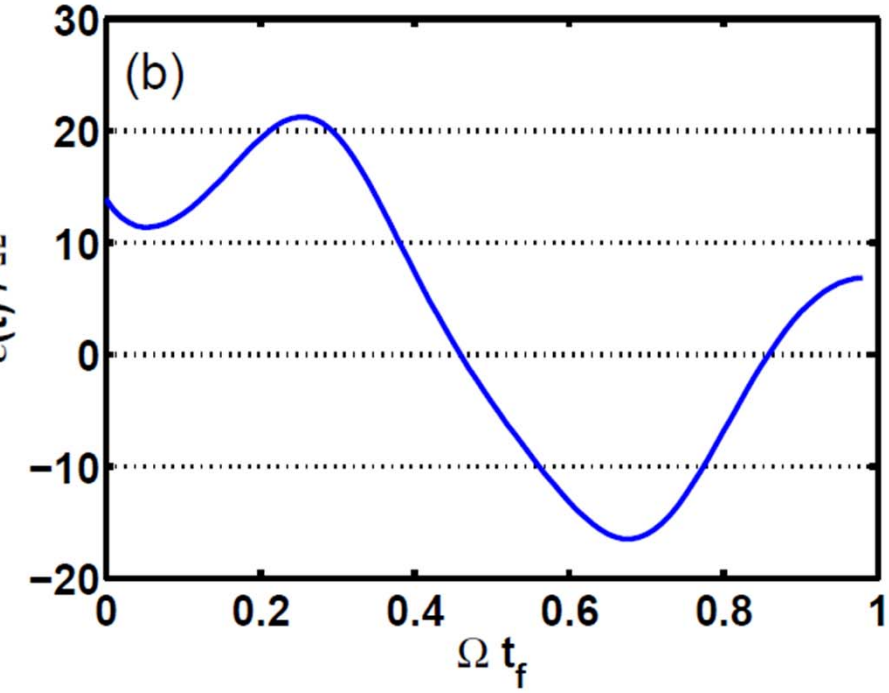
Z-gate in an Ohmic bath with $\omega_c = \Omega$



- It is possible to achieve high-fidelity Z-gates with errors < 10^{-5} at large times
- The longer bath correlation time allows the optimal control sequence to counteract and suppress the contribution from the bath

Z-gate error vs. cutoff frequency

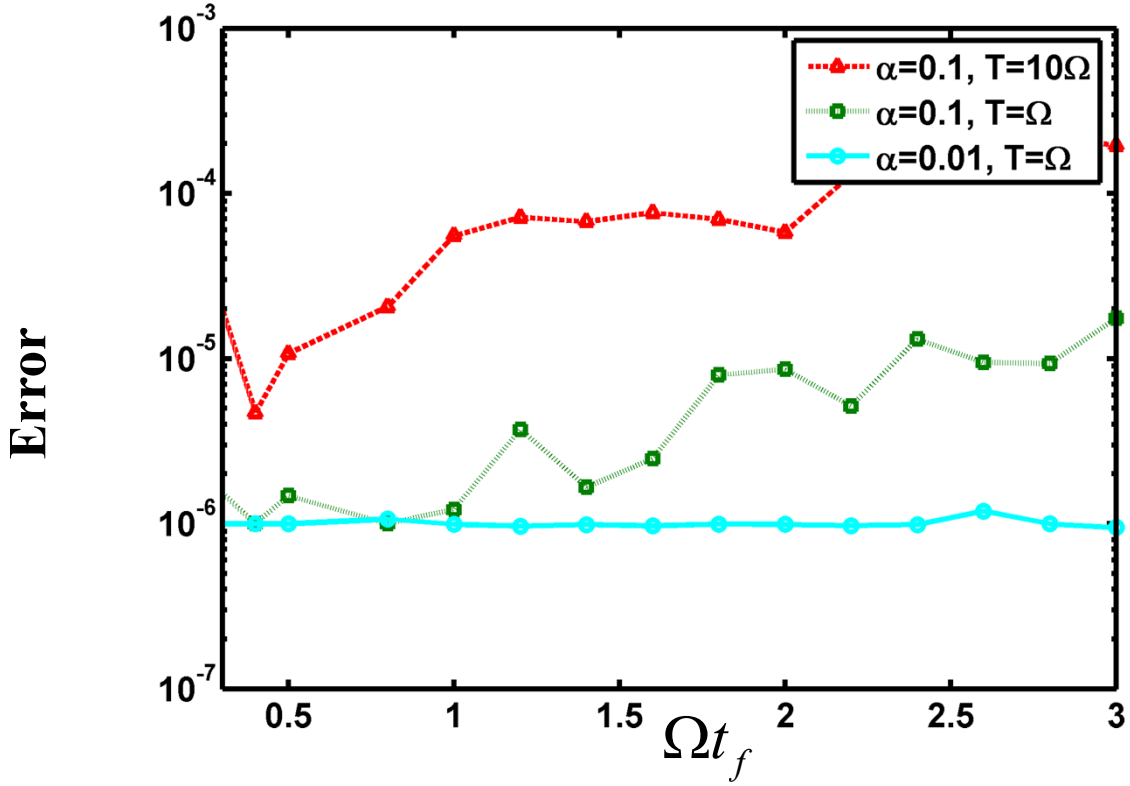




- The error increases as cutoff frequency ω_c becomes bigger.
- Memory effect plays an important role in determining the gate error

- The optimal control sequence that suppresses the decoherence induced by bath is totally different from that of the ideal unitary case .

Error of identity-gate

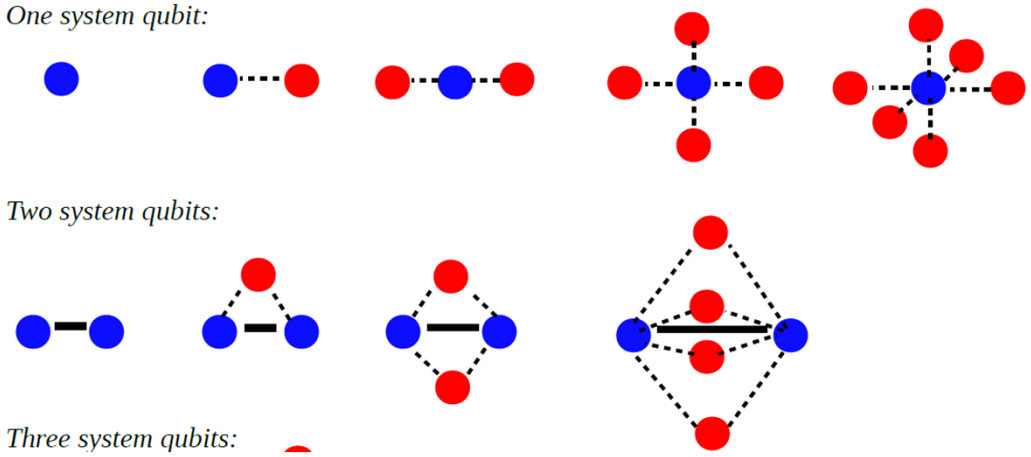


$$\omega_c = \Omega$$

- Achieving a high-fidelity identity gate at long times implies having the capability for arbitrary state preservation, i.e., restoring arbitrary state robustly against the bath.
- The gate errors are expected to be much lower if one has independent control over both σ_z and σ_x terms in the qubit Hamiltonian and if there is no restriction on the control parameter strengths.

More works on quantum optimal control

- Two-qubit gates taking into account more realistic bath spectral densities.
- One-qubit gates with an exact master equation.
- A small number of noise qubits interacting with one or more system qubits (full Hamiltonian approach; also including influence from baths).



Refs:

- F. Floether, P. de Fouquieres, and S. Schirmer, New J. Phys. **14**, 073023 (2012).
- C. Brif, R. Chakrabarti and H. Rabitz, New J. Phys. **12**, 075008 (2010).

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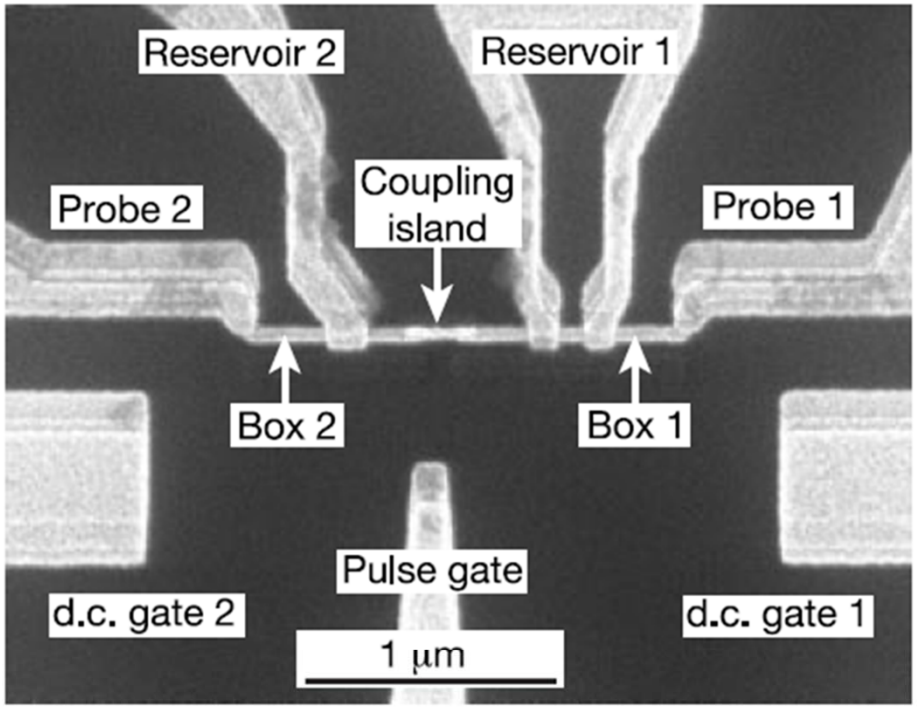

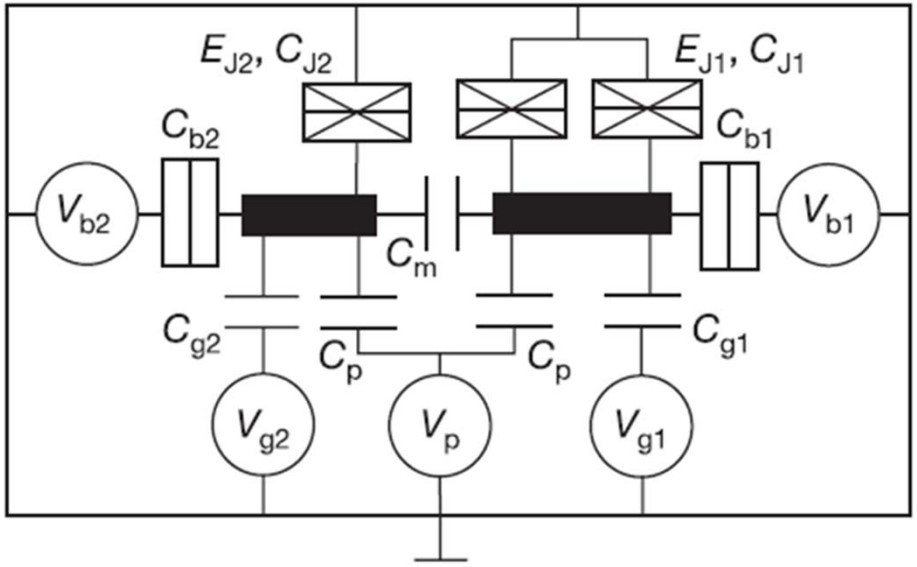

Quantum Noise in the Josephson Charge Qubit

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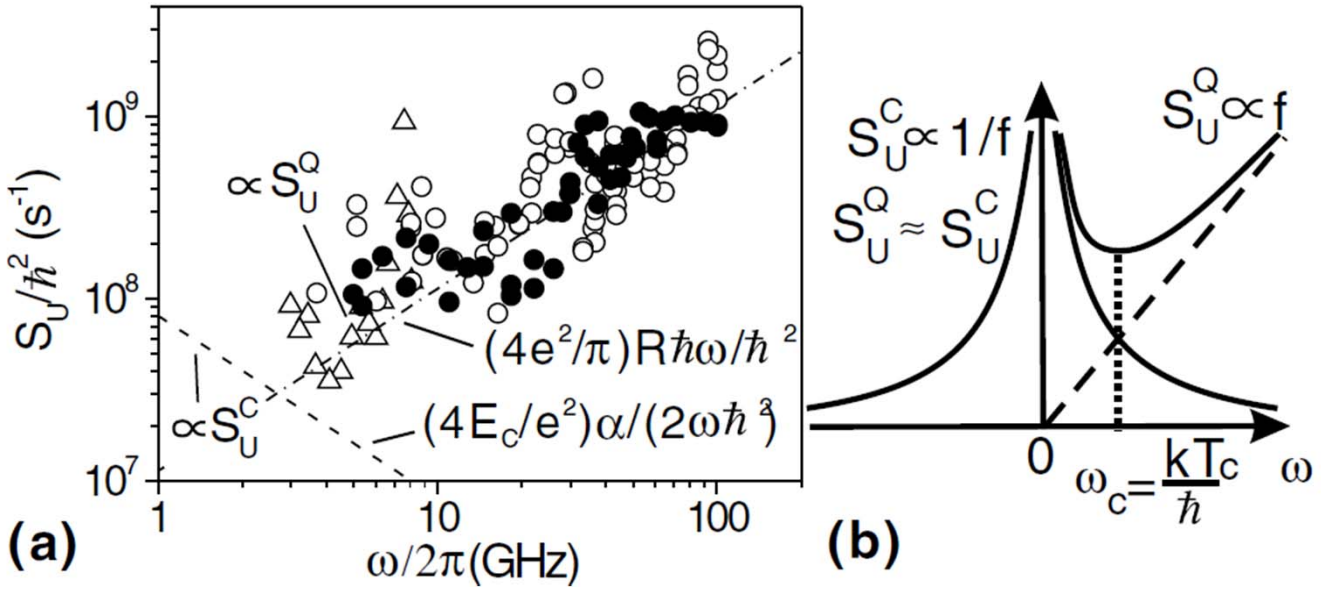


$$J(\omega) = \sum_q |c_q|^2 \, \delta(\omega - \omega_q)$$

$$S(\omega) = J(\omega)\coth\left(\frac{\hbar\omega}{2kT}\right)$$

An open quantum system model with an exact master equation

$$H_{\text{tot}} = H_s + H_B + H_{SB}$$

$$H_s = \frac{\omega + \varepsilon(t)}{2}\sigma_z \equiv \frac{\omega(t)}{2}\sigma_z$$

Control ling the transition frequency

$$H_B = \sum_\lambda \omega_\lambda a_\lambda^+ a_\lambda$$

$$H_{SB} = \sum_\lambda (g_\lambda^* \sigma_- a_\lambda^+ + g_\lambda \sigma_+ a_\lambda)$$

$$\alpha(t-s) \equiv \sum_\lambda |g_\lambda|^2 e^{-i\omega_\lambda (t-s)}$$

$$\dot{\rho}_t = -\frac{i\omega}{2}[\sigma_z, \rho_t] + (F(t) + F^*(t))\left(\sigma_-\rho_t\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho_t\}\right) + \left(\frac{F(t) - F^*(t)}{2}\right)[\sigma_+\sigma_-, \rho_t]$$

$$F(t) \equiv \int_0^t \alpha(t-s) f(t,s) ds$$

$$\partial_t f(t,s) = [i\omega + F(t)] f(t,s)$$

$$f(s,s) = 1.$$

Model

- $H = H_s + H_c(t) + H_I + H_B$
- System:

$$H_s = \sum_{i=1}^{2} \left[-\left(\frac{E_m}{4} + \frac{Ec^i}{4} \right) {\sigma_z}^i - \frac{Ej^i}{2} {\sigma_x}^i \right] + \frac{E_m}{4} {\sigma_z}^1 {\sigma_z}^2$$

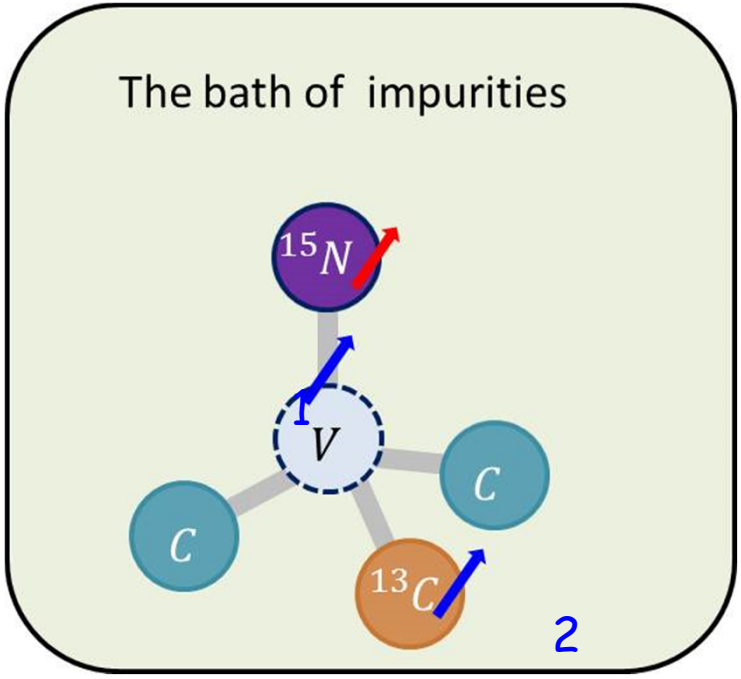
- Control:

$$H_c(t) = \left(\frac{E_m}{2} n^2(t) + \frac{Ec^1}{2} n^1(t) \right) {\sigma_z}^1 + \left(\frac{E_m}{2} n^1(t) + \frac{Ec^2}{2} n^2(t) \right) {\sigma_z}^2$$

$$n^1(t), n^2(t) \in [0,1]$$

Since $n^i = \frac{{V_g}^i {C_g}^i}{2e}$ is the number of the tunneling cooper pair.

Implementation of CNOT gate in NV center



Blue #1 : NV electron spin
Blue #2: ^{13}C nuclear spin
Red: ^{15}N nuclear spin
Bath : from other spins

$$G = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} .$$

How to generate these optimal control pulses experimentally?

- Accurately determination of the parameters in Hamiltonian.
- Calibration of the system response with amplifiers and control fields.
- The optimal pulse sequences look experimentally challenging, but not impossible.
- Commercial devices (e.g., Tektronix AWG70001A) for generating arbitrary wave forms with 10 bits of vertical resolution at a sample rate of 50 GSa/s, a bit rate of 12.5 Gb/s and a rise/fall time smaller than 27 ps are now available.
- Such a device should enable generation of complex signals in a time scale of sub-nanoseconds to nanoseconds.
- In principle the pulses could also be improved using closed-loop optimization where measurement data is immediately fed back to the optimizer to improve the pulses without full knowledge of the system.

Summary I

- A universal and efficient QOCT based on the Krotov method for a time-nonlocal non-Markovian open quantum system has been presented and applied to obtain control sequences and gate errors for Z and identity gates.
- The control-dissipation correlation and the memory effect of the bath are crucial in achieving high-fidelity gates.
- Recent experiments on measuring noise spectral density, [e.g., J. Bylander et al., Nat. Phys. **7**, 565 (2011)], engineering external environment, simulating open quantum system, and observing non-Markovian dynamics could facilitate the experimental realization of the QOCT in non-Markovian open quantum systems in the near future.

Summary II

- The 3-qubit entangled $|W^{-}\rangle$ state can be generalized and stabilized with high fidelity in the circuit QED system in which the cooper pair box qubits are fixed in the transmission line resonator.

- This quantum feedback control scheme is simple, and promising. The scheme is robust against measurement inefficiency and differences in qubit decay rates.