Multimode homodyne detection as a tool for cluster states generation and gaussian quantum computation

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« Gdr Information quantique », 30 Novembre 2012

Can we use our multimode quantum optical experiment as a quantum computer ?

→ This work :

Theoretical study to guide next future quantum optics experiments
(N. Treps, C. Fabre, J. Roslund, R. Medeiros de Araujo, P. Jian, Y. Cai)

→ What we have :

Multimode squeezed states of light in a cavity
+
Multipixel homodyne detection system (next future)

→ What we want to do (this talk):

- Demonstrate quadratures measurement with cluster state statistics
- Study the applications to measurement based quantum computation
- ...

Outlines

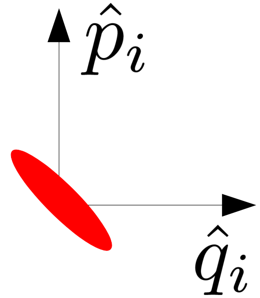
1. The physical system(s) + what we can do

2. Cluster states + an example of implementation

3. Measurement based quantum computation + an example of implementation

Experiment : several co-propagating modes

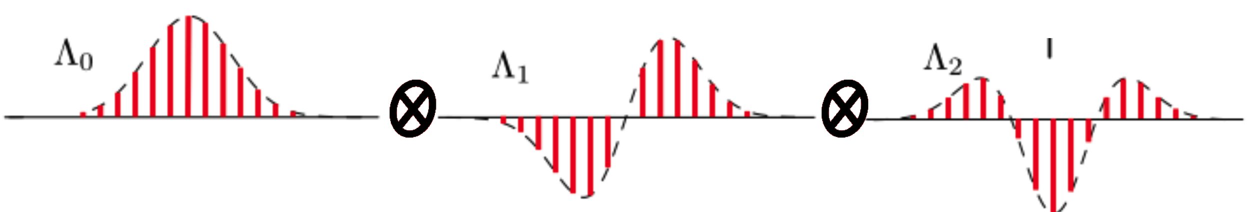
individually squeezed!



Formally described by

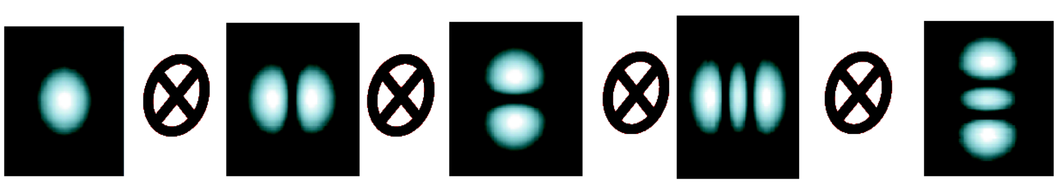
$$\vec{a}_{\mathrm{squ}} = \begin{pmatrix} \hat{a}_{\mathrm{squ}_1} \\ \hat{a}_{\mathrm{squ}_2} \\ \cdots \\ \hat{a}_{\mathrm{squ}_N} \end{pmatrix}$$

Either temporal...



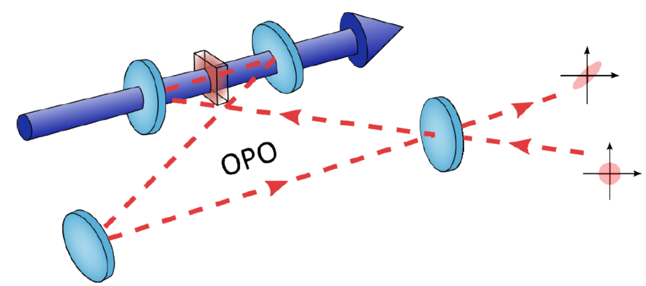
O. Pinel et al, PRL 108, 083601 (2012) ;
G. Patera et al, EPJD 56, 123 (2012) ***PARIS***

...or spatial.



S. Armstrong et al, Nat. Comm. 3, 1026 (2012) ***CANBERRA***

femto-second laser pump
+ non-linear crystal
+ cavity (SPOPO)

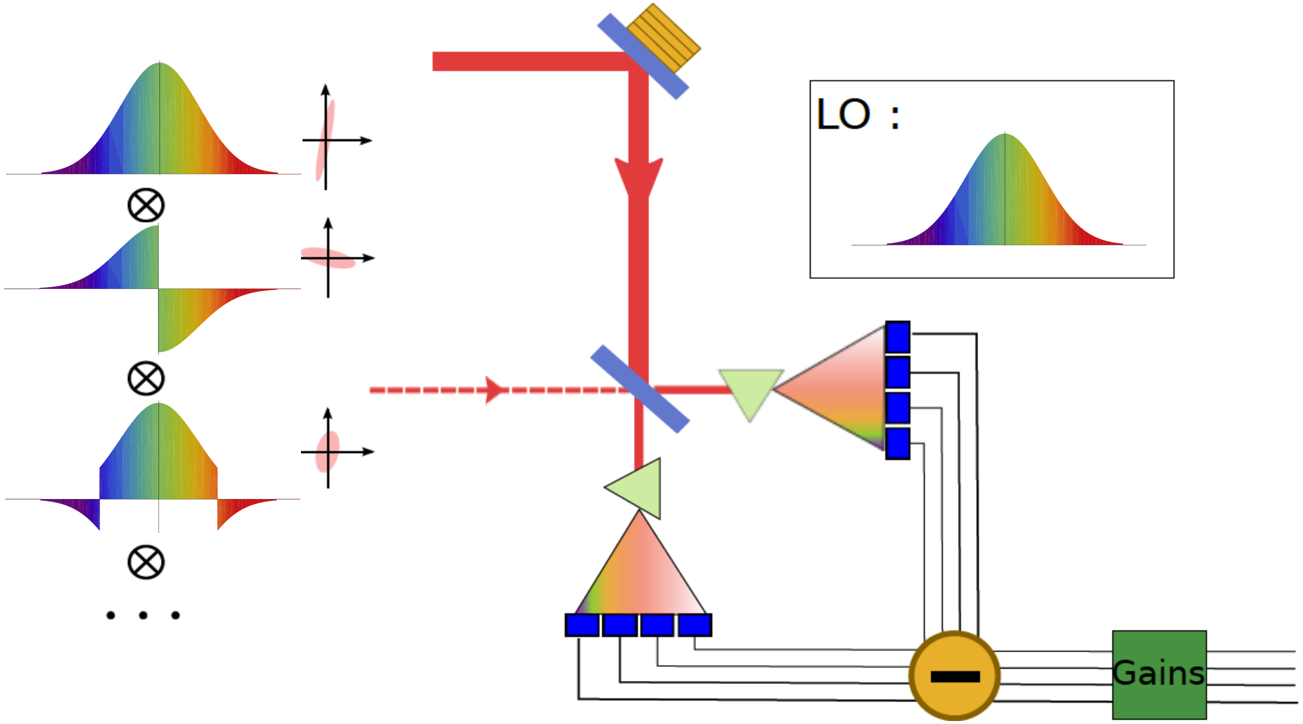


laser pumps
+ non-linear crystals
+ several cavities, then combined (OPOs)

MultiPixel Homodyne Detection (MPHD)

Temporal modes

$\hat{a}_{\mathrm{squ}_i}$



MultiPixel Homodyne Detection (MPHD)

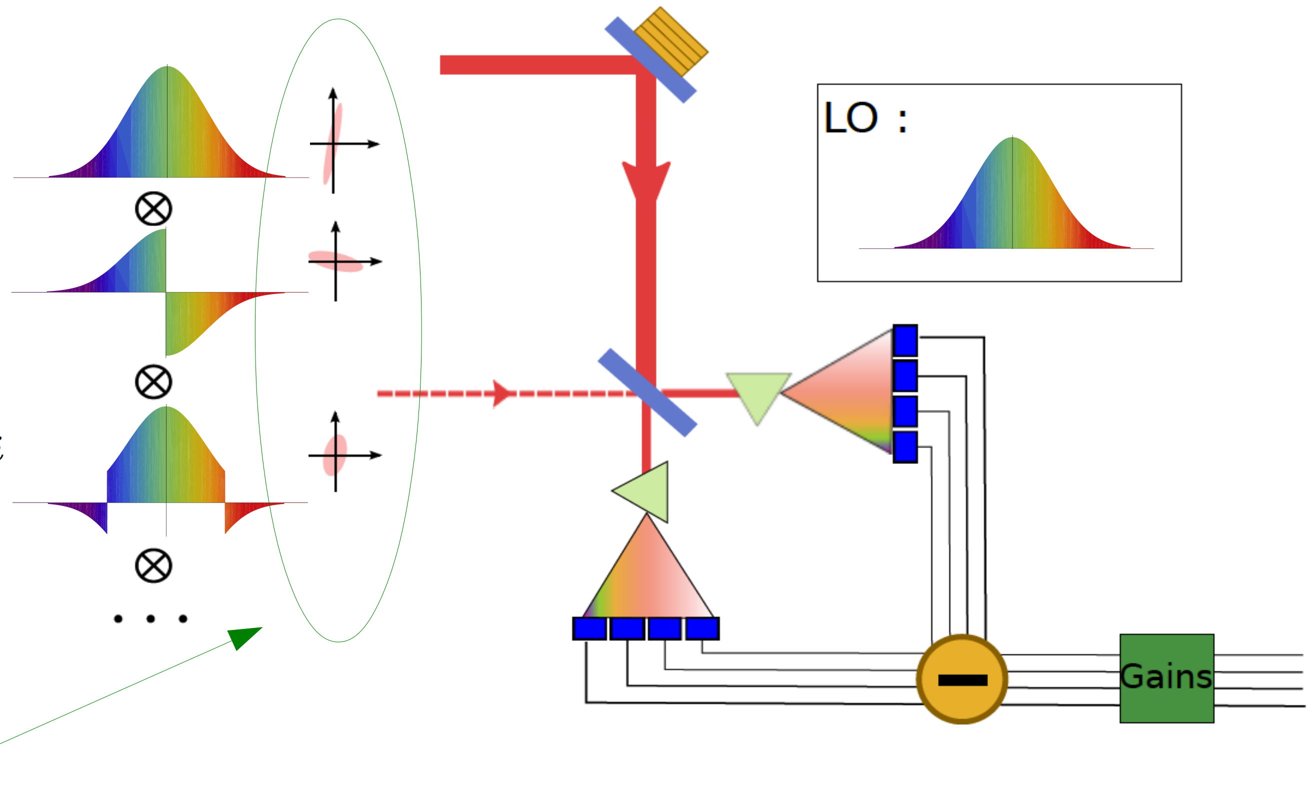
Temporal modes

$\hat{a}_{\mathrm{squ}_i}$

1) input dephasing

$$\hat{a}_{\mathrm{squ'}_i} = e^{i\varphi_i}\hat{a}_{\mathrm{squ}_i}$$

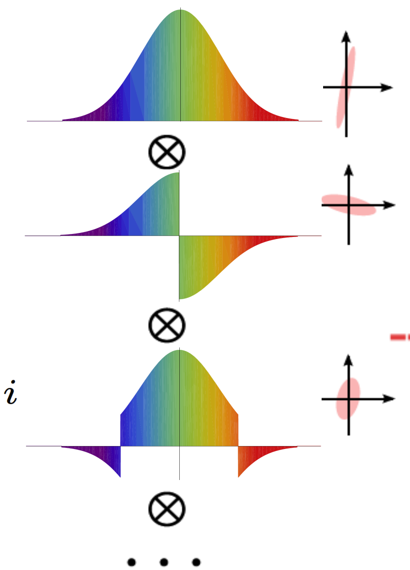
$$D_r$$



MultiPixel Homodyne Detection (MPHD)

Temporal modes

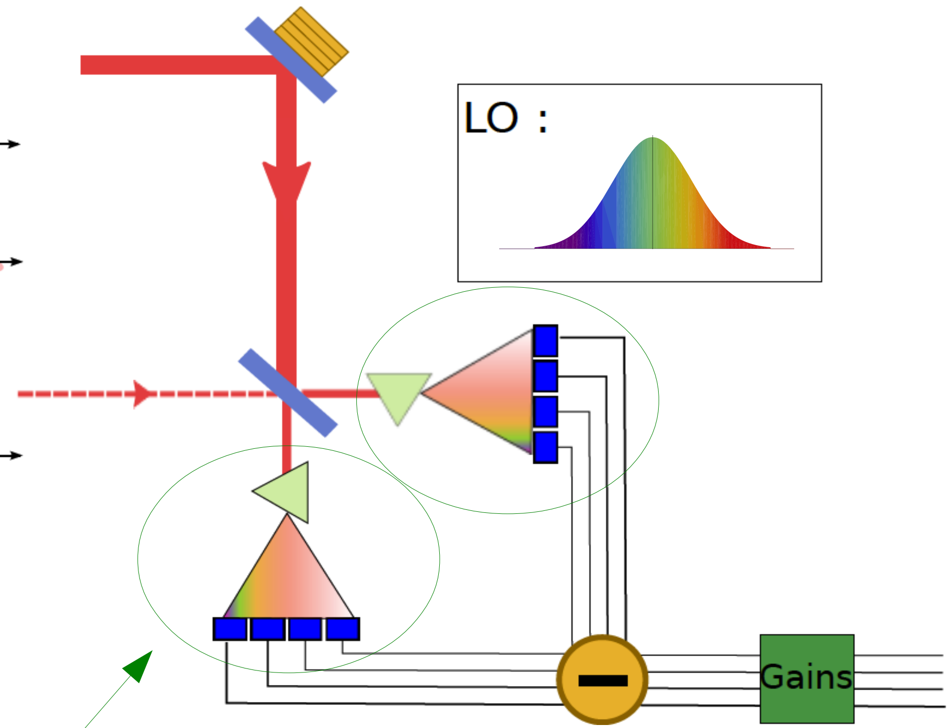
$$\hat{a}_{\text{squ}_i}$$



1) input dephasing

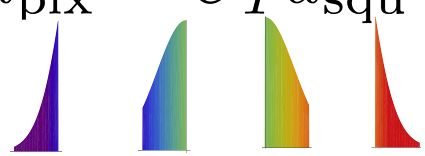
$$\hat{a}_{\text{squ'}_i} = e^{i\varphi_i}\hat{a}_{\text{squ}_i}$$

$$D_r$$

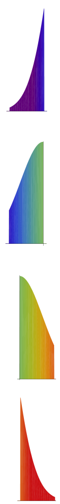


2) detection in the pixel basis

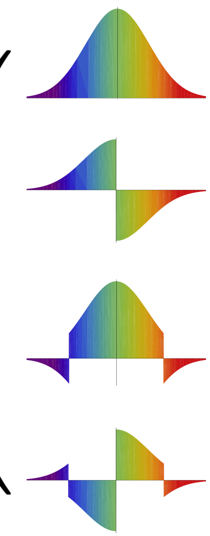
$$\vec{a}_{\text{pix}} = U_T \vec{a}_{\text{squ'}}$$



Detection matrix (example : 4 modes)



$$\left(\; \right) = \kappa \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}^{-1} \left(\; \right)$$



output of the detectors :
pixel modes

detection matrix U_T
(κ normalization constant)

input of the detectors :
flip modes

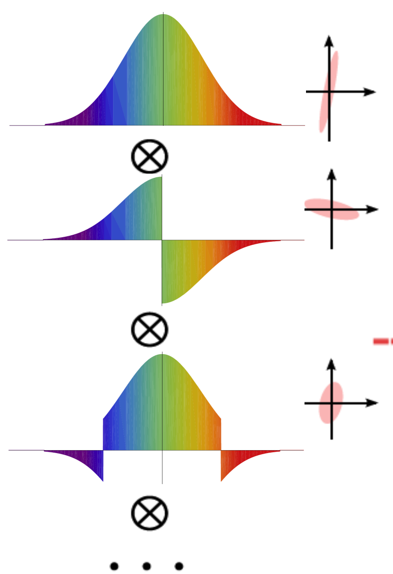
$$I_{-}^{\varphi_0} \propto \hat{q}_{\mathrm{pix}_i}^{\varphi_0} = (\hat{q}_{\mathrm{pix}_i} \cos\varphi_0 - \hat{p}_{\mathrm{pix}_i} \sin\varphi_0)$$

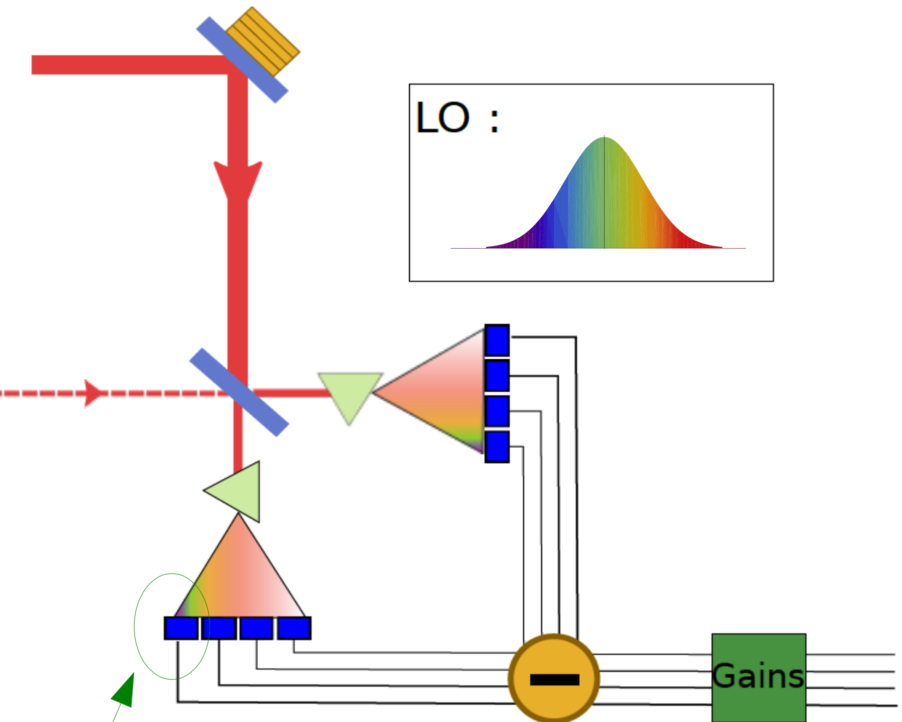
φ_0 global phase of the local oscillator

MultiPixel Homodyne Detection (MPHD)

Temporal modes

$$\hat{a}_{\mathrm{squ}_i}$$





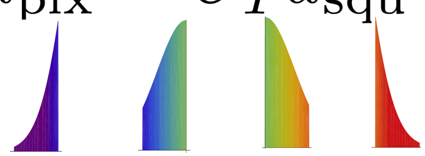
1) input dephasing

$$\hat{a}_{\mathrm{squ'}_i} = e^{i\varphi_i}\hat{a}_{\mathrm{squ}_i}$$

$$D_r$$

2) detection in the pixel basis

$$\vec{a}_{\mathrm{pix}} = U_T \vec{a}_{\mathrm{squ'}}$$



3) pixel dephasing

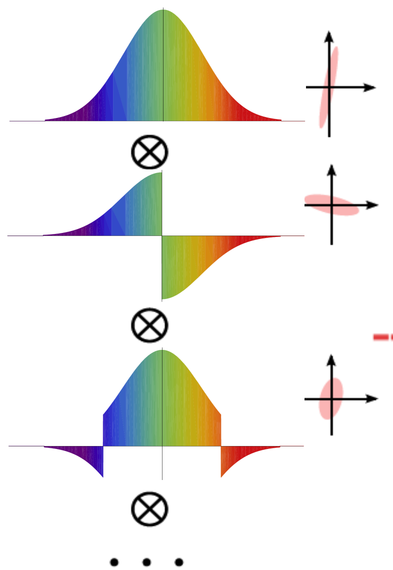
$$\hat{a}_{\mathrm{pix'}_i} = e^{i\phi_i}\hat{a}_{\mathrm{pix}_i}$$

$$\Delta_{\mathrm{LO}}$$

MultiPixel Homodyne Detection (MPHD)

Temporal modes

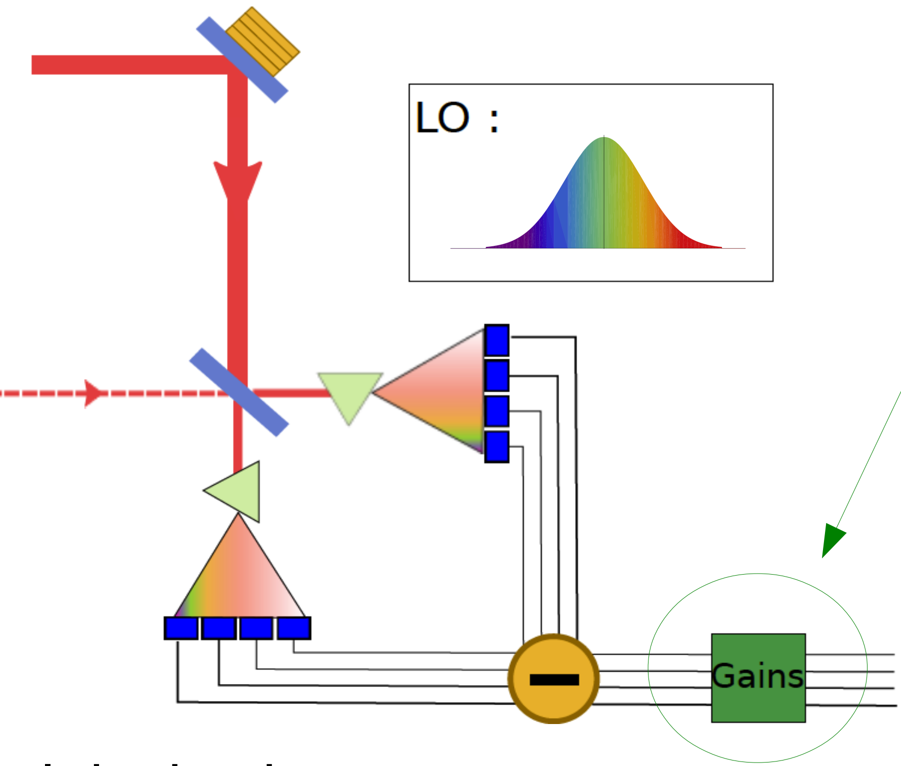
$$\hat{a}_{\mathrm{squ}_i}$$



1) input dephasing

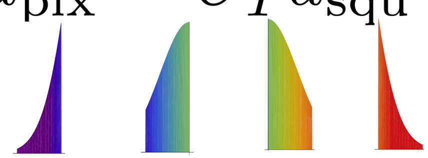
$$\hat{a}_{\mathrm{squ'}_i} = e^{i\varphi_i}\hat{a}_{\mathrm{squ}_i}$$

$$D_r$$



2) detection in the pixel basis

$$\vec{a}_{\mathrm{pix}} = U_T \vec{a}_{\mathrm{squ'}}$$



3) pixel dephasing

$$\hat{a}_{\mathrm{pix'}_i} = e^{i\phi_i}\hat{a}_{\mathrm{pix}_i}$$

$$\Delta_{\mathrm{LO}}$$

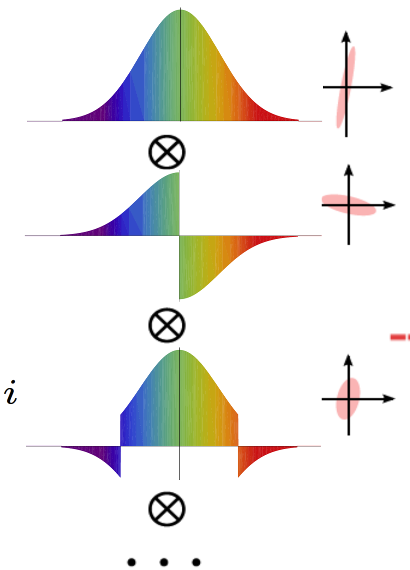
4) digital recombination

$$\vec{a}_{\mathrm{out}} = O\vec{a}_{\mathrm{pix'}}$$

MultiPixel Homodyne Detection (MPHD)

Temporal modes

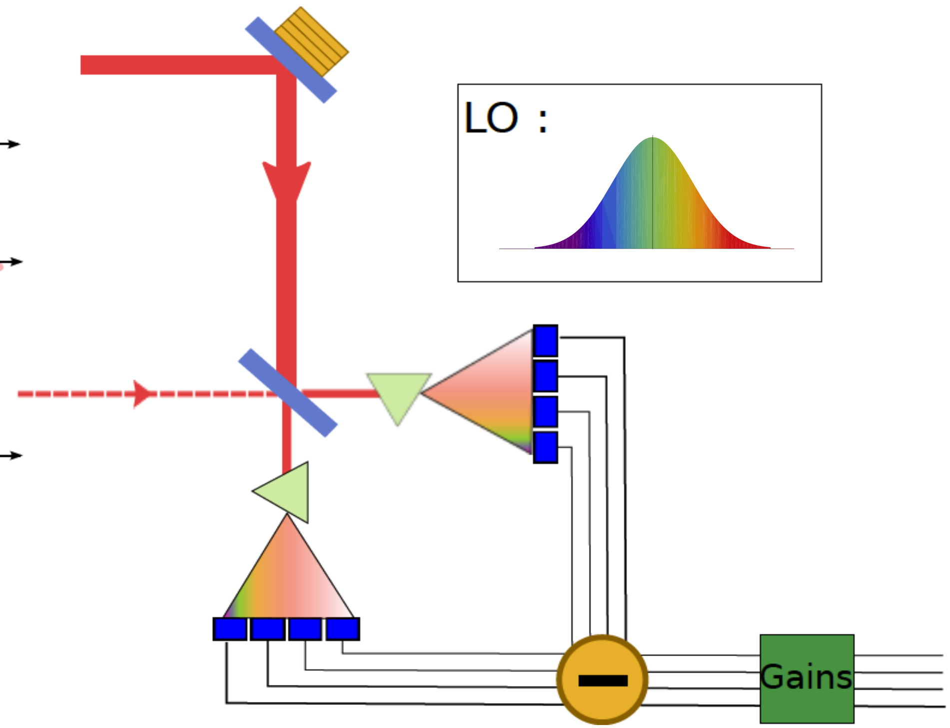
$$\hat{a}_{\mathrm{squ}_i}$$



1) input dephasing

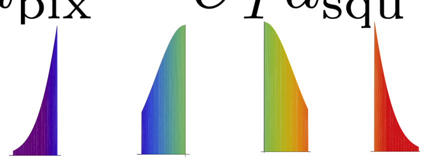
$$\hat{a}_{\mathrm{squ'}_i} = e^{i\varphi_i}\hat{a}_{\mathrm{squ}_i}$$

$$D_r$$



2) detection in the pixel basis

$$\vec{a}_{\mathrm{pix}} = U_T \vec{a}_{\mathrm{squ'}}$$



3) pixel dephasing

$$\hat{a}_{\mathrm{pix'}_i} = e^{i\phi_i}\hat{a}_{\mathrm{pix}_i}$$

$$\Delta_{\mathrm{LO}}$$

4) digital recombination

$$\vec{a}_{\mathrm{out}} = O\vec{a}_{\mathrm{pix'}}$$

$$\rightarrow \hat{p}_{\mathrm{out}_i}$$

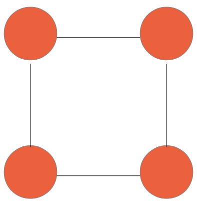
$$\vec{a}_{\mathrm{out}} = O\Delta_{\mathrm{LO}} U_T D_r \vec{a}_{\mathrm{squ}} = U_{\mathrm{MPHD}} \vec{a}_{\mathrm{squ}}$$

UNITARY MATRIX

M.Beck, PRL 84 5748 (2000) ; J-F. Morizur, PhD thesis (2011)

Why is this interesting? Cluster states :

Operational definition :



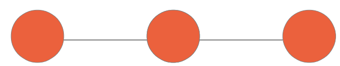
- Take *N* (infinitely) squeezed modes

- Apply Cz gates according to graph matrix *V*

e.g. :

$$C_Z(|\psi\rangle_i \otimes |\psi\rangle_j) = e^{i\hat{q}_i \otimes \hat{q}_j}(|\psi\rangle_i \otimes |\psi\rangle_j)$$

$$V = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$



Symplectic description : $\begin{pmatrix} \vec{a}_{\mathrm{clu}} \\ \vec{a}^{\dagger}_{\mathrm{clu}} \end{pmatrix} = \begin{pmatrix} I & 0 \\ V & I \end{pmatrix} \begin{pmatrix} \vec{a}_{\mathrm{squ}} \\ \vec{a}^{\dagger}_{\mathrm{squ}} \end{pmatrix}$

due to Bloch-Messiah decomposition : $\begin{pmatrix} \vec{a}_{\mathrm{clu}} \\ \vec{a}^{\dagger}_{\mathrm{clu}} \end{pmatrix} = \begin{pmatrix} U & 0 \\ 0 & U^* \end{pmatrix} \begin{pmatrix} \vec{a}_{\mathrm{squ}} \\ \vec{a}^{\dagger}_{\mathrm{squ}} \end{pmatrix}$ UNITARY MATRIX

P. Van Loock et al, PRA 76, 032031 (2007) ;

→ If $U_{\mathrm{MPHD}} = U$ we can mimic cluster states statistics with our detection system!

Which transformations can we implement?

Given a certain transformation *U,* can I implement it in the lab, i .e.

$\exists\ U_{\mathrm{MPHD}}$ such that $U = U_{\mathrm{MPHD}}$?

YES ! $\Leftrightarrow$ $\boxed{{U'}^T U' = D}$ with $U' = U(U_T D_r)^\dagger$

D any diagonal matrix with unitary modulus elements

Necessity :

$$U = U_{\mathrm{MPHD}} = O\Delta_{\mathrm{LO}} U_T D_r$$

$\Rightarrow$ $U' = O\Delta_{\mathrm{LO}}$ and ${U'}^T U' = \Delta_{\mathrm{LO}}^T O^T O \Delta_{\mathrm{LO}} = \Delta_{\mathrm{LO}}^2 = D$

Sufficiency :

${U'}^T U' = D$ $\Rightarrow$ $\begin{aligned} \Delta_{\mathrm{LO}} &= D^{\frac{1}{2}} \\ O &= U' \Delta_{\mathrm{LO}}^{-1} \end{aligned}$ used in practice to find the experimental parameter to used!

An example : four-mode linear cluster state

$$\vec{a}_{\rm clu} = U_{\rm lin}\vec{a}_{\rm squ}$$

with $$U_{\rm lin} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{10}} & \frac{2i}{\sqrt{10}} & 0 \\ \frac{i}{\sqrt{2}} & -\frac{i}{\sqrt{10}} & \frac{2}{\sqrt{10}} & 0 \\ 0 & -\frac{2}{\sqrt{10}} & \frac{i}{\sqrt{10}} & \frac{i}{\sqrt{2}} \\ 0 & -\frac{2i}{\sqrt{10}} & -\frac{1}{\sqrt{10}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

We can find a solution for the experimental parameters

$$U_{\rm lin} = U_{\rm MPHD} = O\Delta_{\rm LO}U_T D_r$$

$$\gamma \equiv \arctan(1/2)$$

$$\Delta_{\rm LO} = \begin{pmatrix} e^{i\frac{\gamma+\pi}{2}} & 0 & 0 & 0 \\ 0 & e^{-i\frac{\gamma}{2}} & 0 & 0 \\ 0 & 0 & e^{i\frac{\gamma}{2}} & 0 \\ 0 & 0 & 0 & e^{-i\frac{\gamma+\pi}{2}} \end{pmatrix}$$

$$O = \begin{pmatrix} -\frac{1}{2}\sqrt{1-\frac{2}{\sqrt{5}}} & \frac{1}{2}\sqrt{1+\frac{2}{\sqrt{5}}} & \sqrt{\frac{1}{4}+\frac{1}{2\sqrt{5}}} & -\frac{1}{2}\sqrt{1-\frac{2}{\sqrt{5}}} \\ \sqrt{\frac{1}{4}+\frac{1}{2\sqrt{5}}} & -\frac{1}{2}\sqrt{1-\frac{2}{\sqrt{5}}} & \frac{1}{2}\sqrt{1-\frac{2}{\sqrt{5}}} & -\frac{1}{2}\sqrt{1+\frac{2}{\sqrt{5}}} \\ \sqrt{\frac{1}{4}+\frac{1}{2\sqrt{5}}} & \frac{1}{2}\sqrt{1-\frac{2}{\sqrt{5}}} & \frac{1}{2}\sqrt{1-\frac{2}{\sqrt{5}}} & \frac{1}{2}\sqrt{1+\frac{2}{\sqrt{5}}} \\ -\frac{1}{2}\sqrt{1-\frac{2}{\sqrt{5}}} & -\frac{1}{2}\sqrt{1+\frac{2}{\sqrt{5}}} & \sqrt{\frac{1}{4}+\frac{1}{2\sqrt{5}}} & \frac{1}{2}\sqrt{1-\frac{2}{\sqrt{5}}} \end{pmatrix}$$

U_T fixed detection matrix ; $D_r = \mathrm{diag}(1, i, i, 1)$

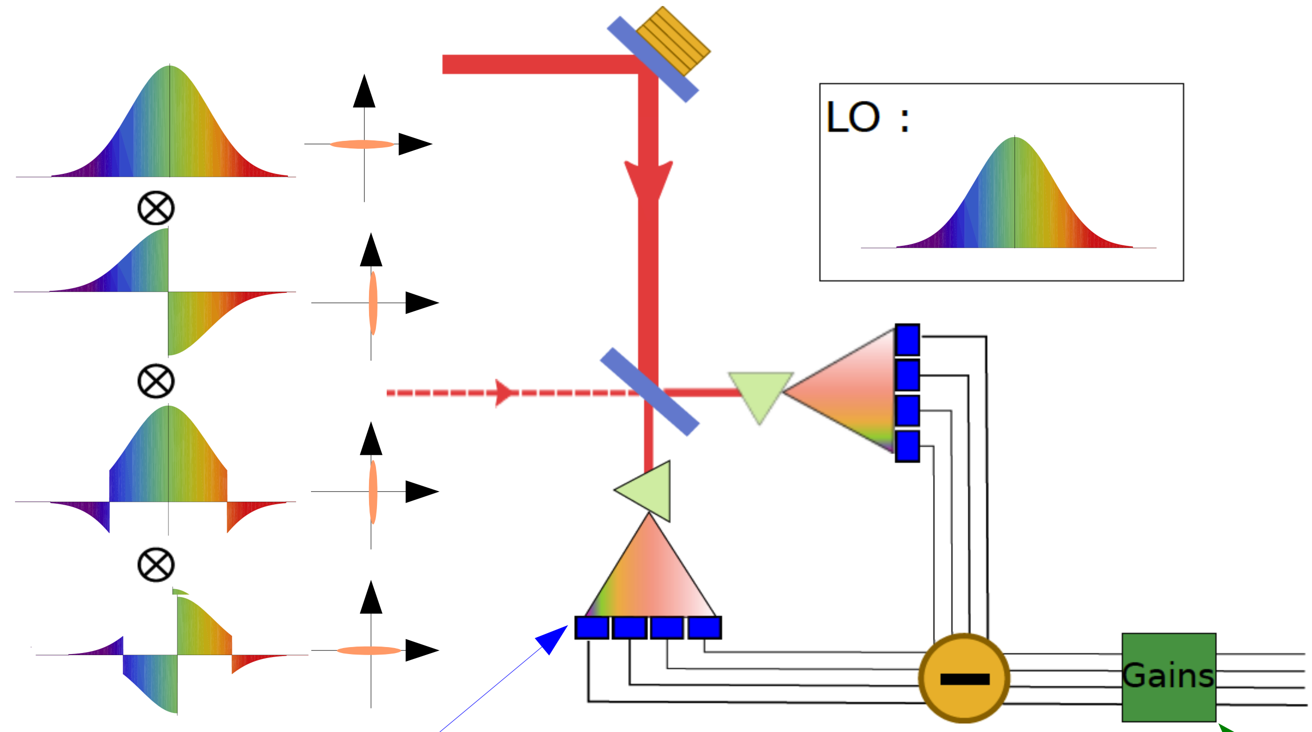
So in practice we have to set...

Temporal modes

a_{squ_i}

input dephasing

$D_r = \mathrm{diag}(1, i, i, 1)$

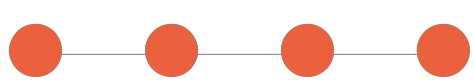


pixel dephasing

$$\Delta_{\mathrm{LO}} = \begin{pmatrix} e^{i1.8} & 0 & 0 & 0 \\ 0 & e^{-i0.23} & 0 & 0 \\ 0 & 0 & e^{i0.23} & 0 \\ 0 & 0 & 0 & e^{i1.8} \end{pmatrix}$$

digital recombination

$$O = \begin{pmatrix} -0.16 & 0.69 & 0.69 & -0.16 \\ 0.69 & -0.16 & 0.16 & -0.69 \\ 0.69 & 0.16 & 0.16 & 0.69 \\ -0.16 & -0.69 & 0.69 & 0.16 \end{pmatrix}$$

$\Rightarrow$ $\hat{p}_{\mathrm{out}_i}$ as for 

CV quantum computation

Definition :

« A system is a universal quantum computer if it can simulate the action of a Hamiltonian consisting of a general polynomial of $\hat{q}$ and $\hat{p}$ to any fixed accuracy »

$$|\psi\rangle_{\text{out}} = e^{iH(\hat{q},\hat{p})t}|\psi\rangle_{\text{in}}$$

M. Gu et al, PRA 79, 062318 (2009)

How is this reached ?

→ Universal gate set : $\left\{ e^{i\hat{q}s}, e^{i\hat{q}^2 s}, e^{i\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}, e^{i\hat{q}_1\otimes\hat{q}_2}, e^{i\hat{q}^3 s} \right\}$

$\{D_{f(\hat{q})}\}$ $\qquad F$

Single-mode gaussian $\qquad$ Two-mode $\qquad$ non-gaussian

→ Short time evolution (Trotter expansion)

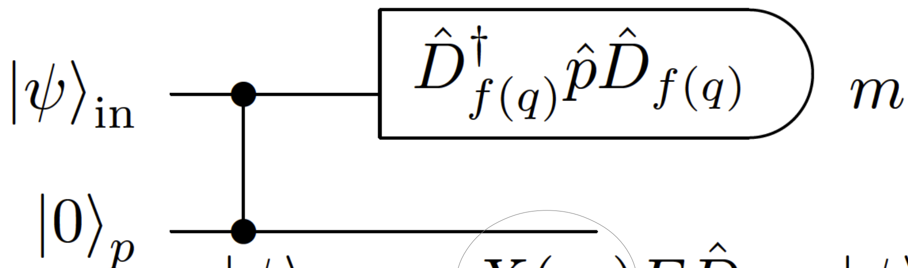
$$e^{iA\delta t}e^{iB\delta t}e^{-iA\delta t}e^{-iB\delta t} = e^{-[A,B]\delta t^2+O(\delta t^3)}$$

$$\{H_i\} \quad \Rightarrow \quad \pm i[H_i, H_j], \ \pm[H_i,[H_j,H_k]] \ \ldots$$

S. Lloyd and S. Braunstein, PRL 82, 1784 (1999)

Measurement based quantum computation

I want to implement the operation $D_{f(\hat{q})}$:



$$|\psi\rangle_{\text{out}} = X(m) F \hat{D}_{f(q)} |\psi\rangle_{\text{in}}$$

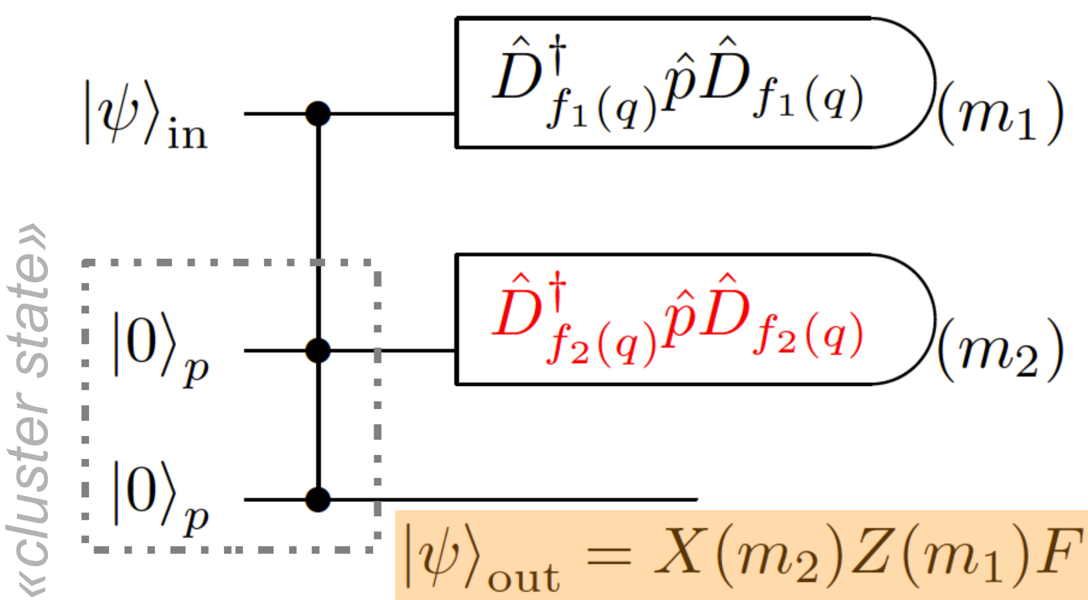
with  $\equiv e^{i\hat{q}_1 \otimes \hat{q}_2}$

(Cz gate)

and $X(s) = e^{-is\hat{p}}$

« by-product operator »

For more operations, repeat the previous circuit :

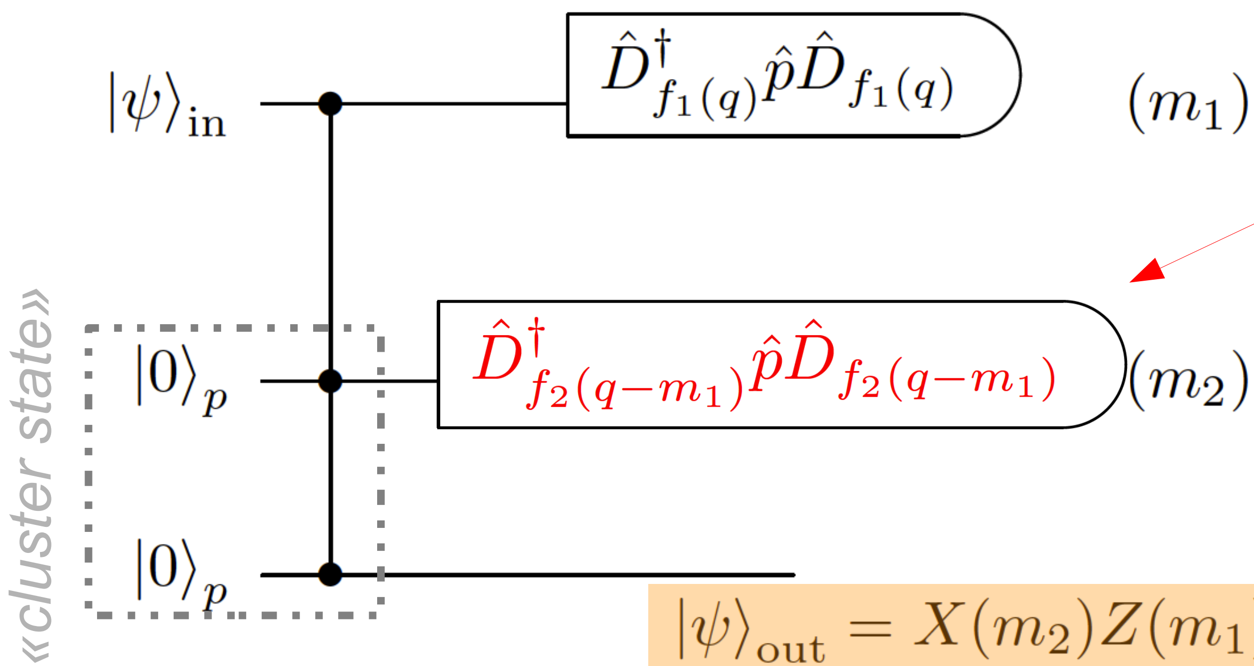


depends on the outcome of the first measurement !

$$|\psi\rangle_{\text{out}} = X(m_2) Z(m_1) F \hat{D}_{f_2(q+m_1)} F \hat{D}_{f_1(q)} |\psi\rangle_{\text{in}}$$

Measurement based quantum computation

To obtain deterministic operations one has to adapt the measurement basis depending on the outcomes of previous measurements



$$|\psi\rangle_{\rm out} = X(m_2)Z(m_1)F\hat{D}_{f_2(q)}F\hat{D}_{f_1(q)}|\psi\rangle_{\rm in}$$

M. Gu et al, PRA 79, 062318 (2009)

Implementation of Gaussian unitaries

→ by homodyne detection ; e.g : $D_{f(\hat{q})} = e^{is\hat{q}^2}$

$$e^{-is\hat{q}^2}\hat{p}e^{is\hat{q}^2} = \hat{p} + s\hat{q} = g(\hat{q}\sin\theta + \hat{p}\cos\theta)$$

with $g = \sqrt{1+s^2}$ and $\theta = \arctan s$

If I have to adapt my measurement basis :

$$e^{-is(\hat{q}+m)^2}\hat{p}e^{is(\hat{q}+m)^2} = \hat{p} + s\hat{q} + ms$$

Can be done while post-processing!

➡ *ALL GAUSSIAN MEASUREMENTS CAN BE PERFORMED SIMULTANEOUSLY («PARALLELISM»)*

M. Gu et al, PRA 79, 062318 (2009)

This is not the case for : cubic phase gate $D_{f(\hat{q})} = e^{i\gamma\frac{\hat{q}^3}{3}}$

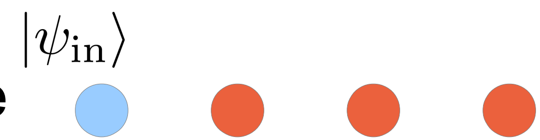
D. Gottesman et al, PRA 64, 012310 (2001)

A simple example : Fourier transform

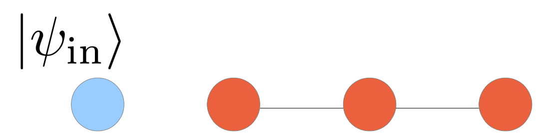
$$\begin{pmatrix} \hat{q}_{\text{out}} \\ \hat{p}_{\text{out}} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \hat{q}_{\text{in}} \\ \hat{p}_{\text{in}} \end{pmatrix} = \begin{pmatrix} -\hat{p}_{\text{in}} \\ \hat{q}_{\text{in}} \end{pmatrix}$$

How to realize that in MBQC?

1) Start with 3 squeezed modes and an input mode

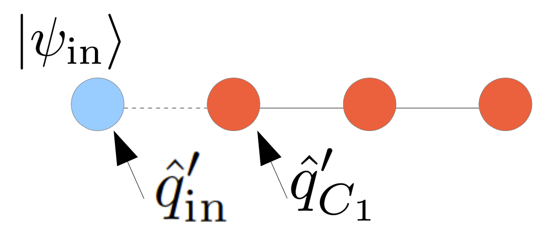


2) Create a 3-mode cluster state $\vec{a}_C = U_{\text{CL}}^{1,2,3}\vec{a}_{\text{squ}}$

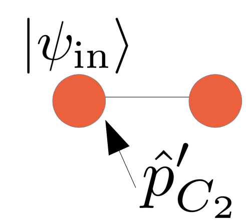


3) Mix the input state to the 1st mode of the cluster with a beam splitter

$$\begin{pmatrix} \hat{a}'_{\text{in}} \\ \hat{a}'_{\text{C}_1} \end{pmatrix} = U_{\text{BS}}^{\text{in},C_1} \begin{pmatrix} \hat{a}_{in} \\ \hat{a}_1 \end{pmatrix}$$



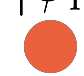
4) Measure $\hat{q}'_{\text{in}}$ and $\hat{q}'_{C_1}$ to teleport the input state



5) Measure $\hat{p}'_{C_2}$ on the 2nd mode of the cluster

6) Read out in arbitrary direction

$|\psi_{\text{out}}\rangle = F|\psi_{\text{in}}\rangle$



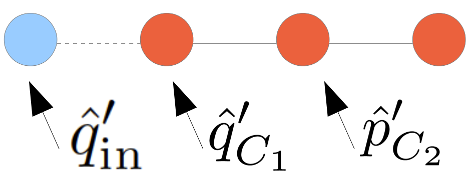
R. Ukai et al, PRA 81, 032315 (2010) ; **experiment :** *R. Ukai et al, PRL 81, 240504 (2011)*

Fourier transform of an input state

Can we implement it with our experiment ?

Starting from 3 squeezed modes + an input mode to be processed :

« Theory » :



$$
\begin{aligned}
U_{\mathrm{FT}} &= D_{\mathrm{meas}}(U_{\mathrm{BS}}^{\mathrm{in},1} \otimes \mathcal{I}^{2,3})(\mathcal{I}^{\mathrm{in}} \otimes U_{\mathrm{CL}}^{1,2,3}) \\
&= \begin{pmatrix} \frac{i}{\sqrt{2}} & 0 & \frac{1}{\sqrt{3}} & \frac{i}{\sqrt{6}} \\ -\frac{1}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 0 & -\frac{i}{\sqrt{2}} & -\frac{i}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & -\frac{i}{\sqrt{3}} \end{pmatrix}
\end{aligned}
$$

$\theta_3 = 0$

« Experiment » : $U_{\mathrm{MPHD}} = O\Delta_{\mathrm{LO}} U_T D_r$

CAN WE SET $U_{\mathrm{MPHD}} = U_{\mathrm{FT}}$?

LET'S CHECK WITH OUR METHOD !

Solution for the Fourier Transform

Compute ${U'}^T U' = D$, then $\Delta_{\mathrm{LO}} = D^{\frac{1}{2}} \qquad O = U' \Delta_{\mathrm{LO}}^{-1}$

$$O = \begin{pmatrix} \frac{1}{2}\sqrt{1+\sqrt{\frac{2}{3}}} & \frac{1}{2\sqrt{3+\sqrt{6}}} & -\frac{1}{2\sqrt{3+\sqrt{6}}} & -\frac{1}{2}\sqrt{1+\sqrt{\frac{2}{3}}} \\ \frac{1}{2\sqrt{3+\sqrt{6}}} & -\frac{1}{2}\sqrt{1+\sqrt{\frac{2}{3}}} & -\frac{1}{2}\sqrt{1+\sqrt{\frac{2}{3}}} & \frac{1}{2\sqrt{3+\sqrt{6}}} \\ -\frac{1}{2}\sqrt{1+\sqrt{\frac{2}{3}}} & -\frac{1}{2\sqrt{3+\sqrt{6}}} & -\frac{1}{2\sqrt{3+\sqrt{6}}} & -\frac{1}{2}\sqrt{1+\sqrt{\frac{2}{3}}} \\ \frac{1}{2\sqrt{3+\sqrt{6}}} & -\frac{1}{2}\sqrt{1+\sqrt{\frac{2}{3}}} & \frac{1}{2}\sqrt{1+\sqrt{\frac{2}{3}}} & -\frac{1}{2\sqrt{3+\sqrt{6}}} \end{pmatrix}$$

$$\Delta_{\mathrm{LO}} = \begin{pmatrix} e^{i\frac{(\beta+\pi)}{2}} & 0 & 0 & 0 \\ 0 & e^{i\frac{\beta}{2}} & 0 & 0 \\ 0 & 0 & e^{-i\frac{\beta}{2}} & 0 \\ 0 & 0 & 0 & e^{-i\frac{(\beta+\pi)}{2}} \end{pmatrix}$$

$$\beta = \arctan 1/\sqrt{2}$$

$$D_r = \mathrm{diag}(1, 1, i, -i)$$

To tune the dephasings: act on pump shape or on the length of the crystal !

G. Patera et al, EPJD 66, 241 (2012)

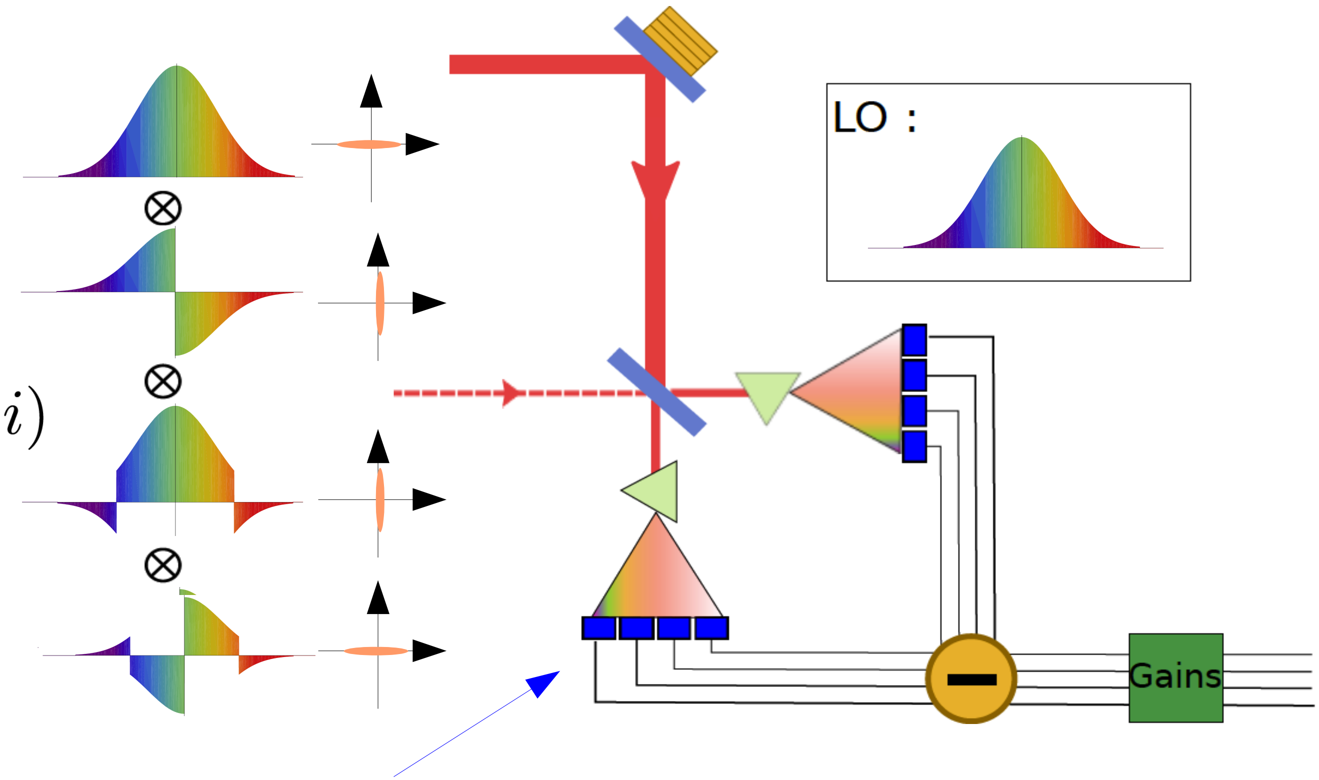
So in practice we have to set...

Temporal modes

a_{squ_i}

input dephasing

$$D_r = \mathrm{diag}(1, 1, i, -i)$$



pixel dephasing

$$\Delta_{\mathrm{LO}} = \begin{pmatrix} e^{i0.48} & 0 & 0 & 0 \\ 0 & e^{i2.0} & 0 & 0 \\ 0 & 0 & e^{-i0.48} & 0 \\ 0 & 0 & 0 & e^{-i2.0} \end{pmatrix}$$

digital recombination

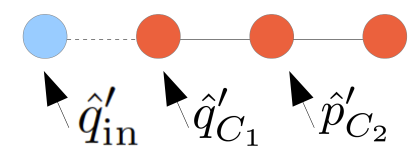
$$O = \begin{pmatrix} 0.63 & 0.33 & 0.63 & 0.33 \\ 0.33 & -0.63 & -0.33 & 0.63 \\ -0.33 & 0.63 & -0.33 & 0.63 \\ -0.63 & -0.33 & 0.63 & 0.33 \end{pmatrix}$$

➡ $\hat{p}_{\mathrm{out}_4}$ as for $F|\psi_{\mathrm{in}}\rangle$

Conclusions

- Possibility of measuring cluster state statistics in a compact way

- Possibility of performing gaussian measurement based quantum computation in a compact way



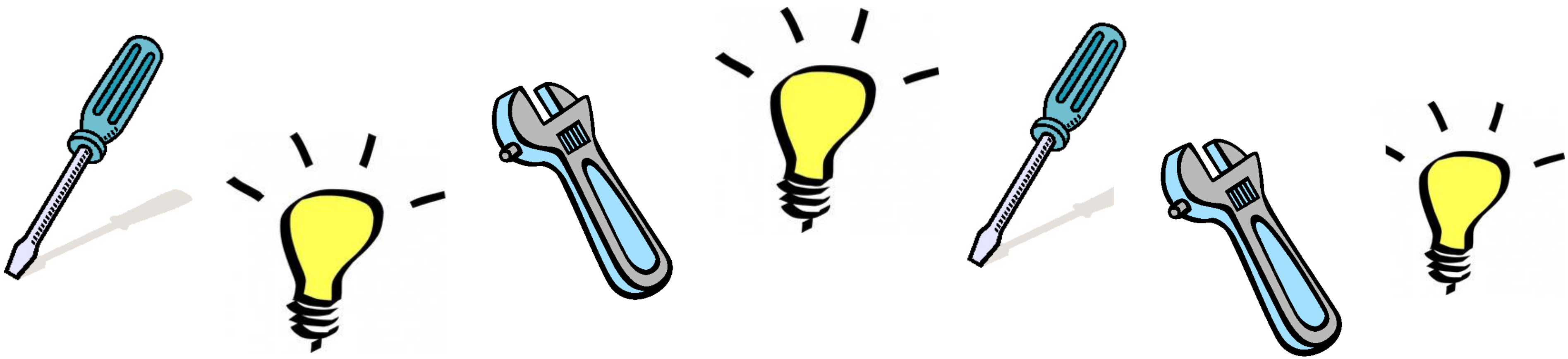
Perspectives

- Inclusion of finite-squeezing effects (in progress)

- Demonstrate more operations (e.g.: two-mode operations)
- Inclusion of a non-gaussian measurement to go beyond gaussian operations

And then...

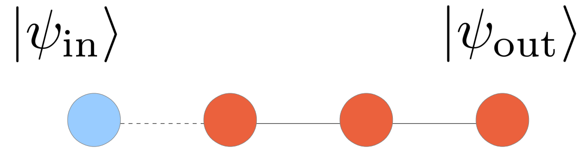
...do the experiment !!!



THANK YOU FOR YOUR ATTENTION !

A simple example : Fourier transform

$$\begin{pmatrix} \hat{q}_{\rm out} \\ \hat{p}_{\rm out} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \hat{q}_{\rm in} \\ \hat{p}_{\rm in} \end{pmatrix} = \begin{pmatrix} -\hat{p}_{\rm in} \\ \hat{q}_{\rm in} \end{pmatrix}$$



How to realize that in MBQC? Theory :

- Create a 3-mode cluster state

$$\vec{a}_C = U_{\rm CL}^{1,2,3} \vec{a}_{\rm squ}$$

$$U_{\rm CL}^{1,2,3} = \begin{pmatrix} 0 & -\sqrt{\frac{2}{3}} & -\frac{i}{\sqrt{3}} \\ -\frac{i}{\sqrt{2}} & -\frac{i}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & -\frac{i}{\sqrt{3}} \end{pmatrix}$$

- Attach the input state to the cluster via teleportation (BS + meas. $\hat{q}'_{\rm in}, \hat{q}'_1$)

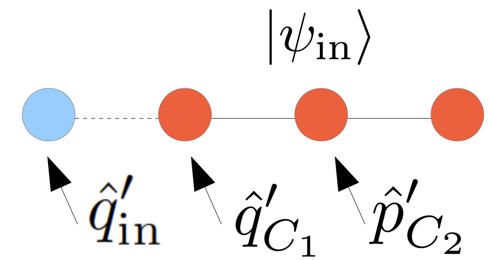
$$\begin{pmatrix} \hat{a}'_{\rm in} \\ \hat{a}'_{\rm C_1} \end{pmatrix} = U_{\rm BS}^{{\rm in},C_1} \begin{pmatrix} \hat{a}_{in} \\ \hat{a}_1 \end{pmatrix}$$

$$U_{\rm BS}^{{\rm in},C_1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

R. Ukai et al, PRA 81, 032315 (2010) ; **experiment :** *R. Ukai et al, PRL 81, 240504 (2011)*

A simple example : Fourier transform

$$\begin{pmatrix} \hat{q}_{\text{out}} \\ \hat{p}_{\text{out}} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \hat{q}_{\text{in}} \\ \hat{p}_{\text{in}} \end{pmatrix} = \begin{pmatrix} -\hat{p}_{\text{in}} \\ \hat{q}_{\text{in}} \end{pmatrix}$$



How to realize that in MBQC? Theory :

- Create a 3-mode cluster state

$$\vec{a}_C = U_{\text{CL}}^{1,2,3} \vec{a}_{\text{squ}} \qquad U_{\text{CL}}^{1,2,3} = \begin{pmatrix} 0 & -\sqrt{\frac{2}{3}} & -\frac{i}{\sqrt{3}} \\ -\frac{i}{\sqrt{2}} & -\frac{i}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & -\frac{i}{\sqrt{3}} \end{pmatrix}$$

- Attach the input state to the cluster via teleportation (BS + meas. $\hat{q}'_{\text{in}}, \hat{q}'_1$)

$$\begin{pmatrix} \hat{a}'_{\text{in}} \\ \hat{a}'_{\text{C}_1} \end{pmatrix} = U_{\text{BS}}^{\text{in},C_1} \begin{pmatrix} \hat{a}_{in} \\ \hat{a}_1 \end{pmatrix} \qquad U_{\text{BS}}^{\text{in},C_1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

- Measure $\hat{p}'_{C_2}$ on the node 2 of the cluster; node 3 contains $|\psi_{\text{out}}\rangle = F|\psi_{\text{in}}\rangle$

Equivalent to applying rotation $\vec{a}'' = D_{\text{meas}} \vec{a}'$ $\quad D_{\text{meas}} = \text{diag}(e^{i\theta_{\text{in}}}, e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_3})$

and measuring $\hat{p}''_i = (\hat{q}'_i \sin\theta_i + \hat{p}'_i \cos\theta_i)$ $\qquad \{\theta_i\} = (\pi/2, \pi/2, 0, \theta_3)$

R. Ukai et al, PRA 81, 032315 (2010) ; experiment : R. Ukai et al, PRL 81, 240504 (2011)

Finite squeezing effects

Cluster state generation with a QND interaction

$$S_{K_{\mathrm{EXP}},V} = \begin{pmatrix} I & 0 \\ V & I \end{pmatrix} \begin{pmatrix} K_{\mathrm{EXP}}^{-\frac{1}{2}} & 0 \\ 0 & K_{\mathrm{EXP}}^{\frac{1}{2}} \end{pmatrix}$$

Cluster state generation with linear optics only (+ squeezing off-line)

$$S_{U',V'} = \begin{pmatrix} X_V & -Y_V \\ Y_V & X_V \end{pmatrix} \begin{pmatrix} K_{\mathrm{EXP}}^{-\frac{1}{2}} & 0 \\ 0 & K_{\mathrm{EXP}}^{\frac{1}{2}} \end{pmatrix}$$

Finite squeezing effects

$$\begin{pmatrix} \vec{x}\,' \\ \vec{p}\,' \end{pmatrix} = \begin{pmatrix} X_V & -Y_V \\ Y_V & X_V \end{pmatrix} \begin{pmatrix} K_{\mathrm{EXP}}^{-\frac{1}{2}} & 0 \\ 0 & K_{\mathrm{EXP}}^{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} \vec{x}^{(0)} \\ \vec{p}^{(0)} \end{pmatrix}$$

$$= \begin{pmatrix} X_V K_{\mathrm{EXP}}^{-\frac{1}{2}} \vec{x}^{(0)} - Y_V K_{\mathrm{EXP}}^{\frac{1}{2}} \vec{p}^{(0)} \\ Y_V K_{\mathrm{EXP}}^{-\frac{1}{2}} \vec{x}^{(0)} + X_V K_{\mathrm{EXP}}^{\frac{1}{2}} \vec{p}^{(0)} \end{pmatrix}.$$

$$\vec{p}^{C} - V\vec{x}^{C} = 0$$

$$(Y_V - VX_V) K_{\mathrm{EXP}}^{-\frac{1}{2}} \vec{x}^{(0)} = 0 \quad \Rightarrow \quad (Y_V - VX_V) = 0;$$

$$(X_V + VY_V) K_{\mathrm{EXP}}^{\frac{1}{2}} \vec{p}^{(0)} \to 0 \qquad \text{excess noise.}$$

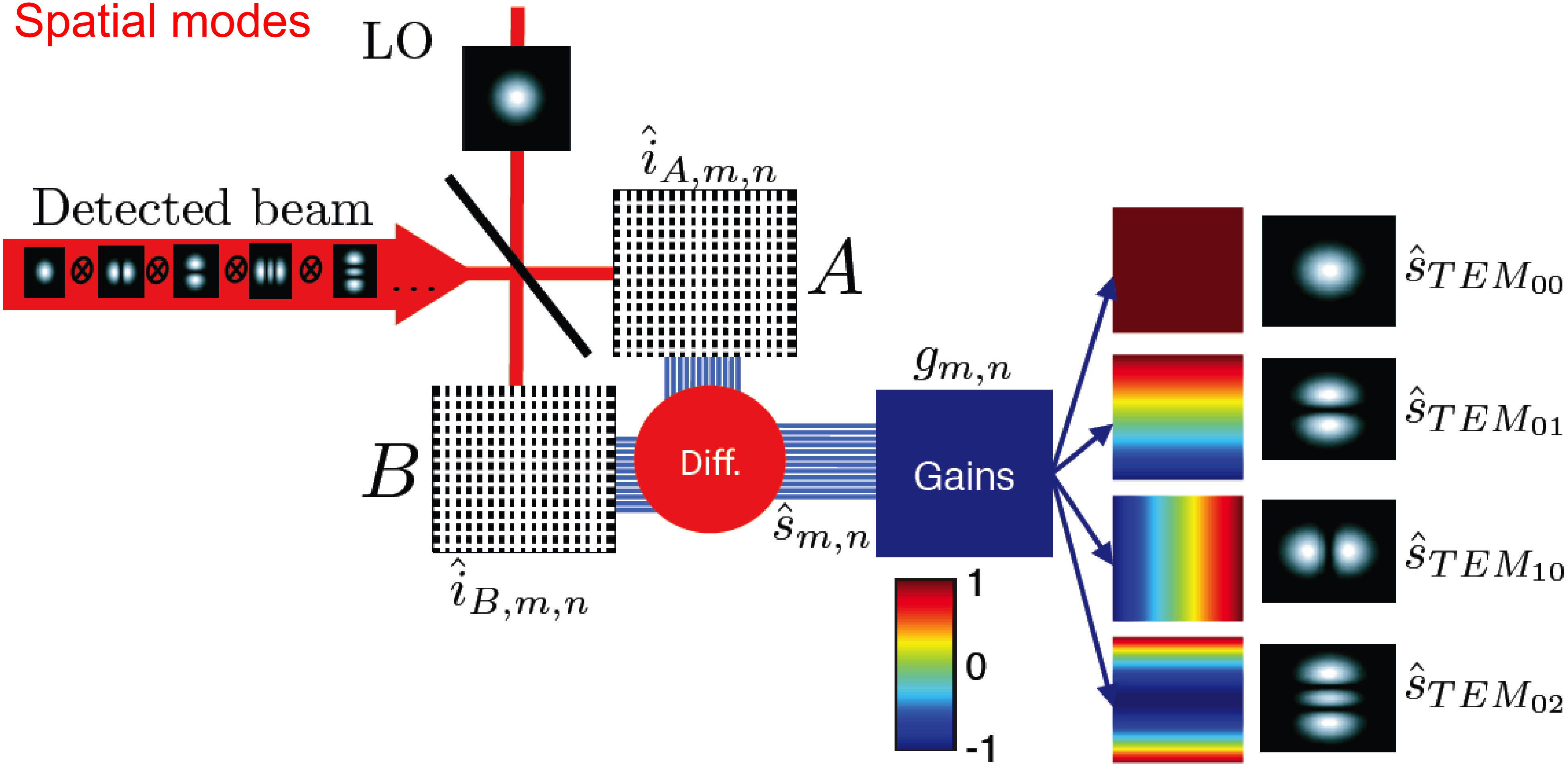
Finite squeezing effects

Excess noise quadratures

$$
\begin{aligned}
\hat{\delta}_1 &= \hat{p}_1^{'} - \sum_l V_{1l}\hat{x}_l^{'} = \hat{p}_1^{'} - \hat{x}_2^{'} = \sqrt{2}e^{-\mathrm{r}1}\hat{p}_1^{(0)} \\
\hat{\delta}_2 &= \hat{p}_2^{'} - \sum_l V_{2l}\hat{x}_l^{'} = \hat{p}_2^{'} - \hat{x}_1^{'} - \hat{x}_3^{'} = \sqrt{\frac{5}{2}}e^{-\mathrm{r}3}\hat{p}_3^{(0)} + \frac{e^{-\mathrm{r}4}}{\sqrt{2}}\hat{p}_4^{(0)} \\
\hat{\delta}_3 &= \hat{p}_3^{'} - \sum_l V_{3l}\hat{x}_l^{'} = \hat{p}_3^{'} - \hat{x}_2^{'} - \hat{x}_4^{'} = \frac{e^{-\mathrm{r}1}}{\sqrt{2}}\hat{p}_1^{(0)} - \sqrt{\frac{5}{2}}e^{-\mathrm{r}2}\hat{p}_2^{(0)} \\
\hat{\delta}_4 &= \hat{p}_4^{'} - \sum_l V_{4l}\hat{x}_l^{'} = \hat{p}_4^{'} - \hat{x}_3^{'} = \sqrt{2}e^{-\mathrm{r}4}\hat{p}_4^{(0)}
\end{aligned}
$$

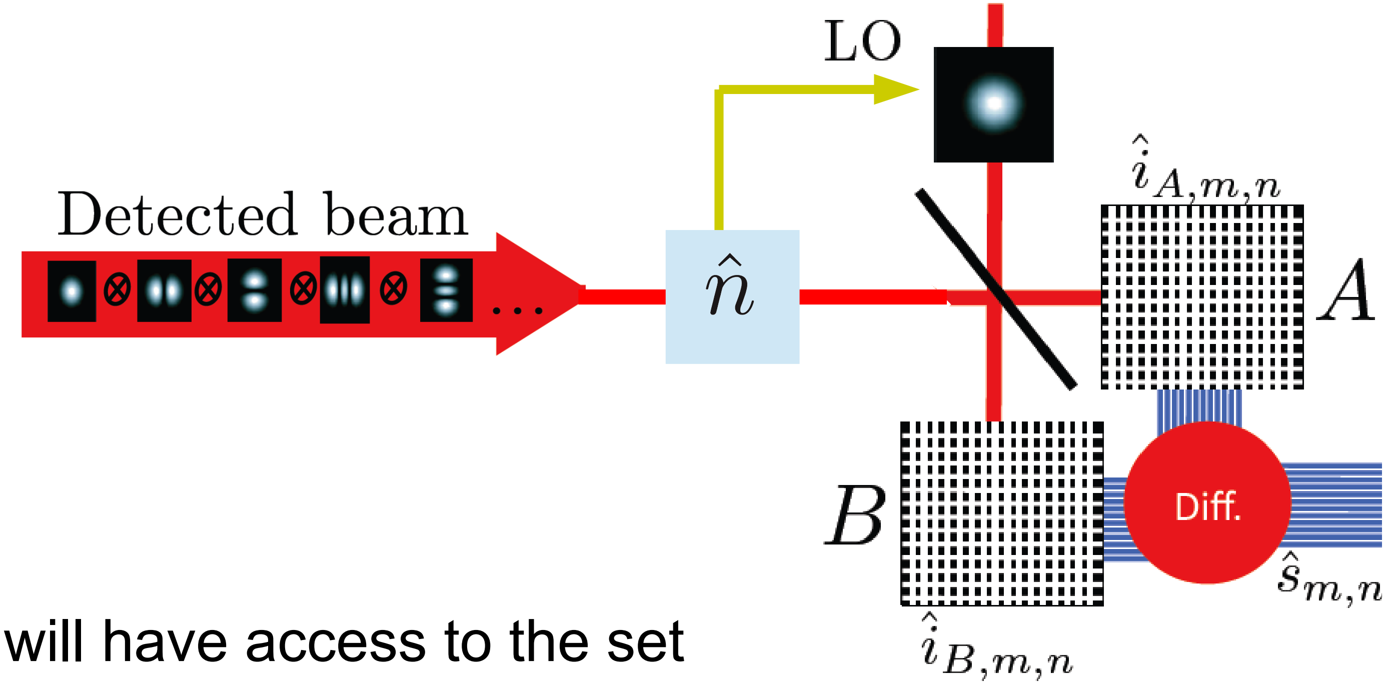
MultiPixel Homodyne Detection (MPHD)

Spatial modes



J-F. Morizur, PhD thesis (2011) ; S. Amstrong et al, Nat. Comm.... (2012)

At best, we will be able to...



Hence we will have access to the set

$$e^{i\gamma\frac{\hat{q}^3}{3}} \cdot \{\text{set of accessible gaussian operations}\}$$

- *Which gaussian operations are we able to implement ?*
- *Which total H are we able to implement ?*
  Do they correspond to any interesting physical phenomenon ?

Implementation of non-Gaussian operations

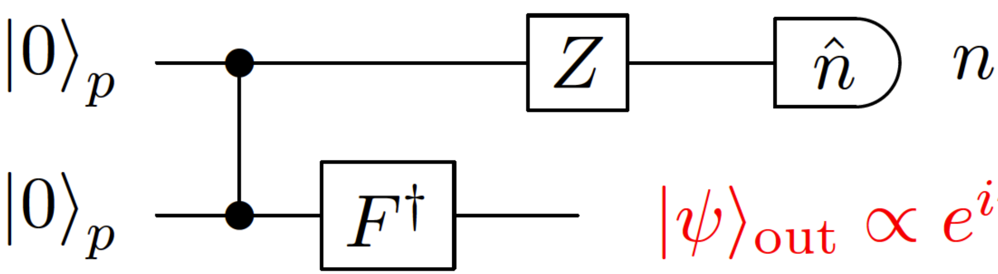
Simplest example : cubic phase gate $D_{f(\hat{q})} = e^{i\gamma \frac{\hat{q}^3}{3}}$

$$\hat{p}_{\gamma(\hat{q}+m)^3/3} = \hat{p} + \gamma\hat{q}^2 + 2ms\hat{q} + m^2\gamma$$ requires actual adaptation...

...which we cannot do! $\rightarrow$ *at best, we will be able to implement it ONCE !*

Acting the gate on a special initial state :

$$Z(s) \quad = \quad e^{is\hat{q}}$$



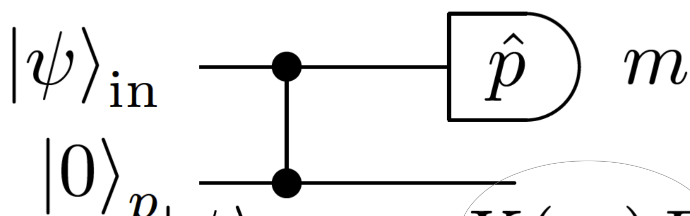
D. Gottesman et al, PRA 64, 012310 (2001)

$$|\psi\rangle_{\text{out}} \propto e^{i\gamma(n)\hat{q}^3}|0\rangle_p \qquad \gamma(n) = \frac{1}{6\sqrt{2n+1}}$$

→ *Non-deterministic non-gaussian state*

Even for a single first implementation, *adaptation is required for determinism* !

Measurement based quantum computation



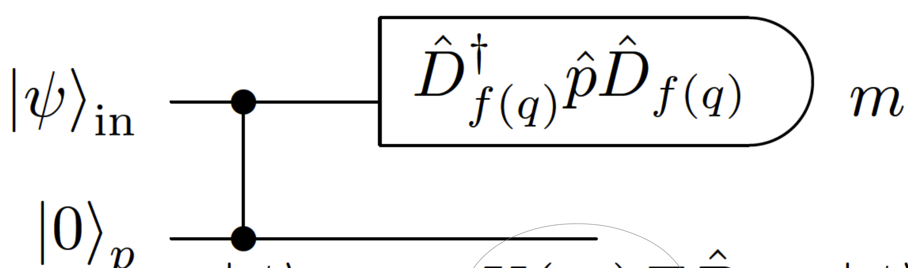
$$|\psi\rangle_{\mathrm{out}} = X(m)F|\psi\rangle_{\mathrm{in}}$$

with  $\equiv \; e^{i\hat{q}_1 \otimes \hat{q}_2}$

(Cz gate)

and $X(s) \; = \; e^{-is\hat{p}}$

« by-product » operators



$$|\psi\rangle_{\mathrm{out}} = X(m)F\hat{D}_{f(q)}|\psi\rangle_{\mathrm{in}}$$

How to compute the quadrature to measure?

Several steps of the computation: $|\psi\rangle_{\text{out}} = ...FD_{f_2(q)}FD_{f_1(q)}|\psi\rangle_{\text{in}}$

Shear $D_{f(\hat{q})} = e^{is\hat{q}^2}$: $\quad e^{-is\hat{q}^2}\hat{p}e^{is\hat{q}^2} = \hat{p} + s\hat{q} = g(\hat{q}\sin\theta + \hat{p}\cos\theta)$

$$\begin{pmatrix} \hat{q}' \\ \hat{p}' \end{pmatrix} = \begin{pmatrix} \hat{q} \\ \hat{p} + s\hat{q} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ s & 1 \end{pmatrix}\begin{pmatrix} \hat{q} \\ \hat{p} \end{pmatrix} \quad \text{with } g = \sqrt{1+s^2} \text{ and } \theta = \arctan s$$

Fourier transform:

$$\begin{pmatrix} \hat{q}' \\ \hat{p}' \end{pmatrix} = \begin{pmatrix} -\hat{p} \\ \hat{q} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \hat{q} \\ \hat{p} \end{pmatrix}$$

Hence, each step of the computation:

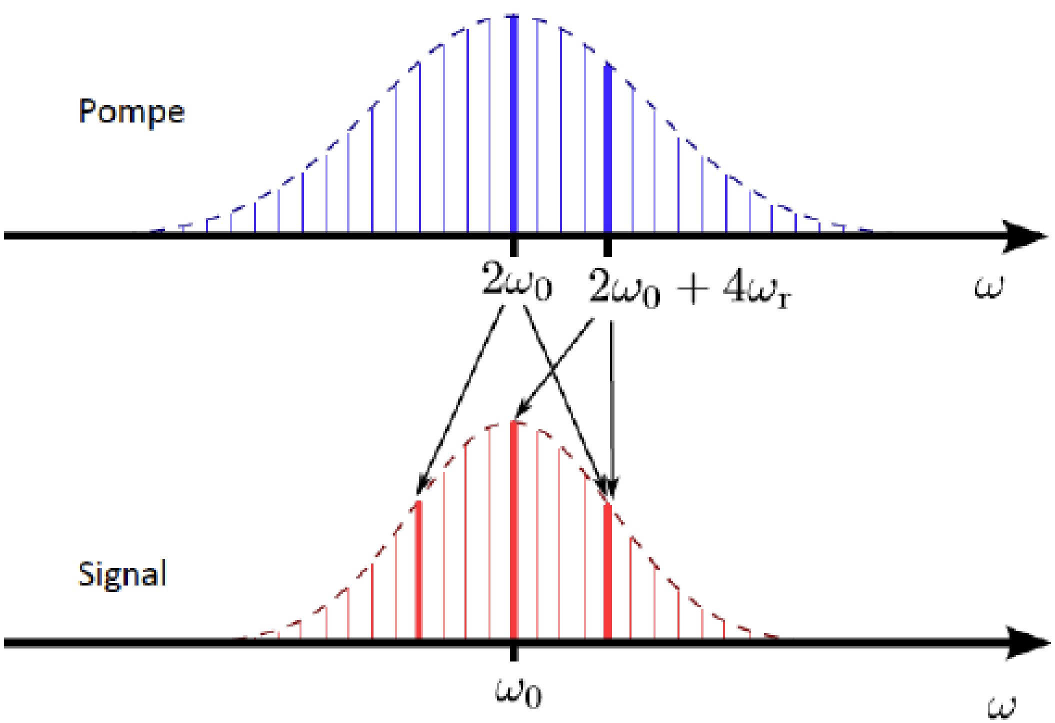
$$M(s) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ s & 1 \end{pmatrix} = \begin{pmatrix} -s & -1 \\ 1 & 0 \end{pmatrix}$$

R. Ukai et al, PRA 81, 032315 (2010)

In order to have $M(s_2) = F$ we have to set $s_2 = 0$

$$\Rightarrow \quad \theta_2 = \arctan s_2 = 0$$

La conversion paramétrique est intrinsèquement multimode



$$\hat{H} \propto \sum_{m,n} \underline{L_{m,n}} \hat{a}_m^\dagger \hat{a}_n^\dagger + \text{h.c.}$$

signal

Matrice de couplage

$$\hat{S}_k = \sum_i U_k^i \hat{a}_i$$

$$\boxed{\hat{H} \propto \sum_k \Lambda_k \hat{S}_k^{\dagger 2} + \text{h.c.}}$$

Hamiltonien diagonal

Les fréquences signal sont toutes couplées entre elles grâce à la pompe multimode en fréquence

De Valcarcel *et al.*, PRA (2006)

L matrice

Λ_k valeurs propres

$\hat{S}_k$ modes propres SUPERMODES

Un SPOPO se comporte comme une assemblée d'OPOs agissant chacun sur un supermode