Reversible information-energy conversions in a quantum hybrid system

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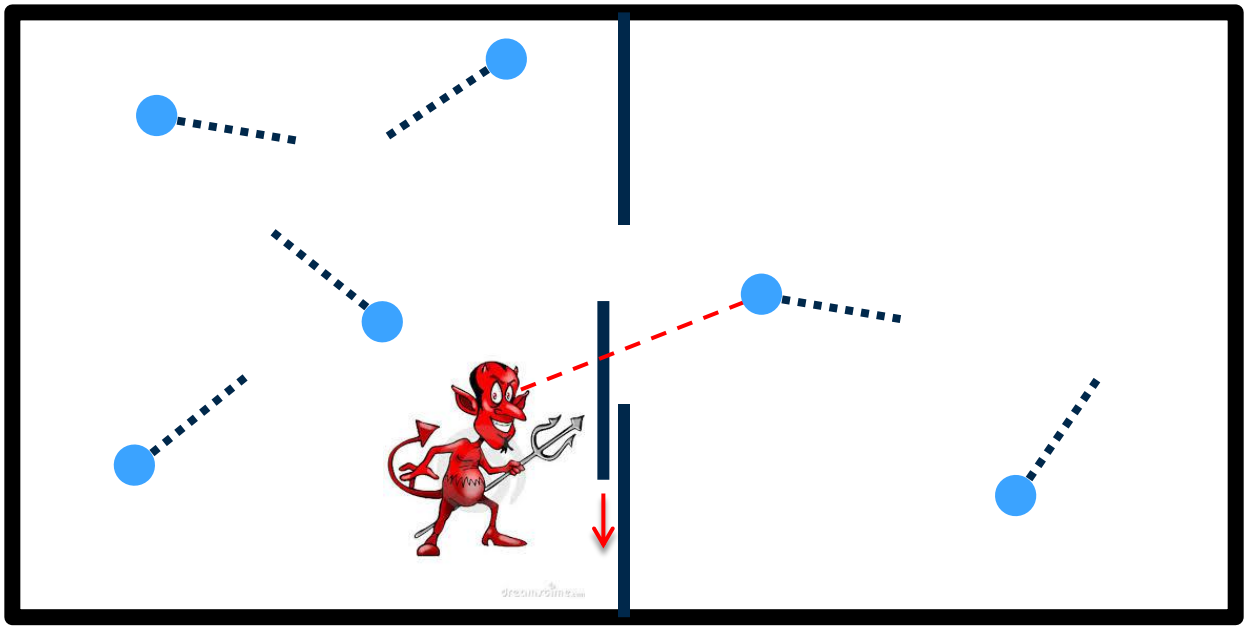
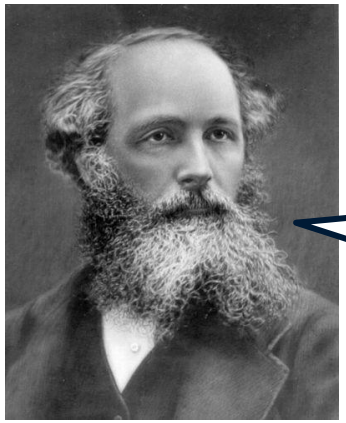
Maxwell's demon paradox

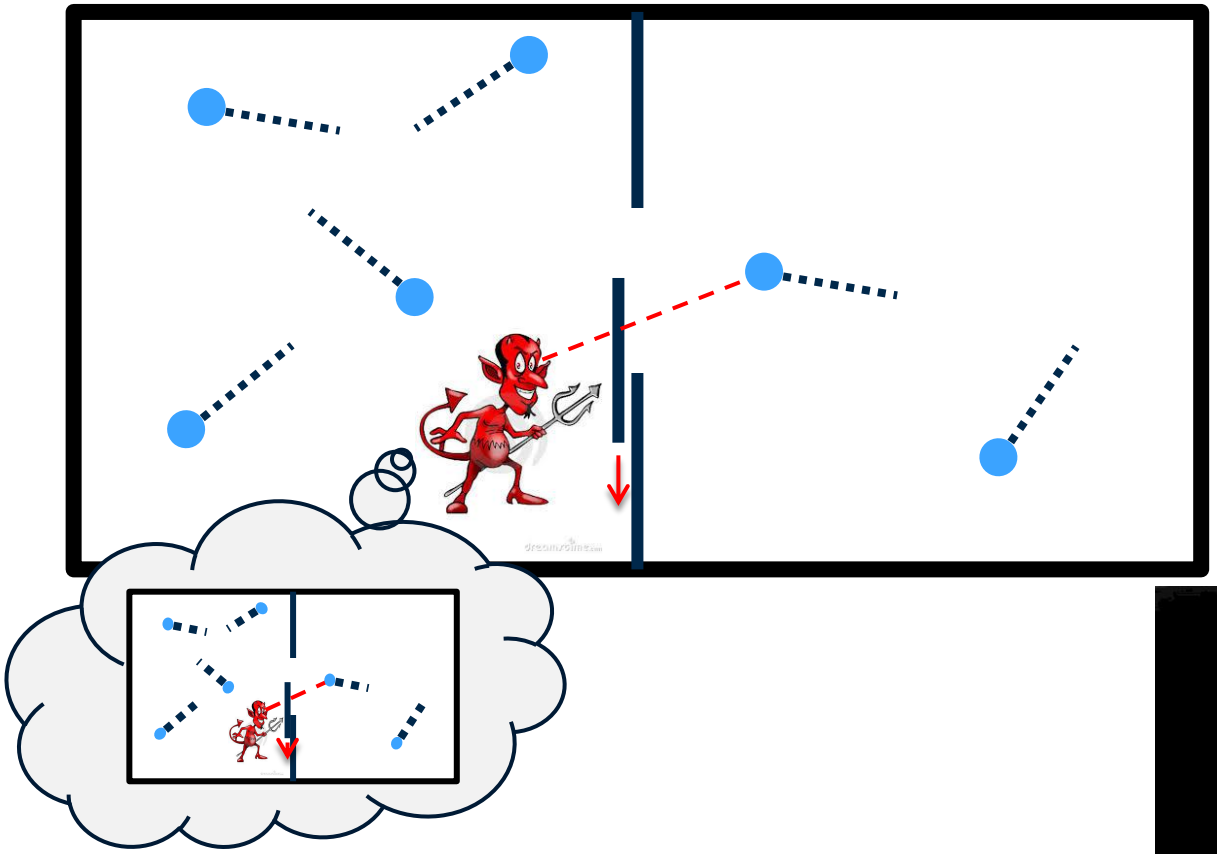




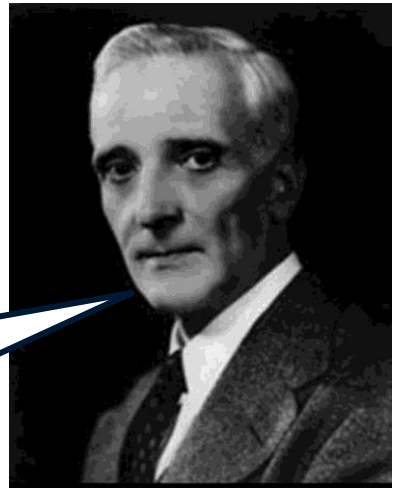
J.C. Maxwell - 1871

The demon can decrease the entropy of the gas without paying work !

The demon's memory

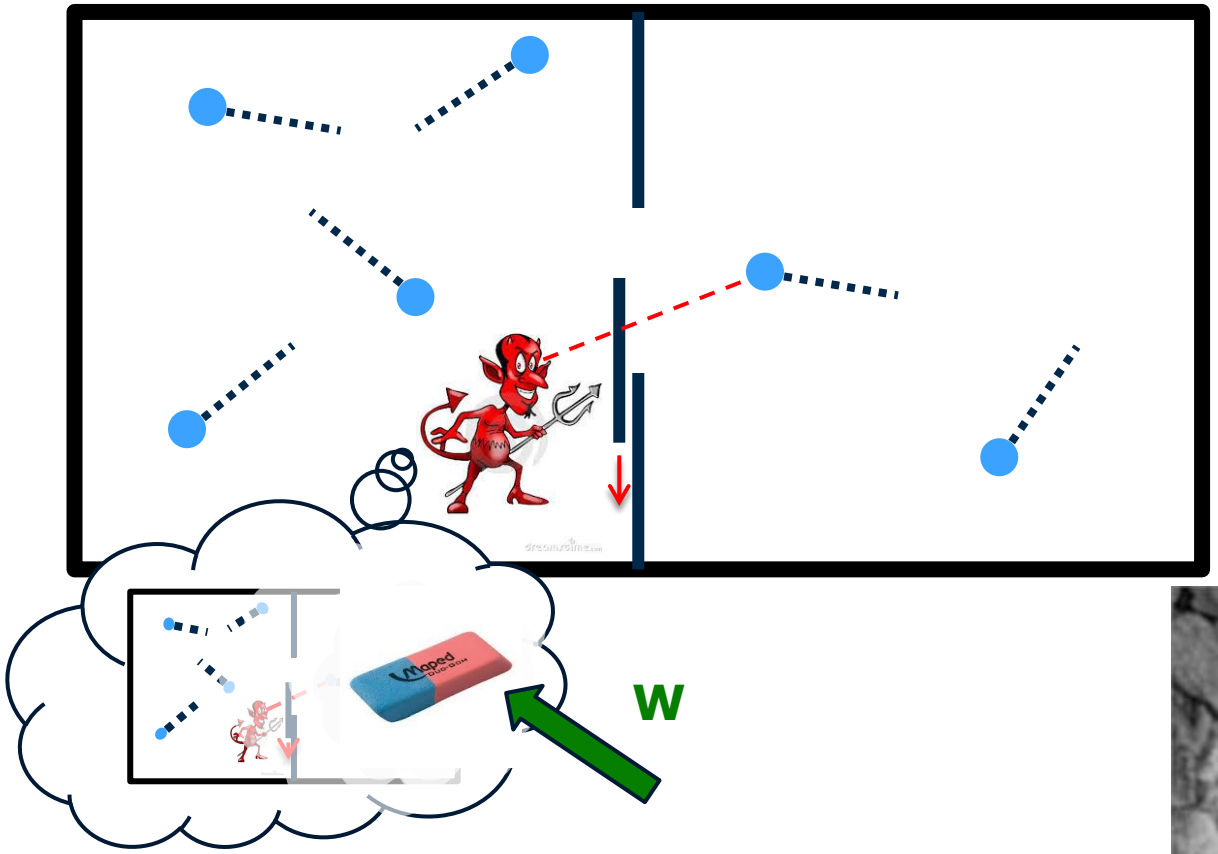


Actually, the demon stores the information about the particles in its memory…



L. Brillouin - 1949

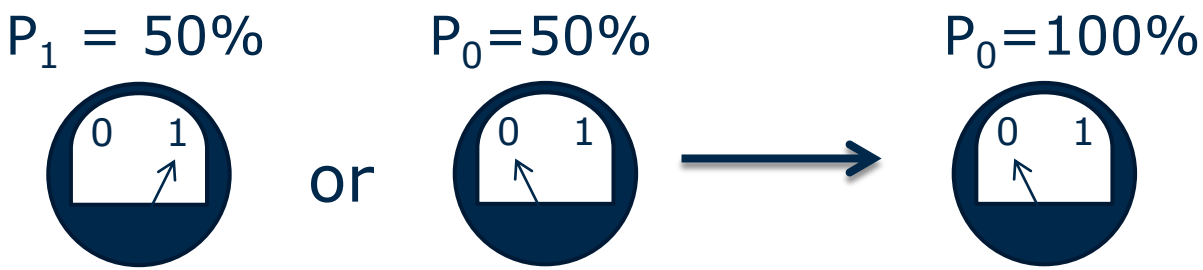
The demon's memory



…and work is required to erase the memory and thus to perform cycles !

R. Landauer - 1961

Landauer's Erasure of 1 bit



$$H_i = 1 \longrightarrow H_f = 0$$

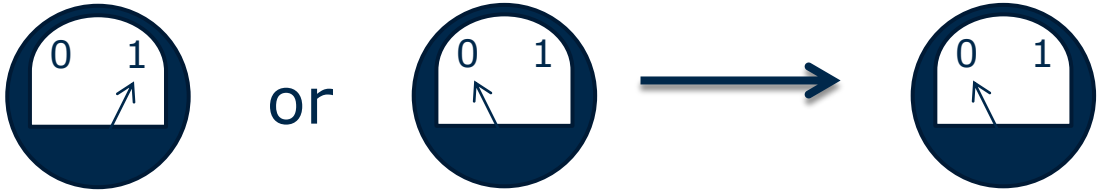
Where Shannon's entropy of the bit is:

$$\boxed{H = -P_1 \log_2 P_1 - P_0 \log_2 P_0}$$

(in bits)

Landauer's Erasure of a bit

$P_1 = 50\%$ $\quad$ $P_0 = 50\%$ $\quad$ $P_0 = 100\%$



$$H_i = 1 \longrightarrow H_f = 0$$

Work required W

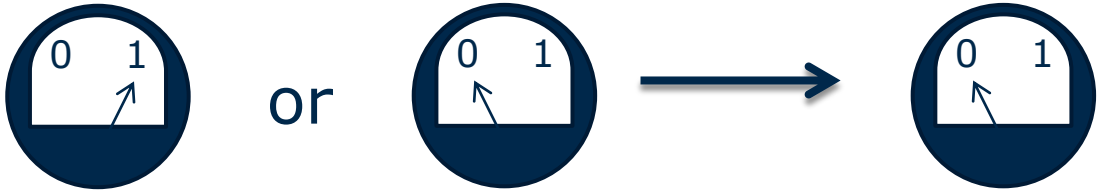
Rolf Landauer

Landauer's principle

$$W \geq W_0 = kT \ln 2$$

Landauer's Erasure of a bit

$P_1 = 50\%$ $\quad$ $P_0 = 50\%$ $\quad$ $P_0 = 100\%$



$$H_i = 1 \longrightarrow H_f = 0$$

Work required W

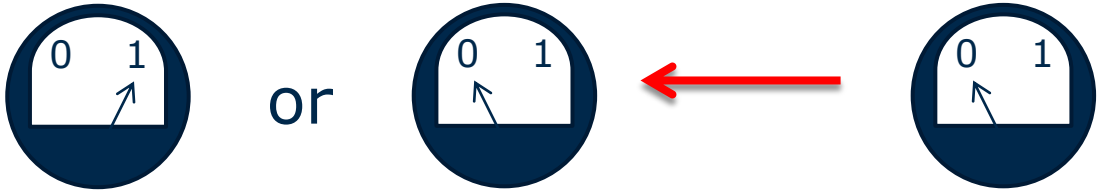
Rolf Landauer

If the erasure is a *reversible* (very slow) transformation:

$$\boxed{W = kT \ln 2}$$

Szilard 's engine

$P_1 = 50\%$ $P_0=50\%$ $P_0=100\%$



$$H_i = 1 \longleftarrow H_f = 0$$

Work *extracted* W

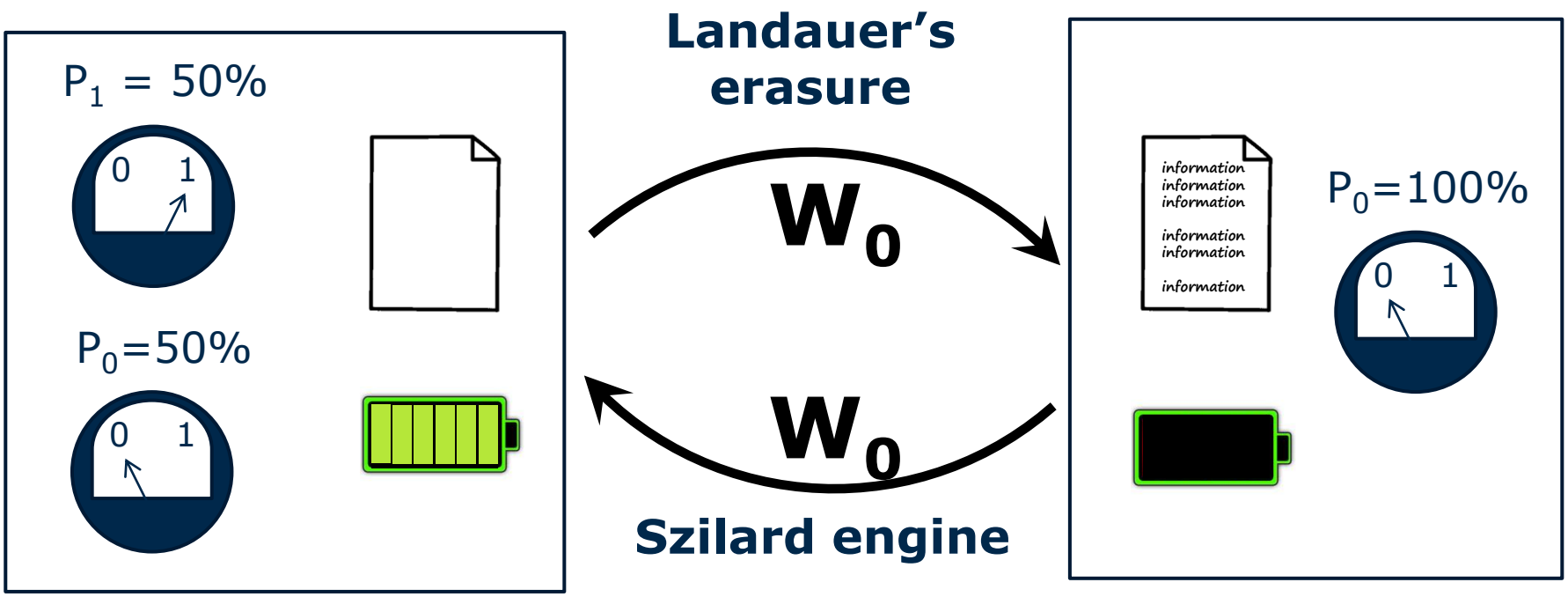
Rolf Landauer

If the erasure is a *reversible* (very slow) transformation:

$$W = kT \ln 2$$

Leo Szilard

Information - energy conversions



W_0 = kT ln 2 is the elementary work corresponding to 1 bit of information

If information becomes quantum…

EPR state

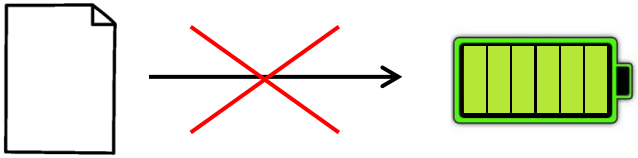
$$\rho_{AB} = |\Psi\rangle\langle\Psi|$$

$$\Psi = (|0_A 1_B\rangle + |1_A 0_B\rangle)/\sqrt{2}$$

Alice's point of view

$$\mathrm{Tr}_B\, \rho_{AB} = \mathbb{I}/2$$

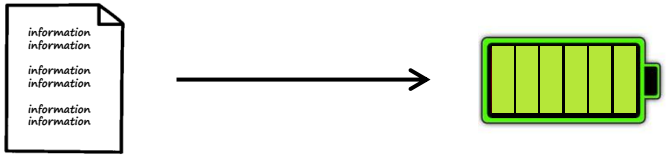
Maximally mixed state,
no work extraction possible



Global point of view

Pure state, H = 0

Can perform a Szilard engine and convert the information into work



If information becomes quantum…

EPR state

$\rho_{AB} = |\Psi\rangle\langle\Psi|$

The peculiarities of quantum information let their imprint on the work we can extract / have to pay

Alice's … f view

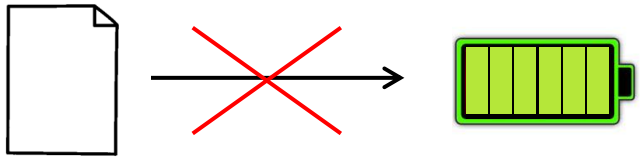
$\mathrm{Tr}_B\, \rho_A$ … = 0

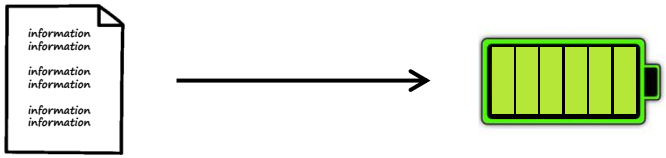
→ Many theoretical results linking work to quantum correlations, discord, entanglement…

L. del Rio et al., Nature 474, 61--63 (2011)
Oppenheim, Horodecki, PRL 89 (2002)
Zurek, PRA 67 (2003) …..

Maximally … no work ex…

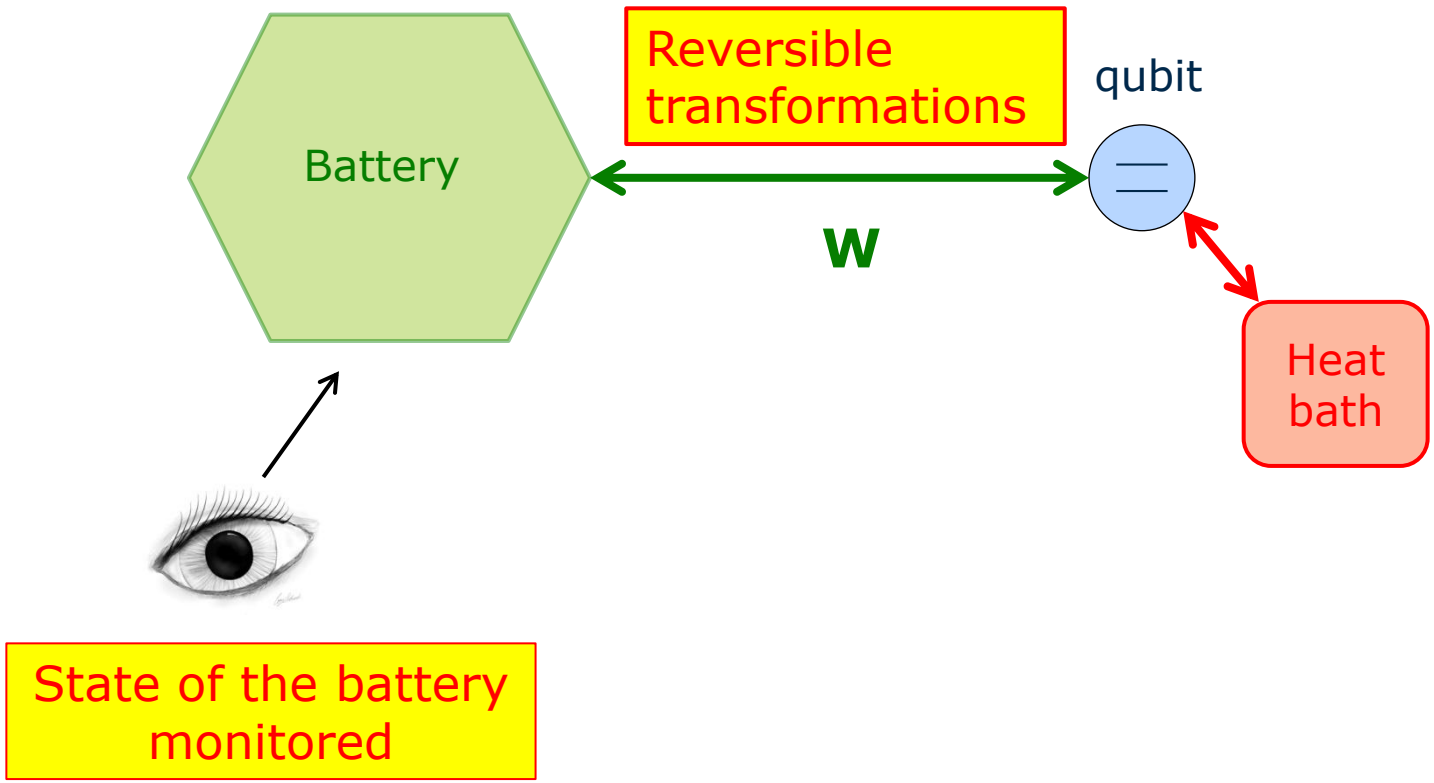
…zilard engine … information into work





Experimental test of these theorems

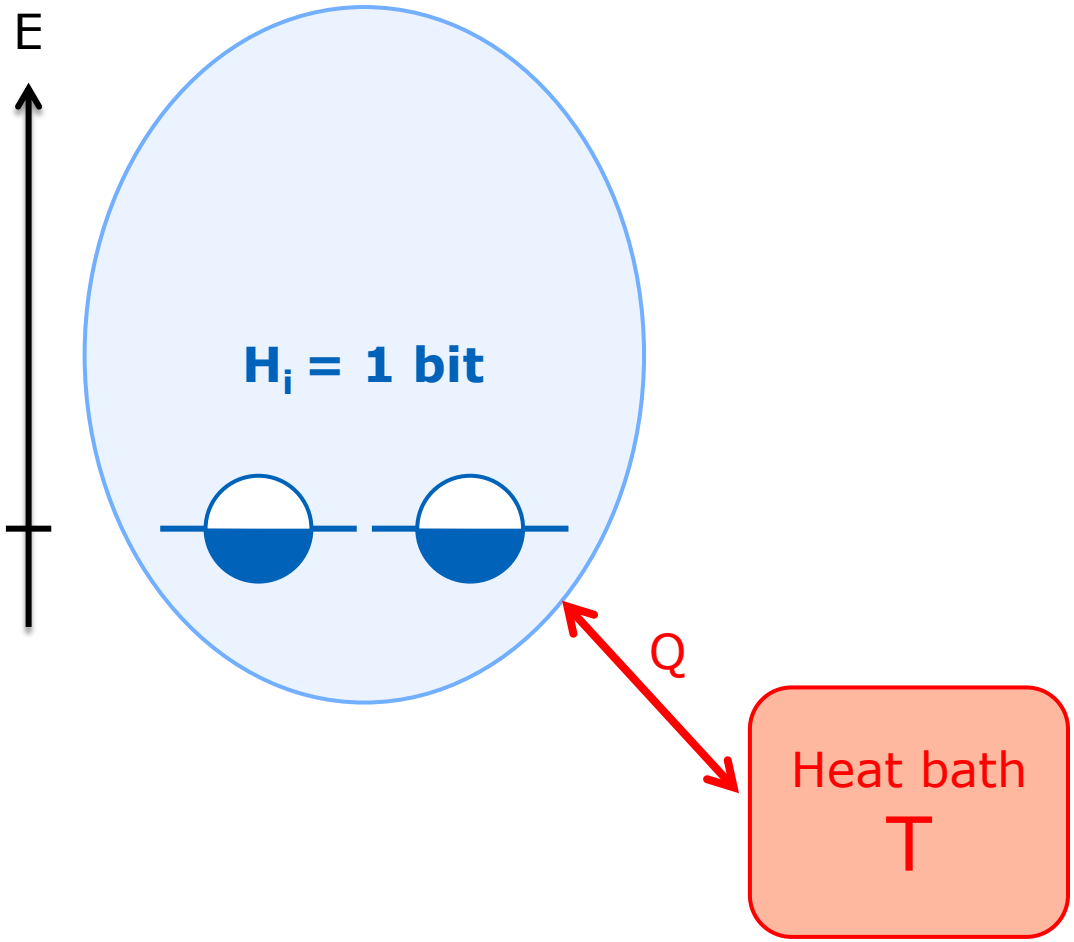
To check quantum info thermodynamics theorems we need:



Outline

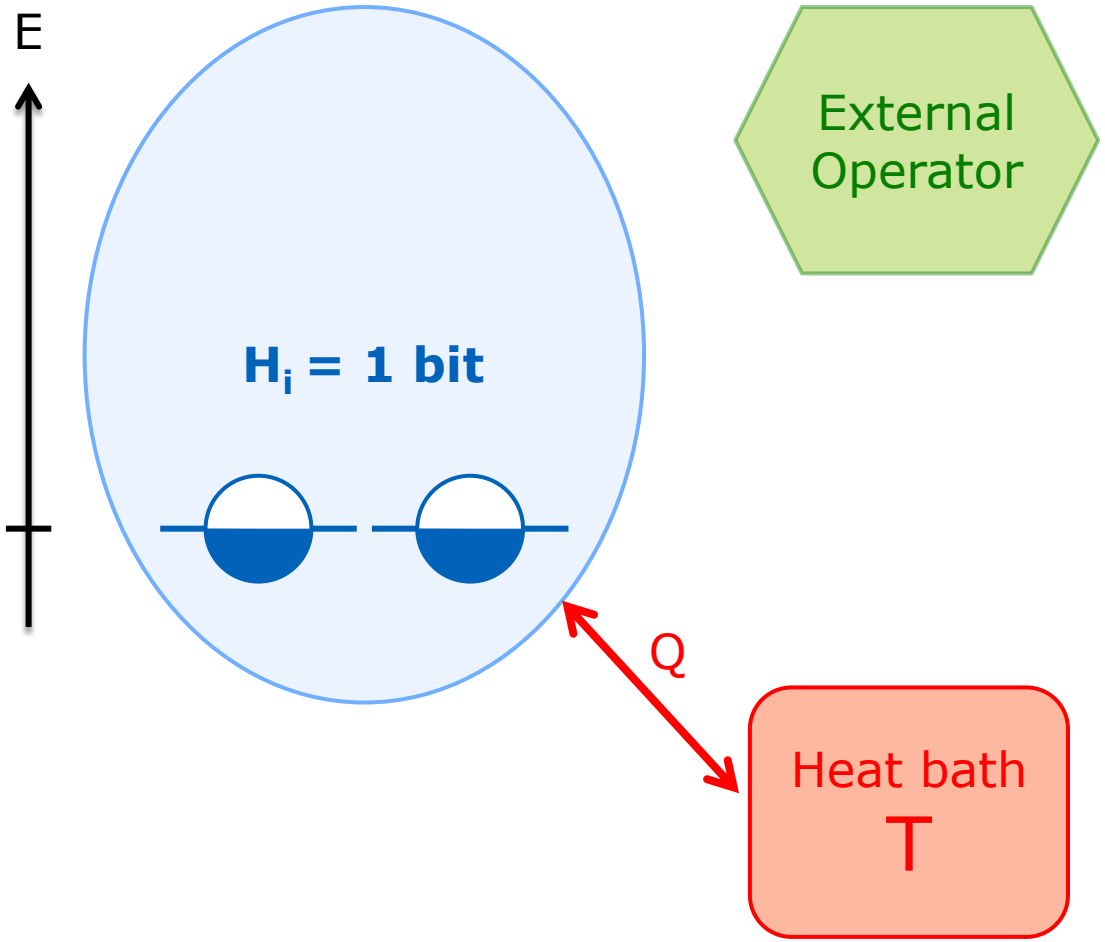
- I) Standard Landauer's erasure protocol
  - Standard protocol
  - Optical protocol
- II) A battery enabling to monitor work exchanges
- III) Full cycle of energy-information conversions
- IV) Proposal for optical Carnot engine

Standard erasure protocol



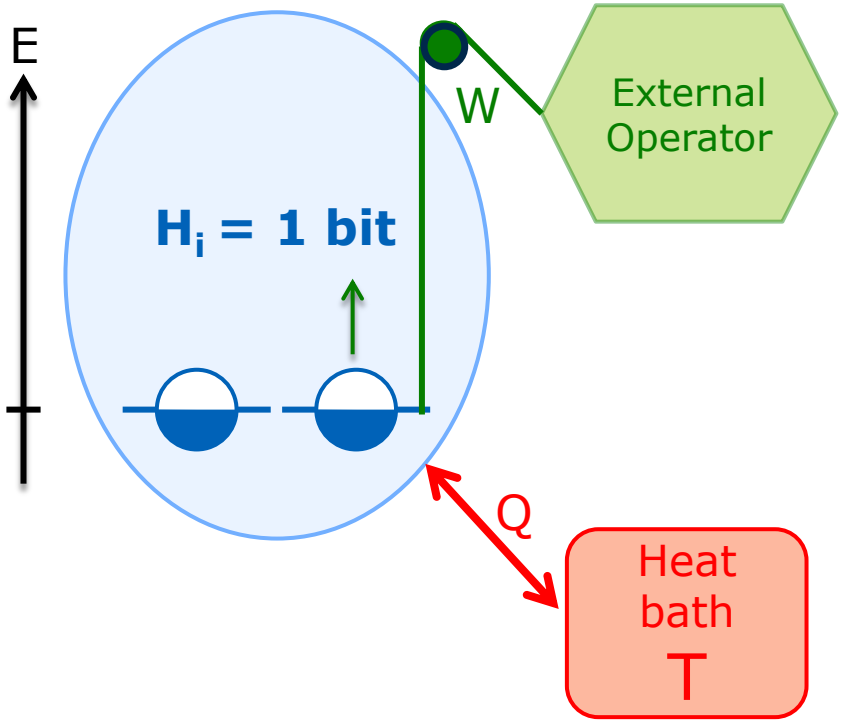
t = 0

Standard erasure protocol



t = 0

Standard erasure protocol



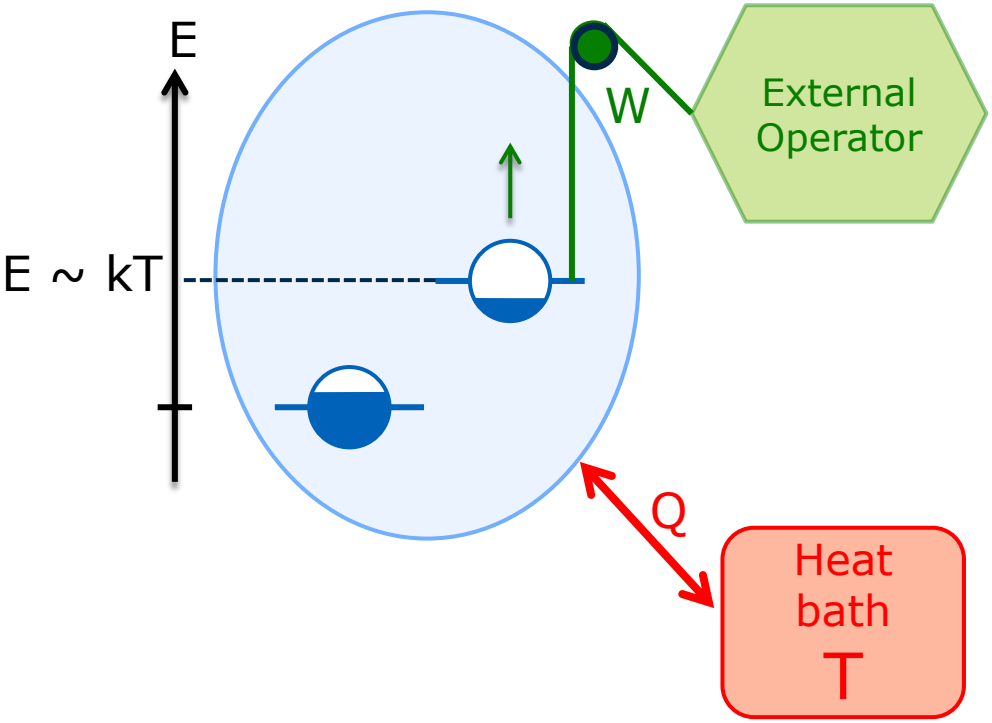
Work performed by the operator while raising one of the states

$$\boxed{W(t) = \int_0^t P(E)\, dE}$$

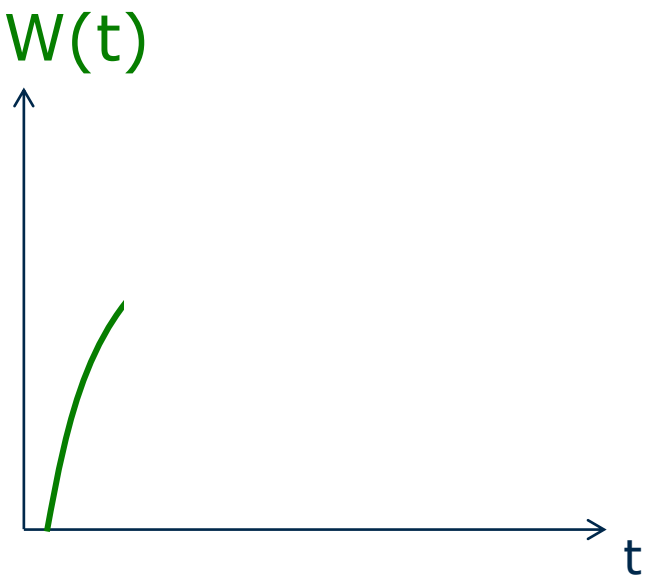
Population of the state

t = 0

Standard erasure protocol



Work performed by the operator

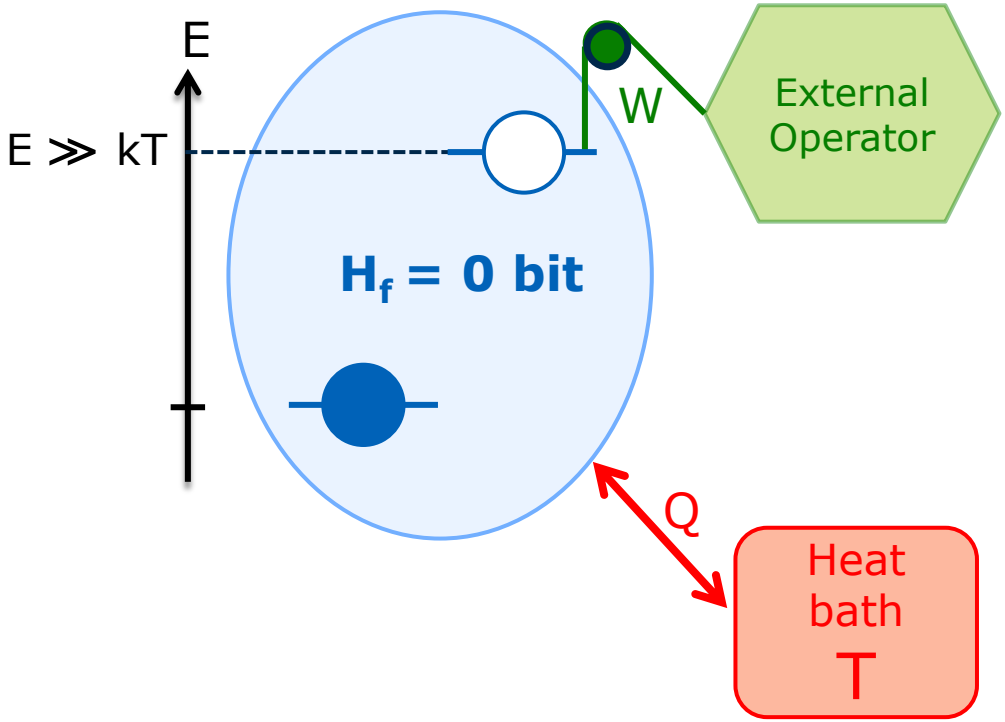


The qubit is at any time in equilibrium with the bath

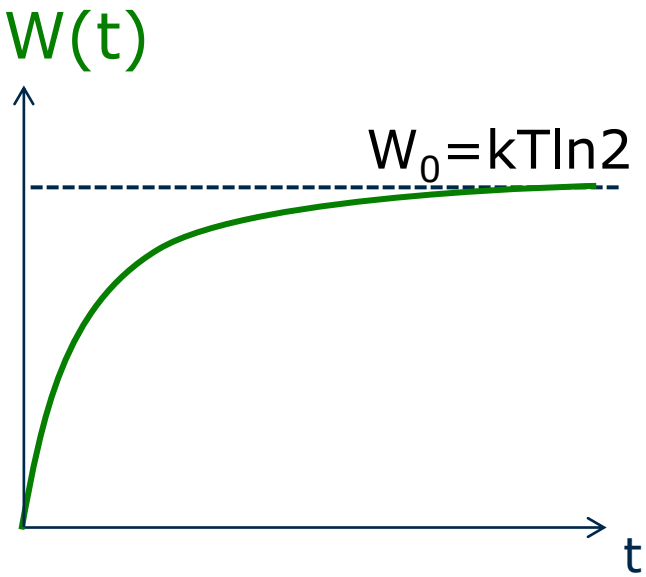
$$P(E) = e^{-E/kT}/Z$$

$$0 < t < t_f$$

Standard erasure protocol



Work performed by the operator

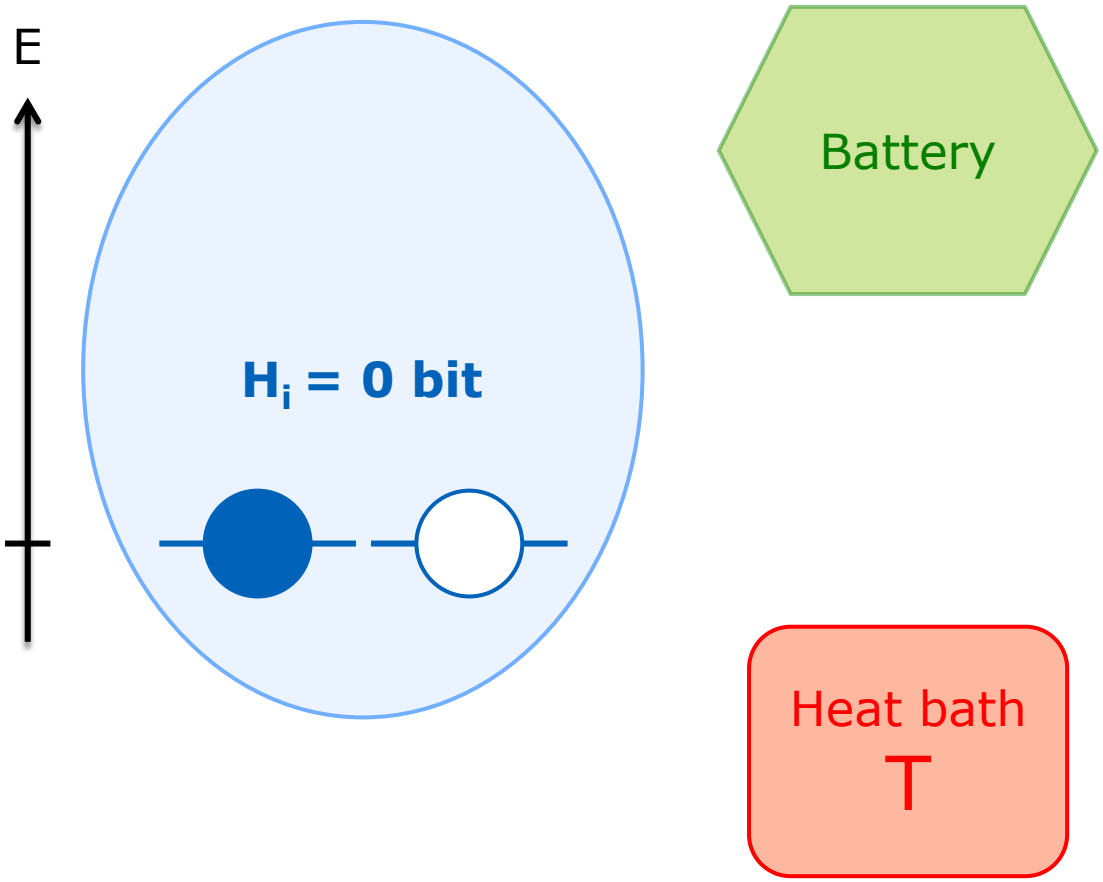


The qubit is at any time in equilibrium with the bath

$$P(E) = e^{-E/kT}/Z$$

$$t = t_f$$

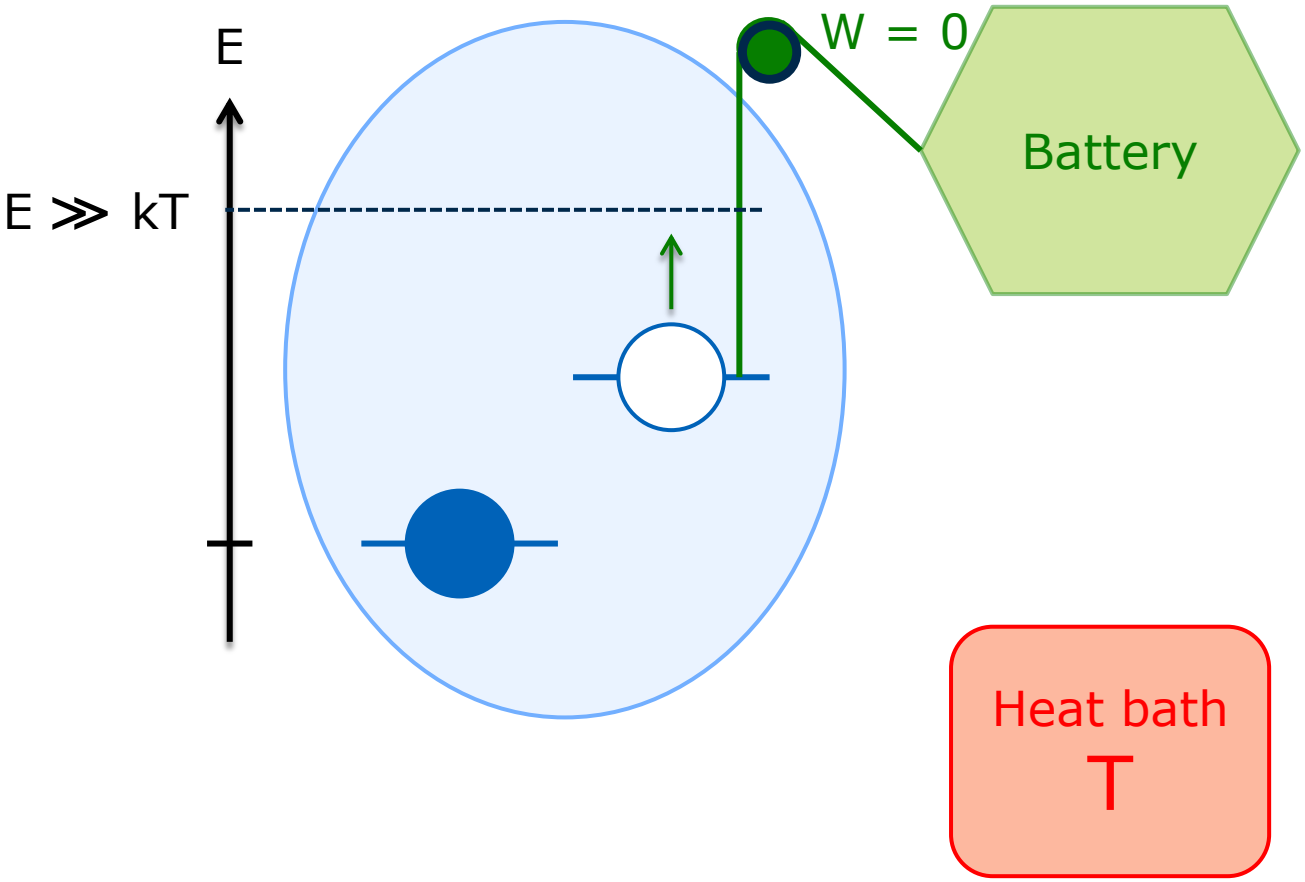
Szilard engine protocol



The qubit is in a known state and isolated from the bath

t < 0

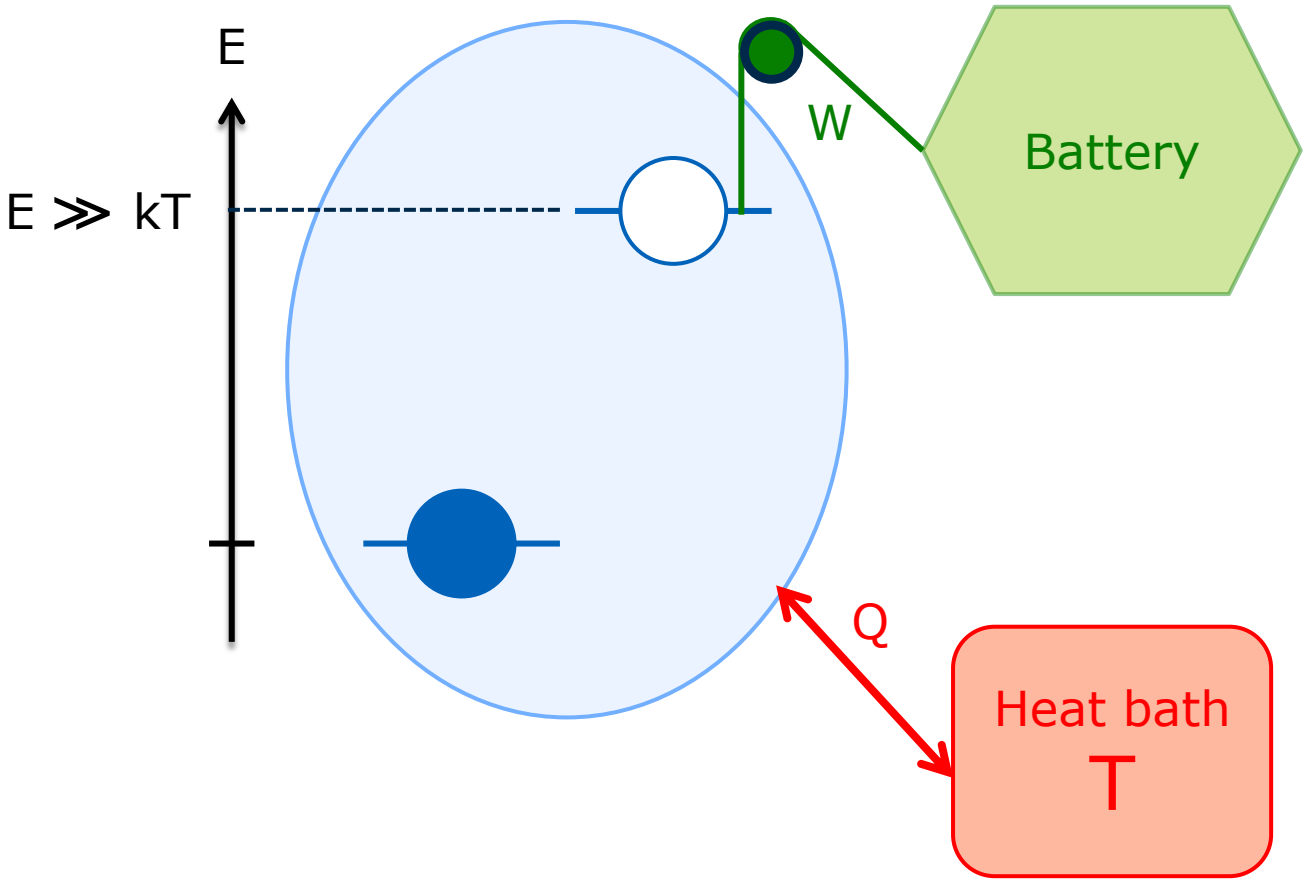
Szilard engine protocol



The empty state is raised with no work cost

t < 0

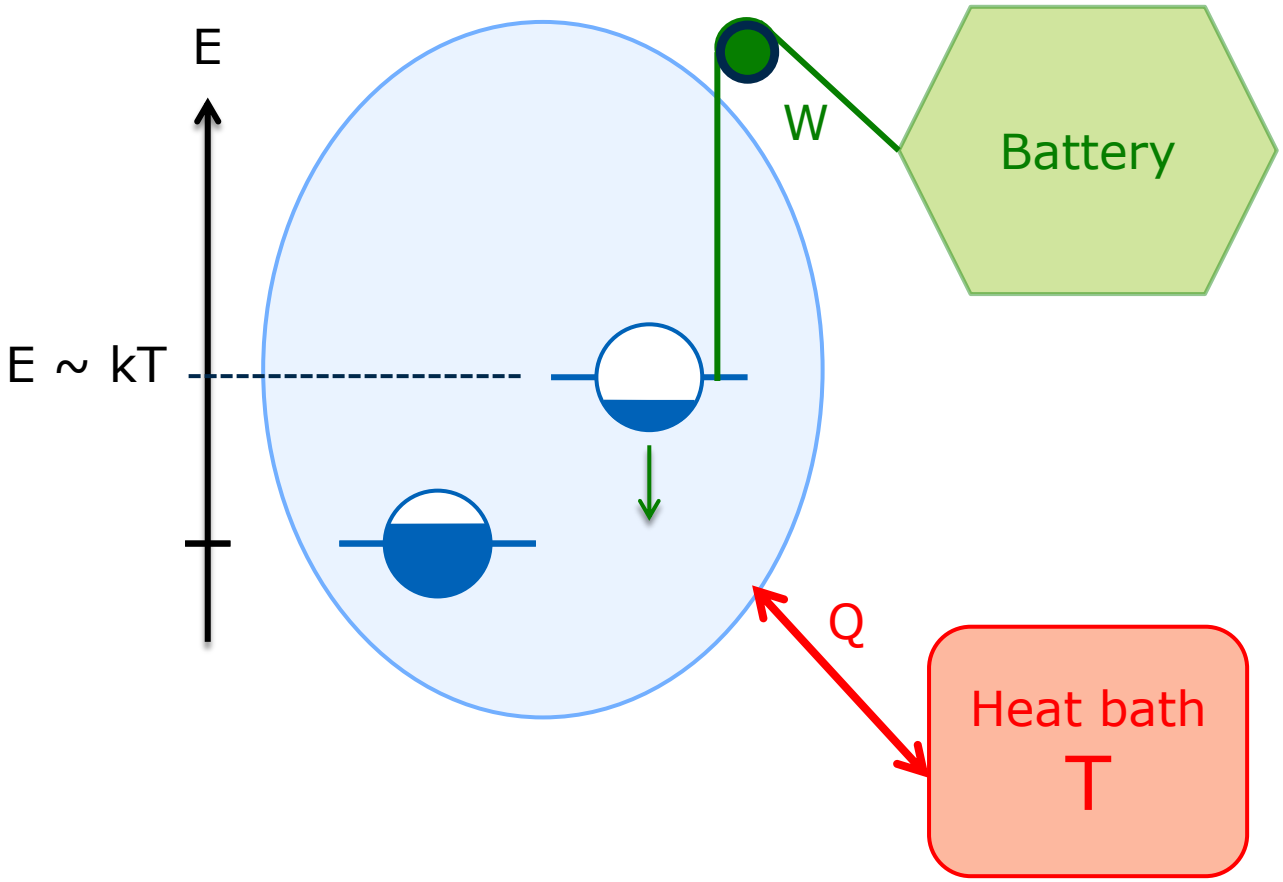
Szilard engine protocol



The qubit is put in equilibrium with the bath

t = 0

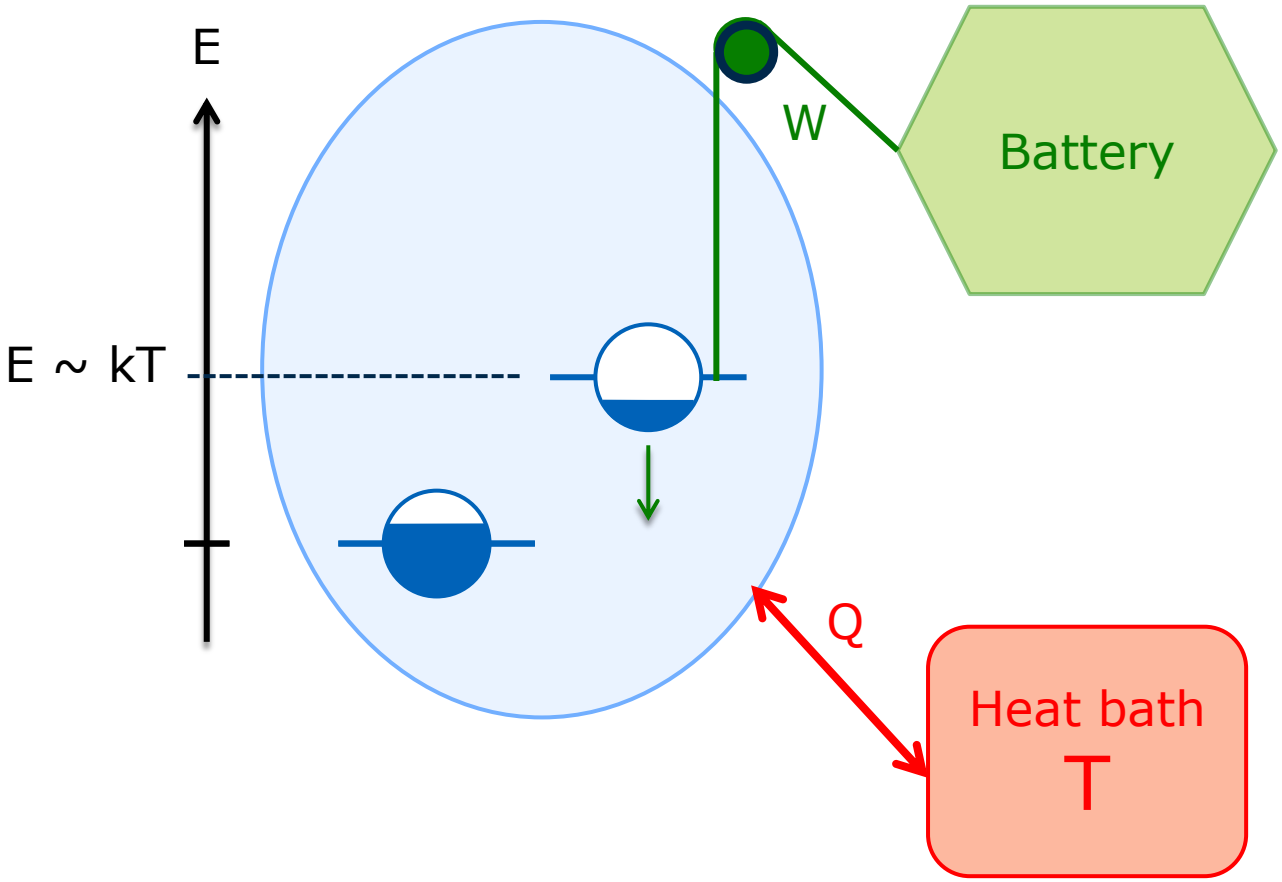
Szilard engine process



The empty state is lowered very slowly. For an energy low enough it gets populated.

$$0 < t < t_f$$

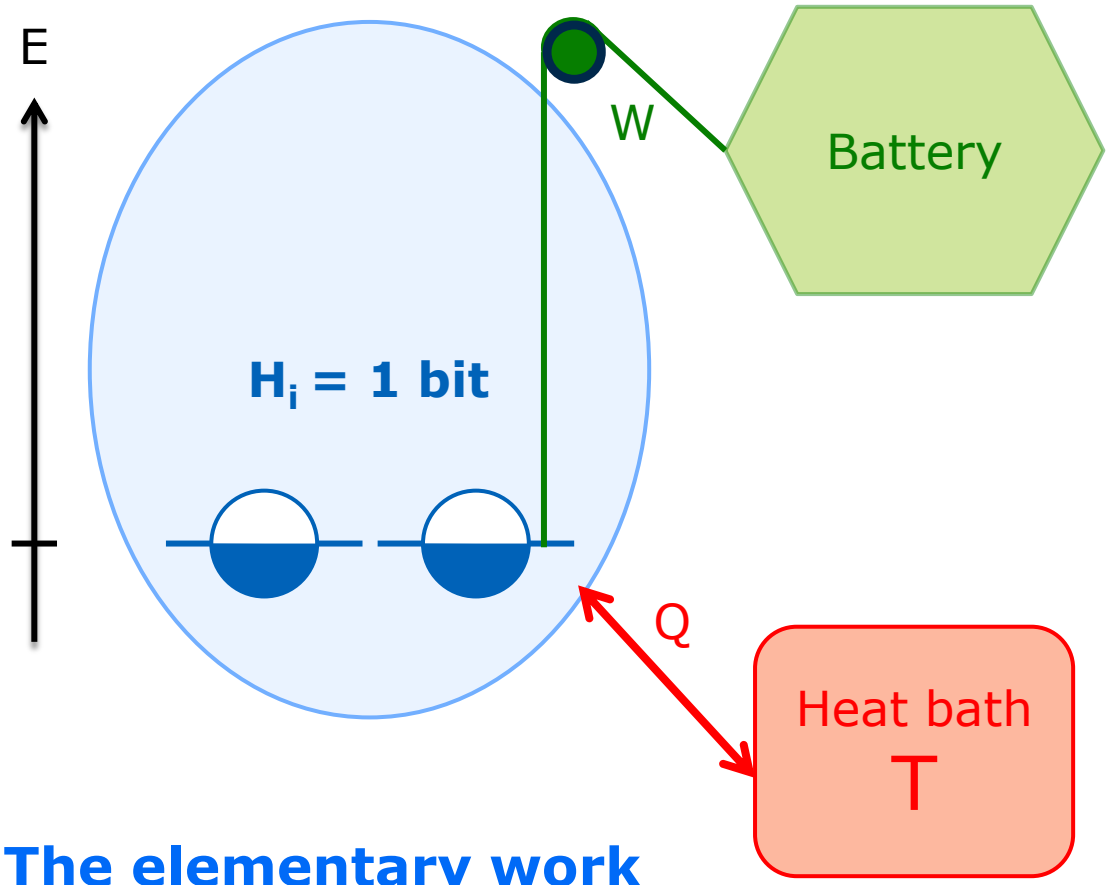
Szilard engine process



Then lowering the occupied state enables to extract work

$$0 < t < t_f$$

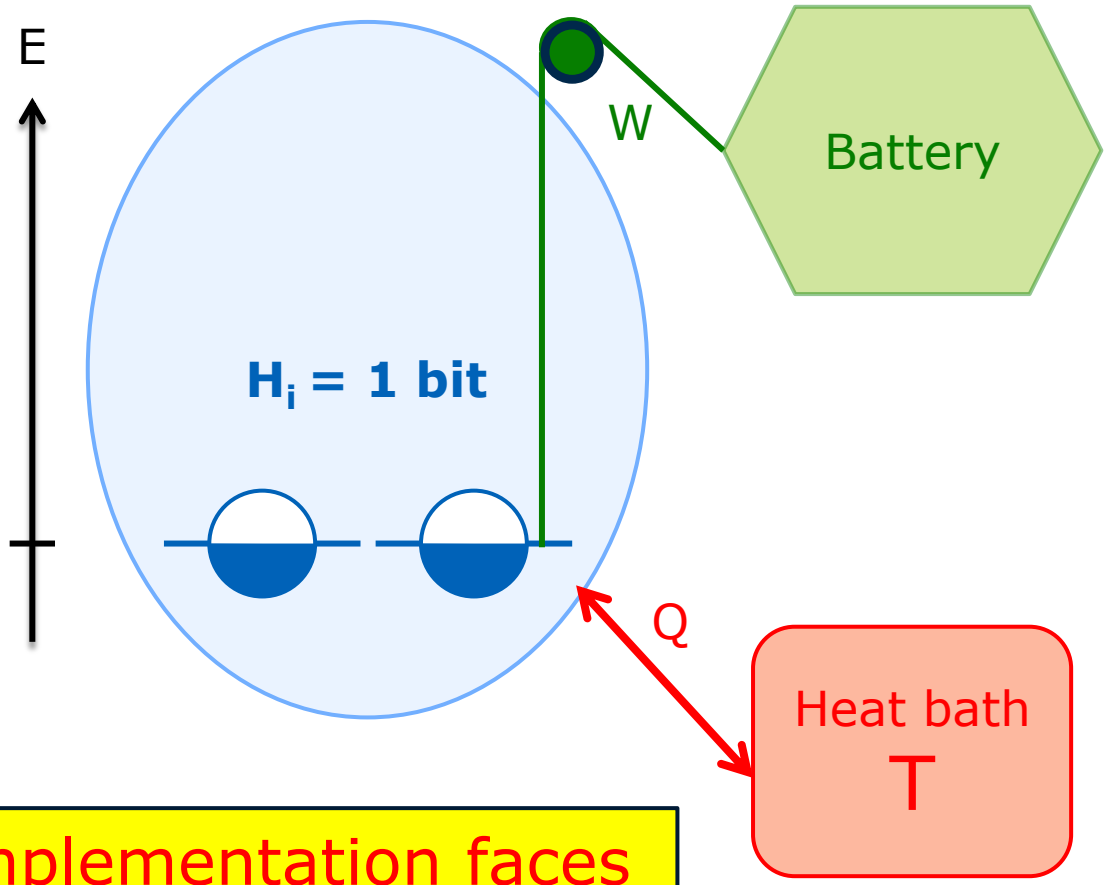
Szilard engine process



The elementary work W_0 is extracted from the bath and stored in the battery

$$t = t_f$$

Szilard engine process



Direct implementation faces challenges:
- reaching reversiblity
- measuring work

$t = t_f$

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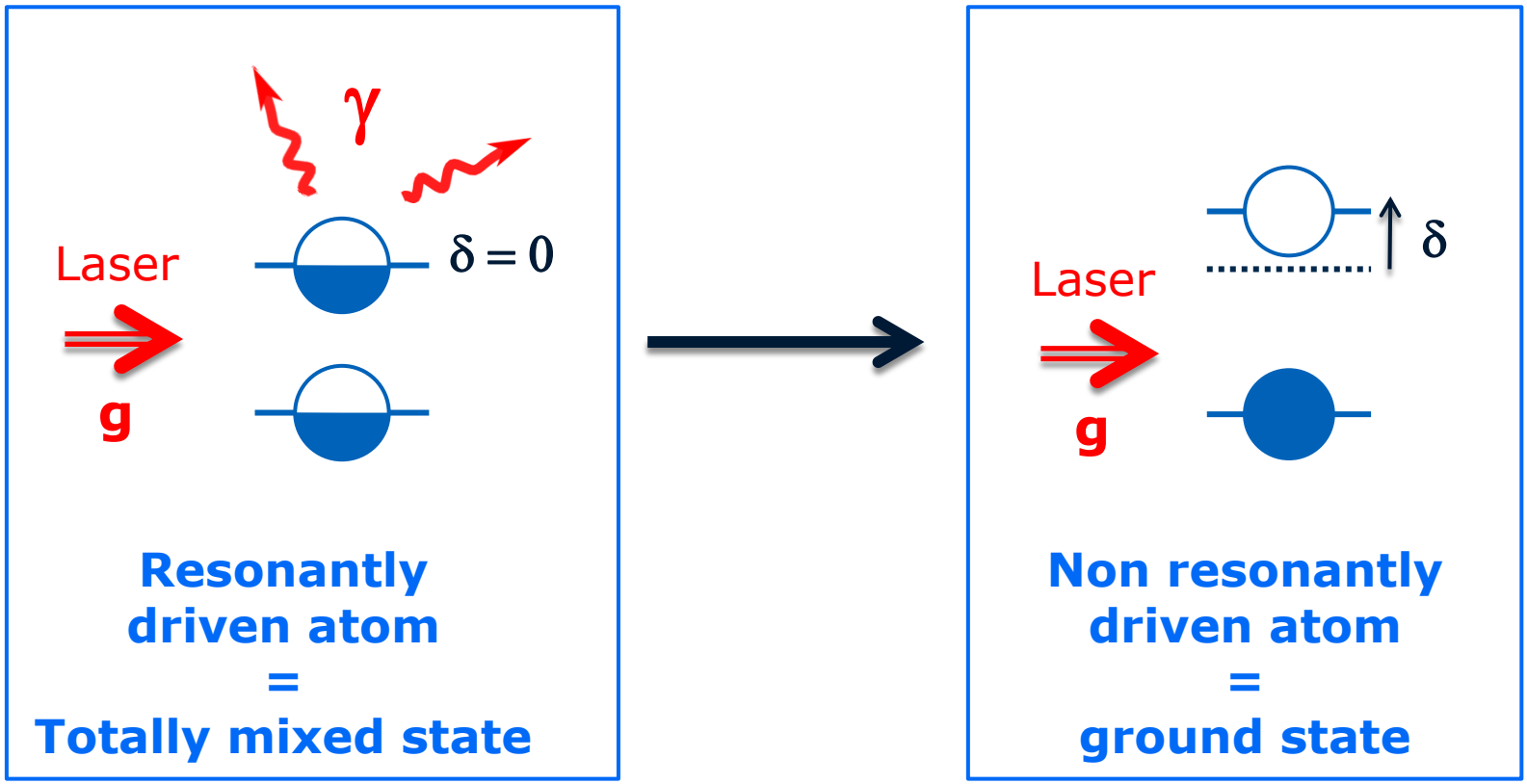
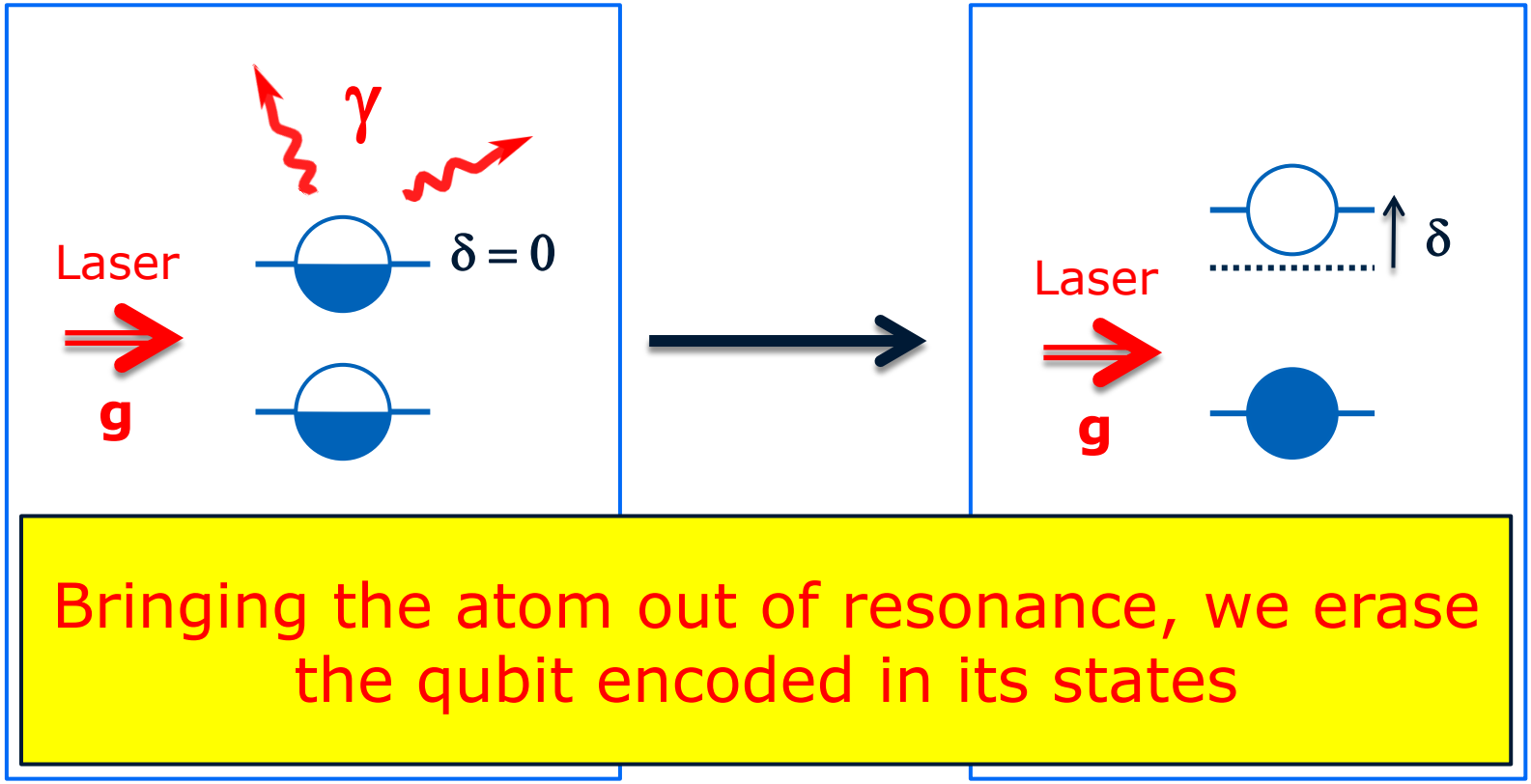
An optical version of the protocol



γ spontaneous emission rate
g classical Rabi frequency
δ atom-laser detuning

Saturated regime
$g \gg \gamma$

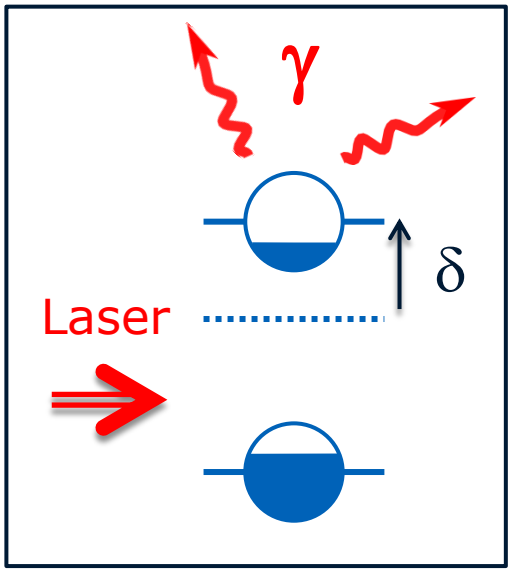
An optical version of the protocol



Bringing the atom out of resonance, we erase the qubit encoded in its states

γ spontaneous emission rate
g classical Rabi frequency
δ atom-laser detuning

The laser mimicks a bath

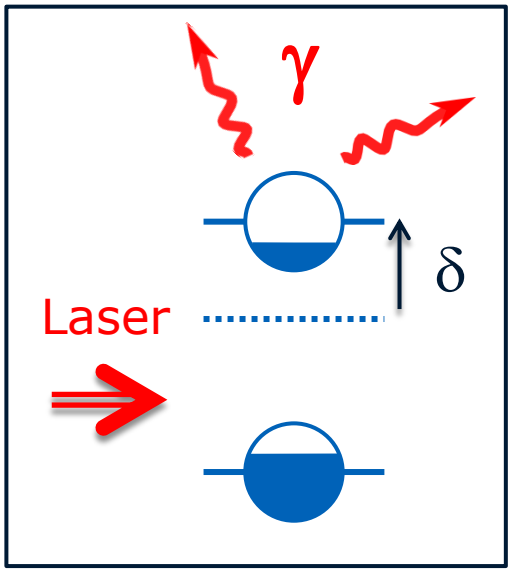


After some 1/γ , the population of the excited state is in the steady state:

$$P_e(\delta) = \frac{1/2}{1+(\delta/g)^2}$$

The Rabi frequency g plays the role of the bath temperature

A very fast thermalization

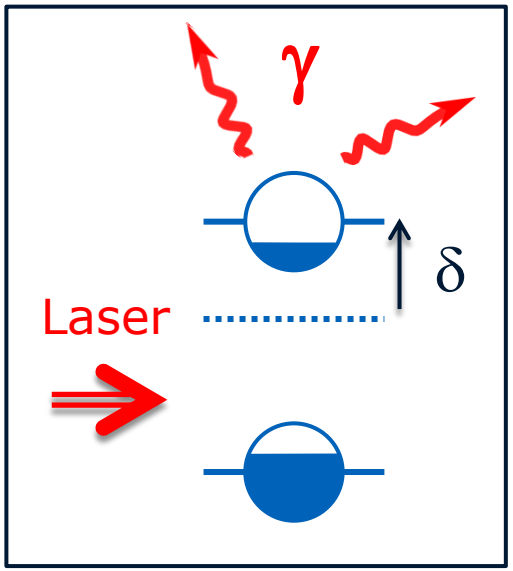


After some 1/γ , the population of the excited state is in the steady state:

$$P_e(\delta) = \frac{1/2}{1+(\delta/g)^2}$$

The effective thermalization time 1/γ is very short
→ Reaching reversibility is easier than with a thermal bath

Quasi-staticity → Adiabaticity



After some 1/γ , the population of the excited state is in the steady state:

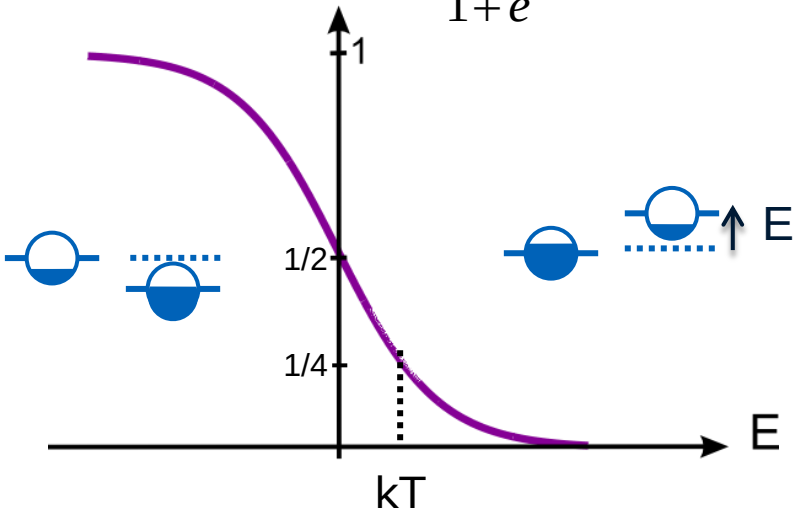
$$P_e(\delta) = \frac{1/2}{1+(\delta/g)^2}$$

δ must vary on a time scale slow with respect to 1/γ

A colored bath

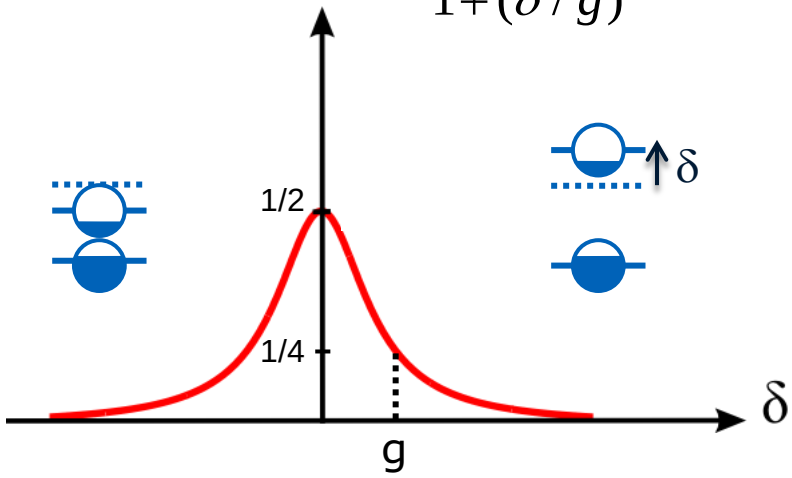
Thermal bath

$$P_e(E) = \frac{e^{-E/kT}}{1+e^{-E/kT}}$$



Optical bath

$$P_e(\delta) = \frac{1/2}{1+(\delta/g)^2}$$

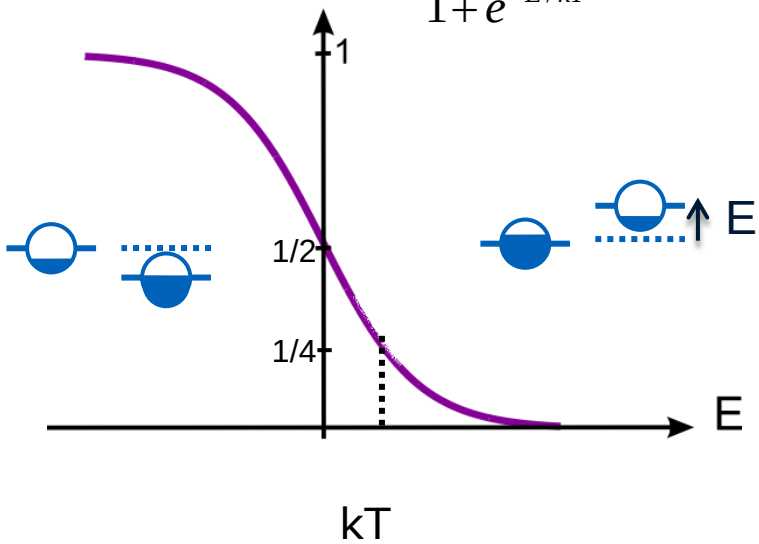


- The right side of the plot is very similar
- Behaviour is different for negative detuning

A new value of the elementary work

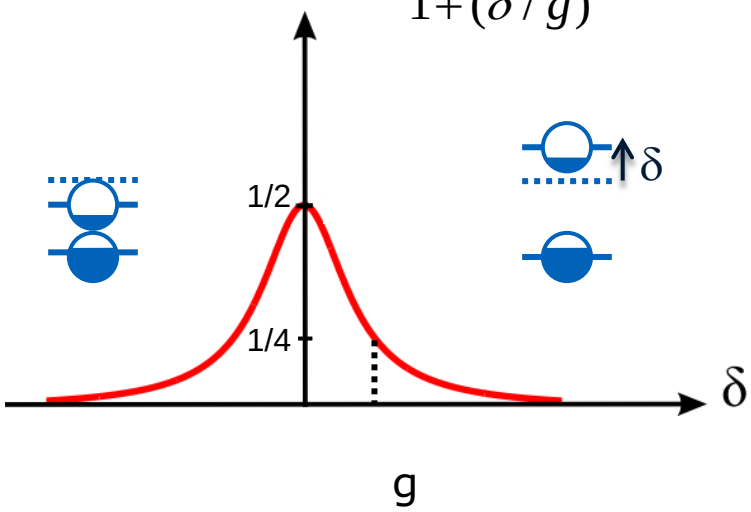
Thermal bath

$$P_e(E) = \frac{e^{-E/kT}}{1+e^{-E/kT}}$$



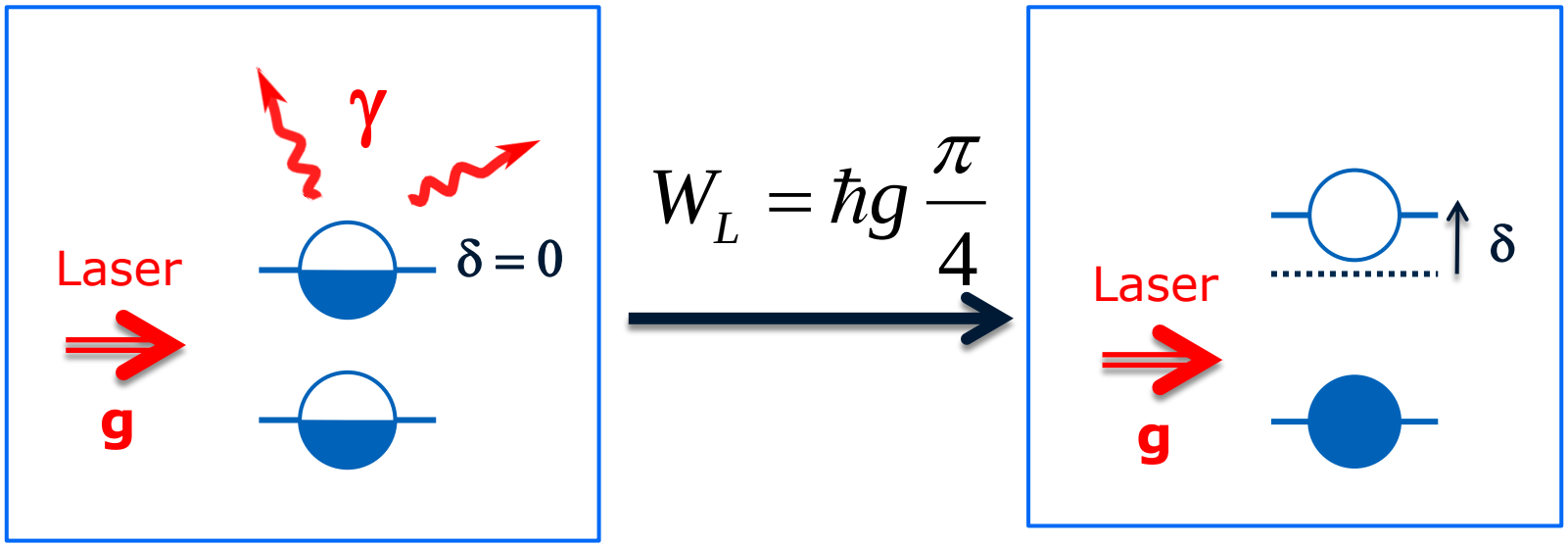
Optical bath

$$P_e(\delta) = \frac{1/2}{1+(\delta/g)^2}$$



$$W_0 = kT\ln 2 \quad \longleftrightarrow \quad \boxed{W_L = \hbar\int_0^\infty P_e(\delta)d\delta = \hbar g\frac{\pi}{4}}$$

Summary



γ spontaneous emission rate
g classical classical Rabi frequency
δ atom-laser detuning

We need an external operator able to increase the detuning adiabatically and that we can monitor

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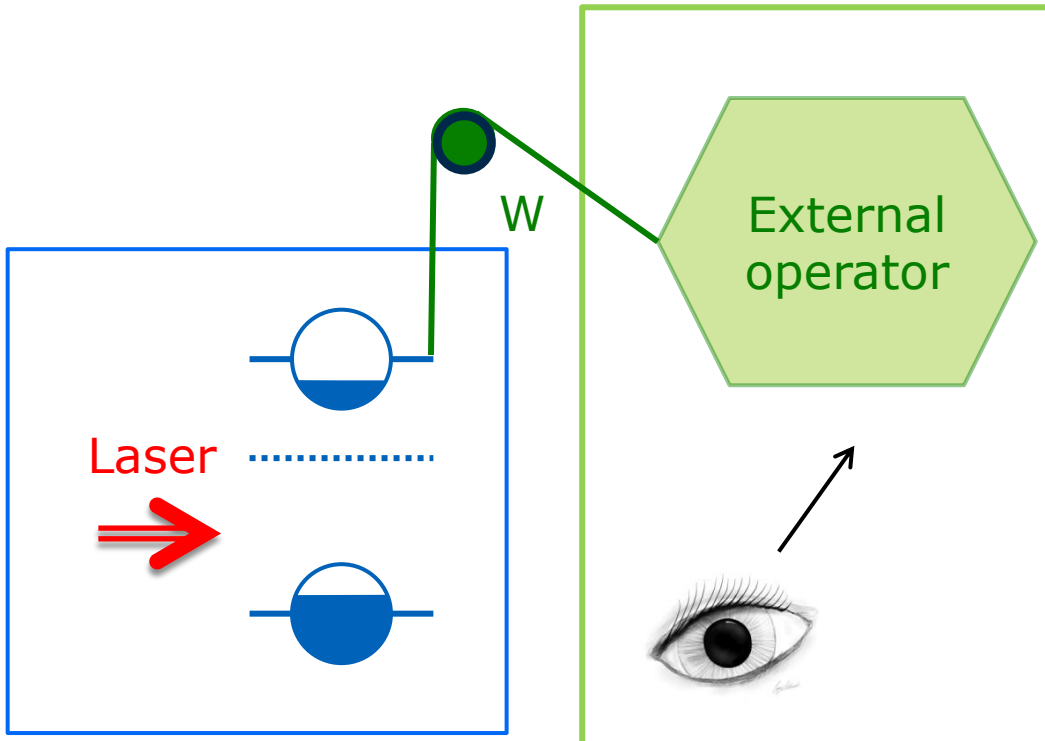
Specifications for the external operator device

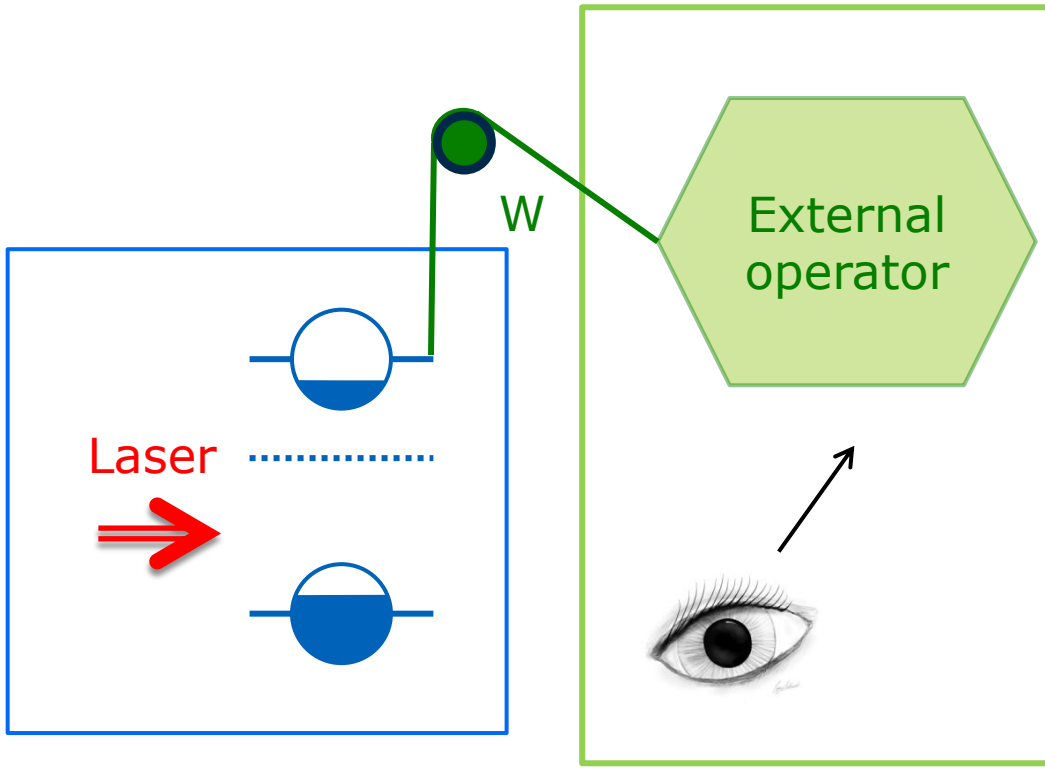


<u>This device must</u>

1) **Evolve very slowly** with respect to the effective thermalization time $1/\gamma$ (adiabtic evolution)

2) **Change its state in a measurable way** when it performs work on the qubit

Specifications for the external operator device

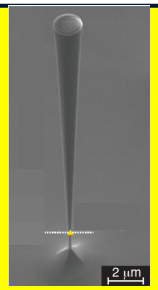


This device must

1) **Evolve very slowly** with respect to the effective thermalization time $1/\gamma$ (adiabtic evolution)

2) **Change its state in a measurable way** when it performs work on the qubit

→ A solution: couple the atom to an oscillating nanowire

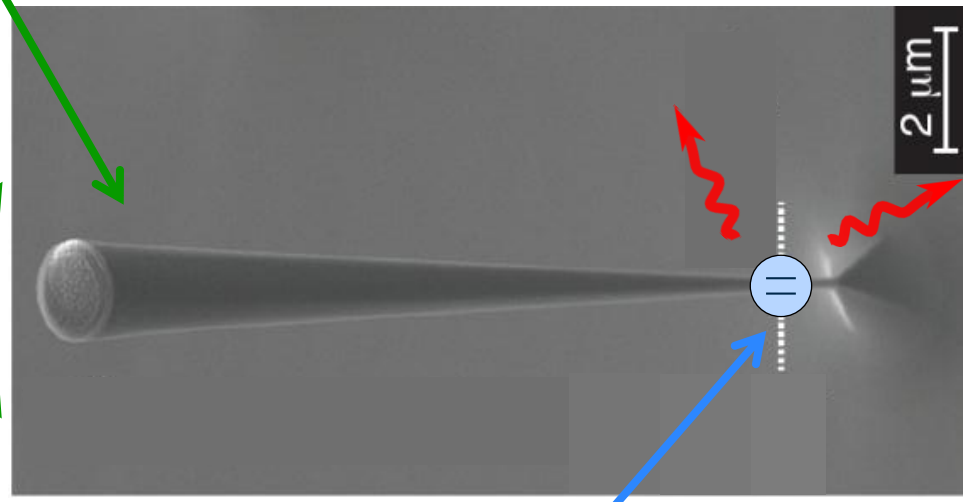


Hybrid optomechanical set up

External operator:
Oscillating nanowire.

Set up : nano 'trumpets'

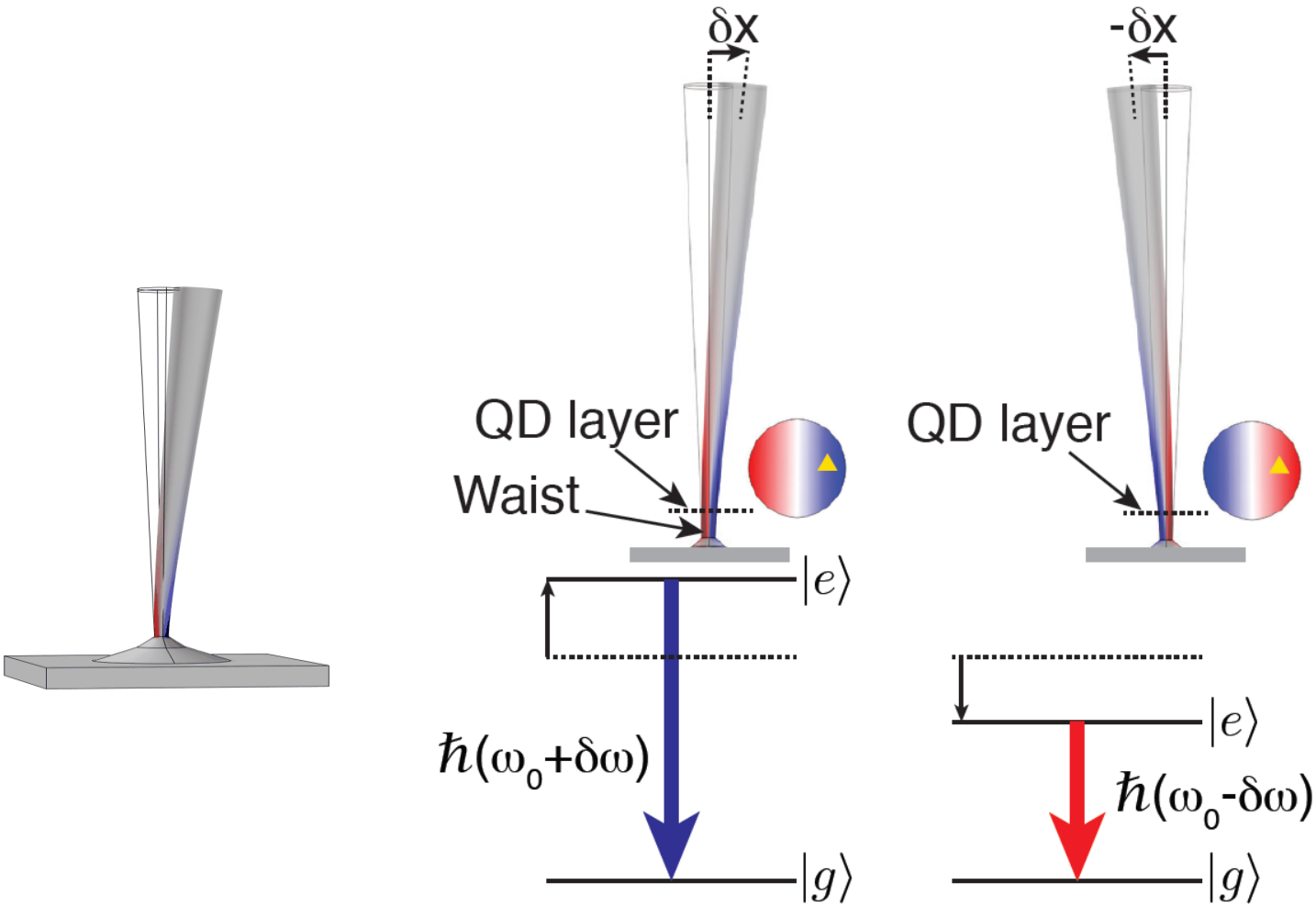
I.Yeo et al., arXiv:1306.4209 (2013)
accepted in Nature Nano



Bath:
Laser resonant with the bare atomic frequency + vacuum

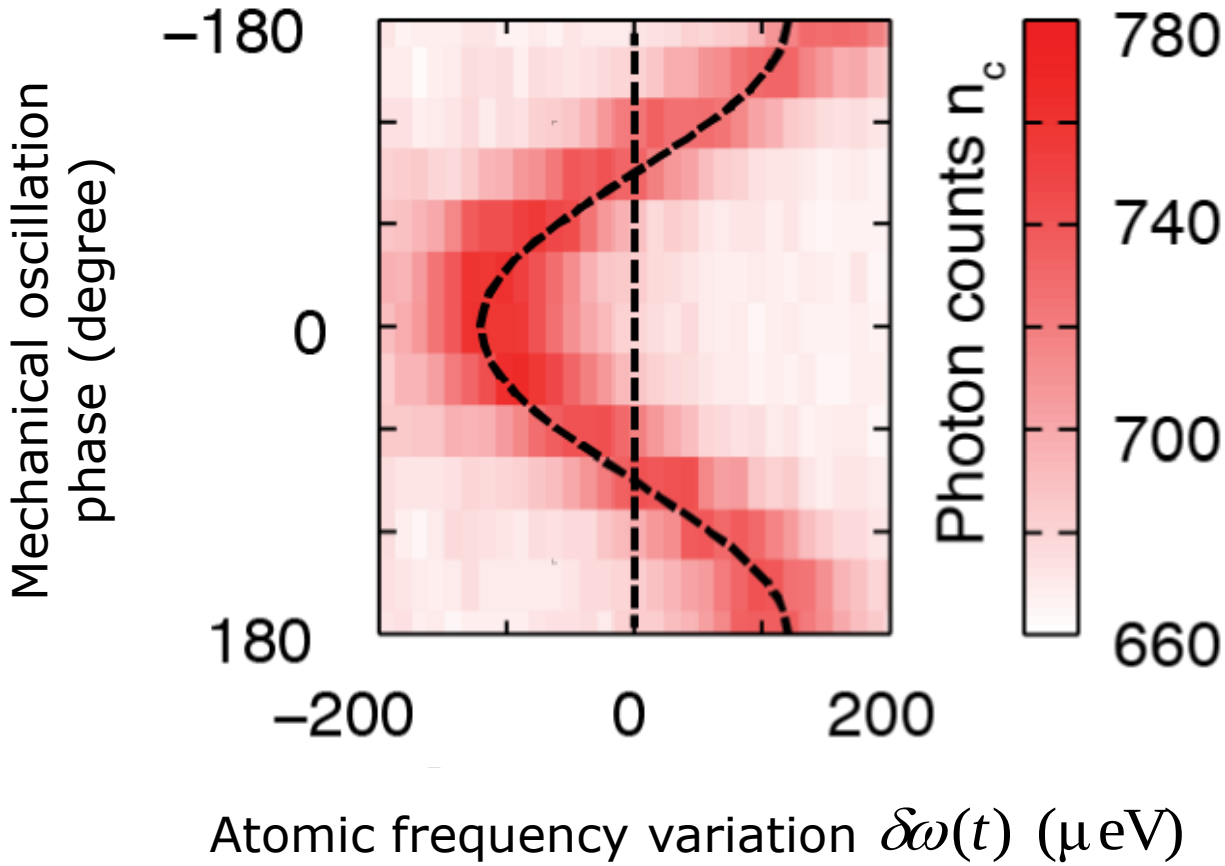
Qubit:
Artificial atom (Quantum dot).

Strain-mediated coupling



Source: I.Yeo et al., arXiv:1306.4209 (2013), *accepted in Nature Nano*

Fluorescence spectroscopy of the embedded atom

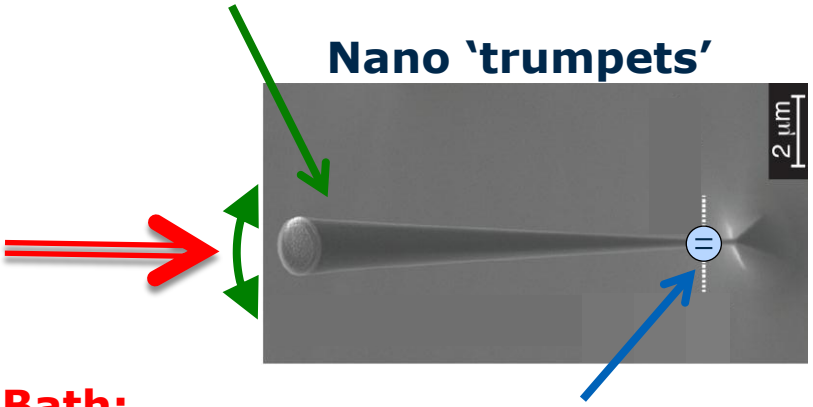


Source: I.Yeo et al., arXiv:1306.4209 (2013), *accepted in Nature Nano*

Using the coupling to change the detuning

External operator:
Oscillating nanowire.

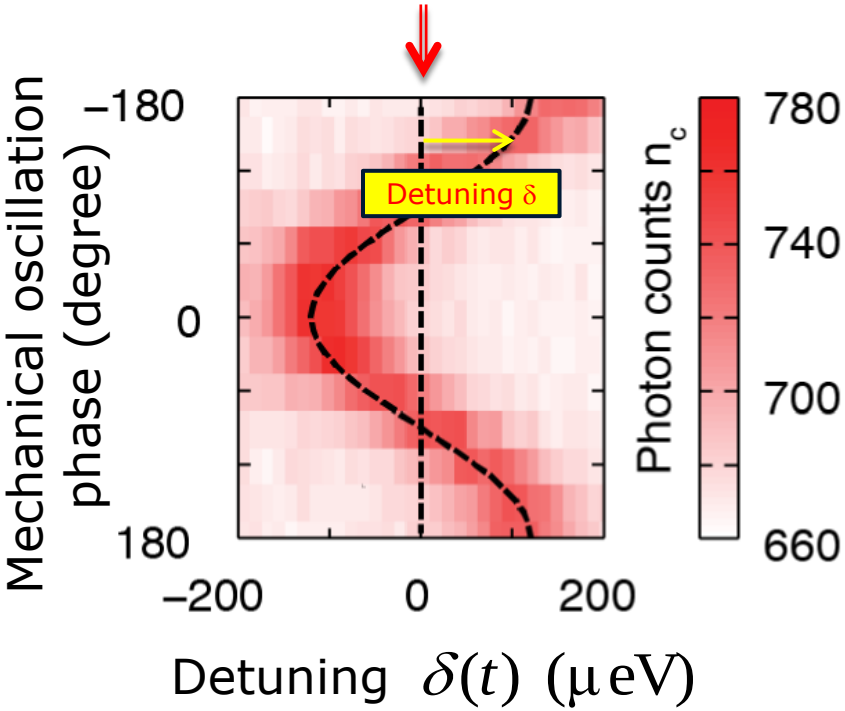
Nano 'trumpets'



Bath:
Laser resonant with the bare atomic frequency + vacuum

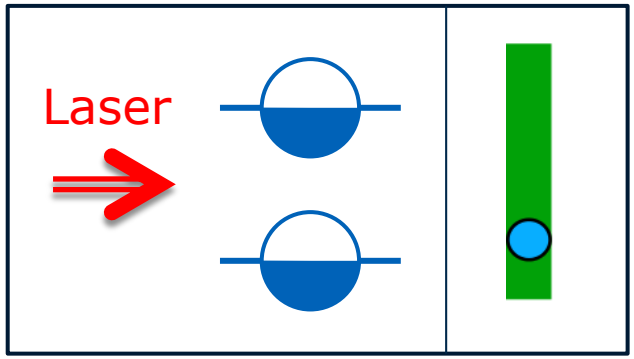
Qubit:
Artificial atom.

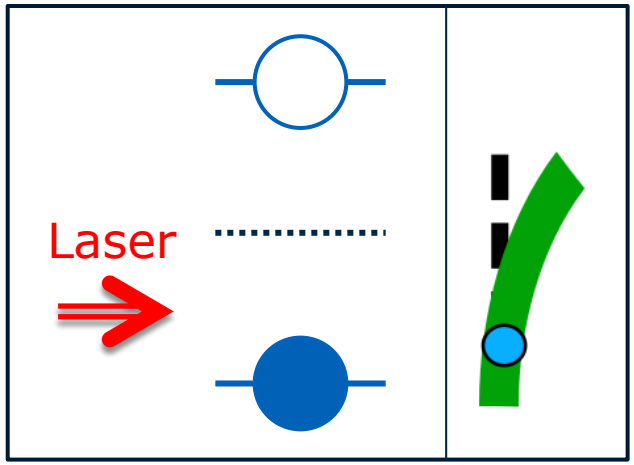
Resonance with the laser

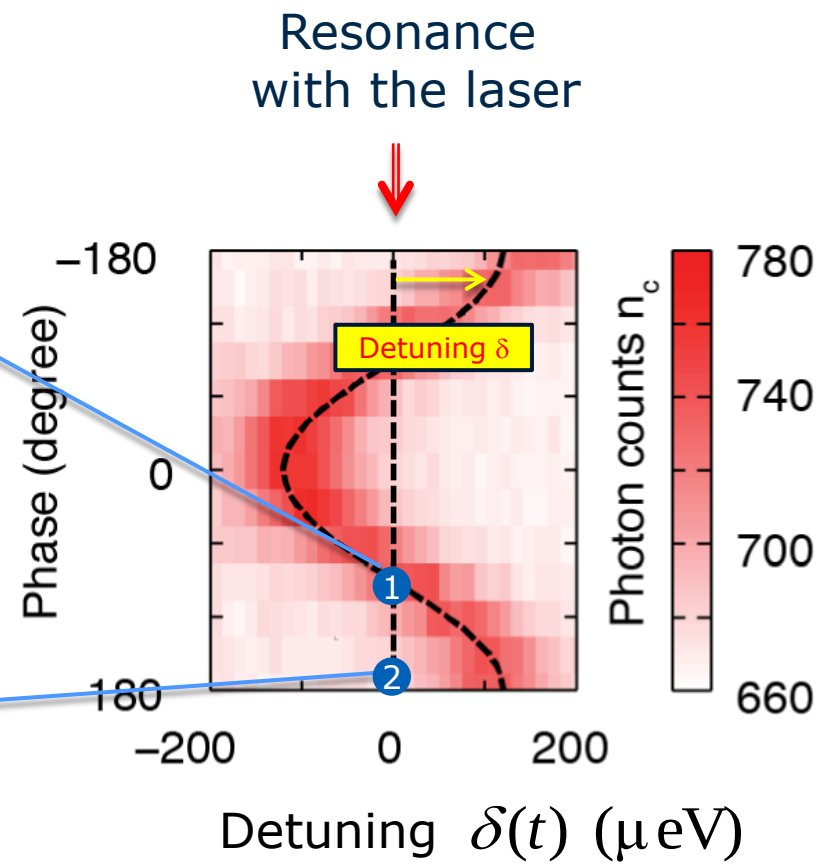


Source: I.Yeo et al., arXiv:1306.4209 (2013), *accepted in Nature Nano*

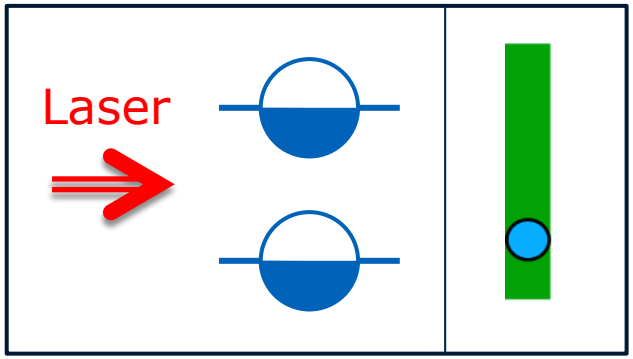
Using the coupling to change the detuning

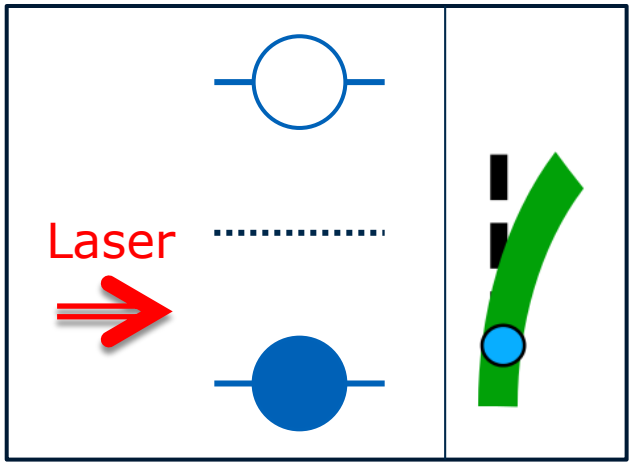




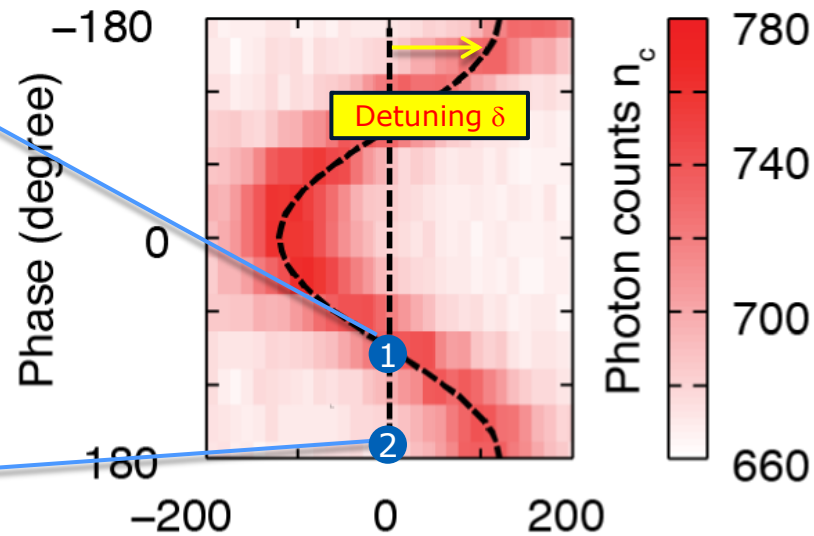


Using the coupling to change the detuning





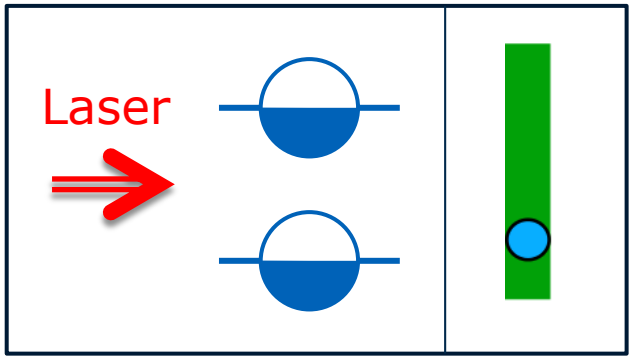
Resonance
with the laser

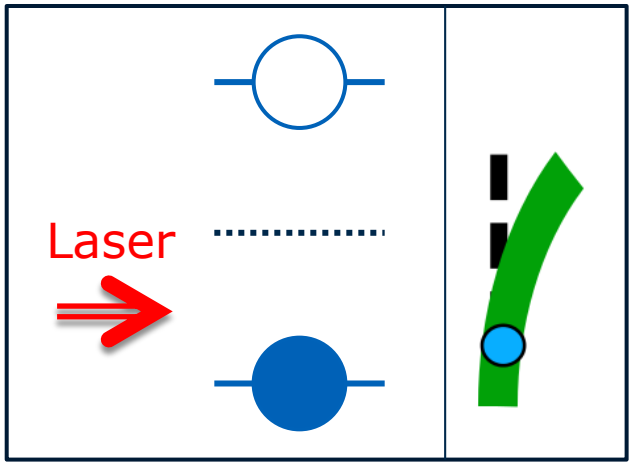


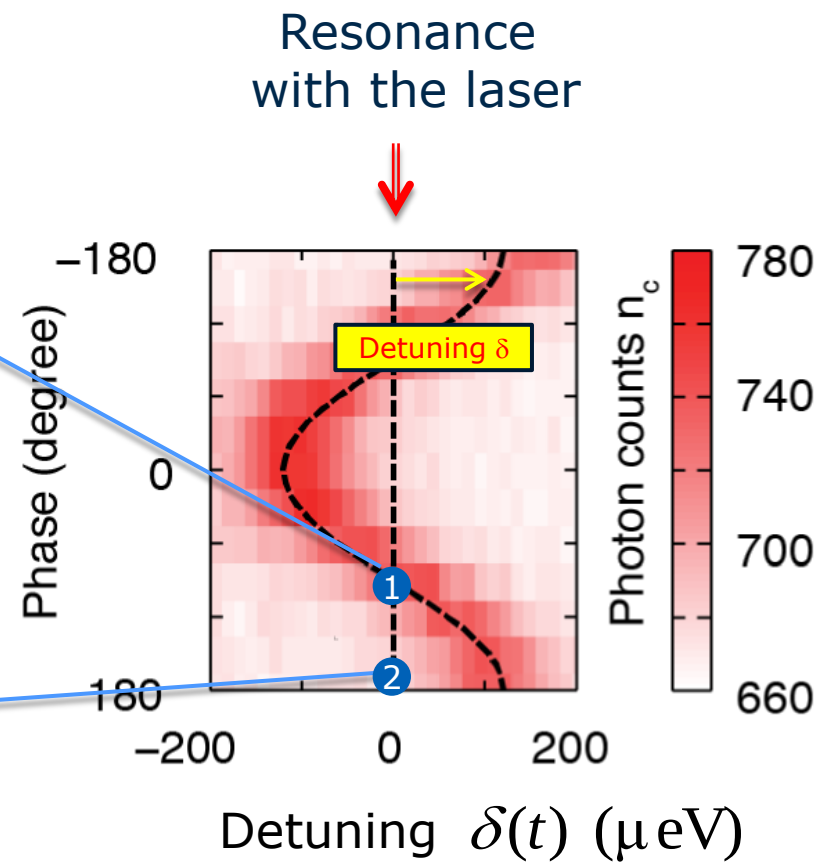
Detuning $\delta(t)$ (μeV)

Adiabatic evolution possible:
the mechanical frequency
is much smaller than γ

Using the coupling to change the detuning







How does the oscillator state change when it performs work ?

Model : spin-boson hamiltonian

$$H = \underbrace{\hbar\omega_0(\sigma_z + 1/2)}_{\text{Atom}} + \underbrace{\hbar g(\sigma_+ e^{-i\omega t} + \sigma_- e^{i\omega t})}_{\text{Coupling with the laser}}$$

$$+ \overbrace{\hbar g_m (b + b^\dagger)\sigma_z}^{\text{Optomechanical coupling}} + \overbrace{\Omega b^\dagger b}^{\text{Mechanical mode}}$$

Coupling strength

$$g_m = \left.\frac{\partial\omega_0}{\partial x}\right|_{x=0} \delta x_{ZPF}$$

Zero point fluctuations

Mechanical frequency

Annihilation operator

I.Yeo et al., arXiv:1306.4209 (2013), *accepted in Nature Nano*
A. Auffèves et al., arXiv:1305.4252 (2013)

Semi-classical approach

We consider a coherent state of the mechanical oscillator → Complex amplitude β(t) = <b>

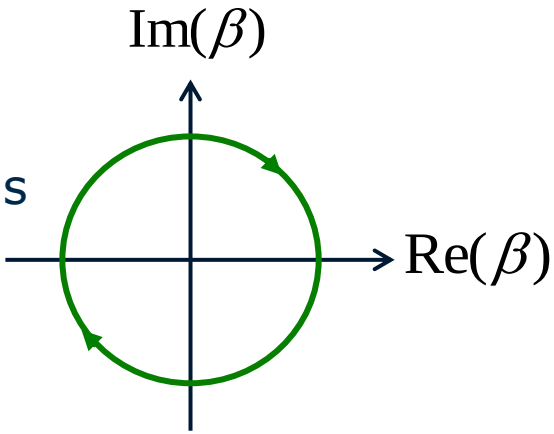
$$x(t) = 2\,\mathrm{Re}(\beta)\delta x_{ZPF}$$

$$\delta(t) = 2 g_m\,\mathrm{Re}(\beta)\delta x_{ZPF}$$

<u>Mechanical energy:</u>

$$E_{MO}(t) = \hbar\Omega\left|\beta(t)\right|^2$$

Ex : free oscillations trajectory



H → Optical Bloch equations

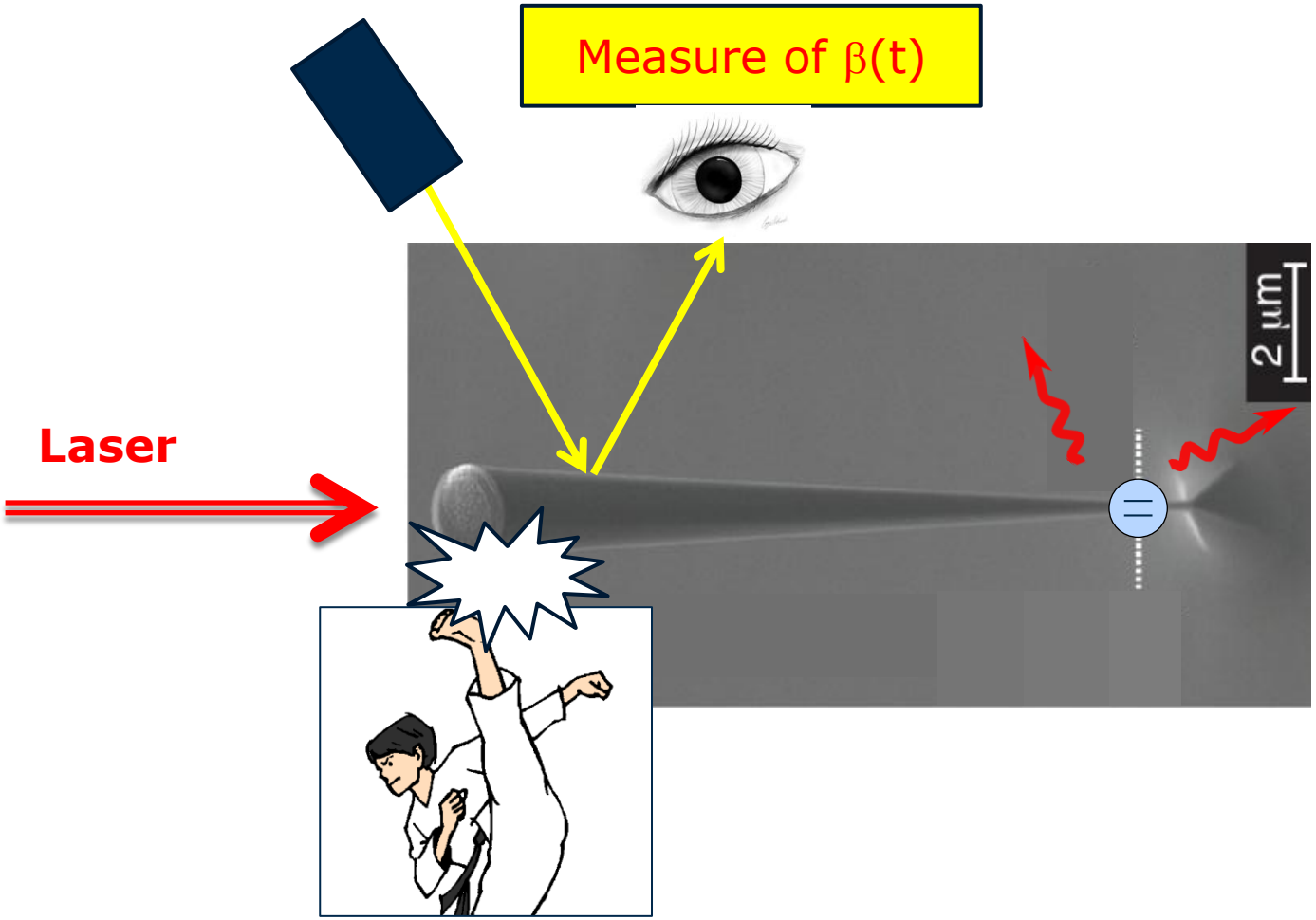
$$E_{MO}(t) = \hbar\Omega\left|\beta(t)\right|^2$$ Mechanical energy

$$E_{MO}(t) - E_{MO}(0) = \int_0^t dt\, \dot{\delta} P_e(t) = w(t)$$

Measuring β(t) gives access to the work performed on the qubit !

→ Light deflexion techniques

Summary



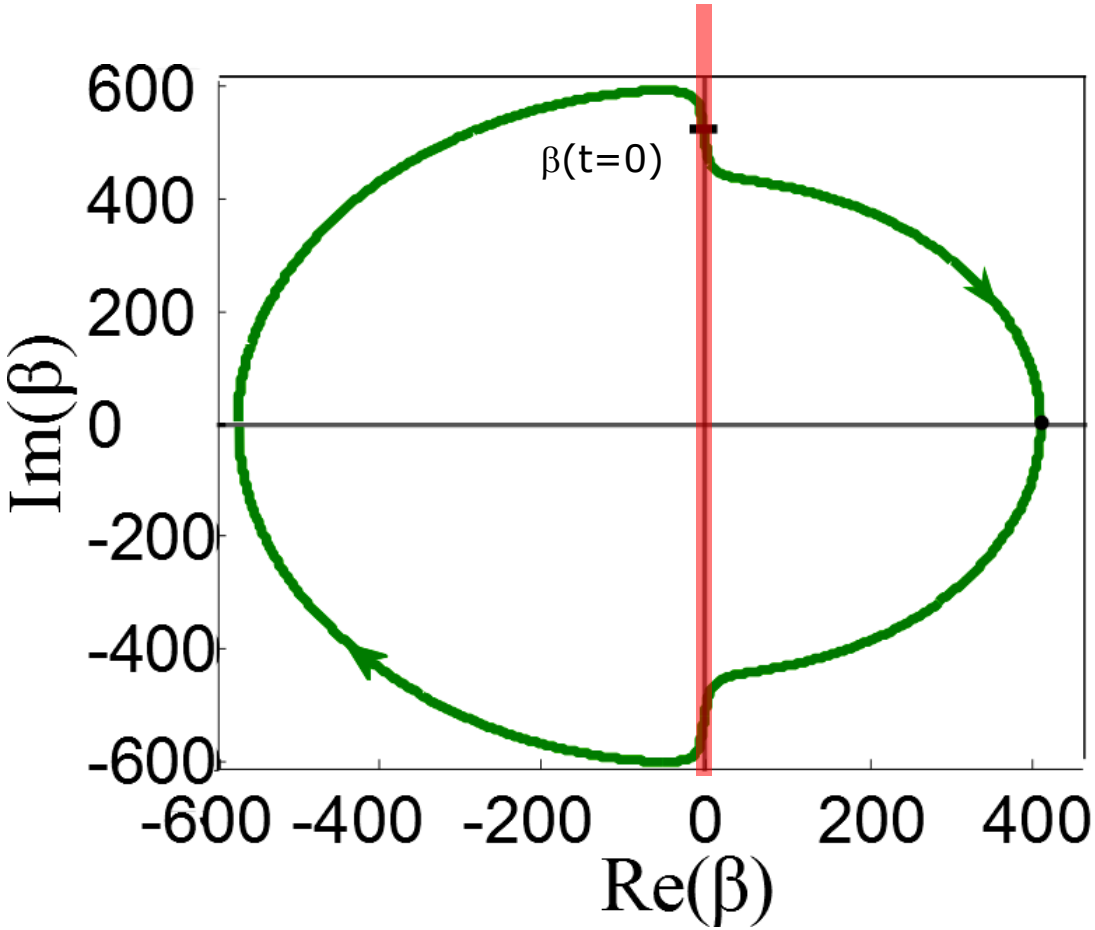
At t=0, we kick the oscillator and let it evolve …

Outline

- I) Standard Landauer's erasure protocol
  - Standard protocol
  - Optical protocol
- II) A battery enabling to monitor work exchanges
- III) Full cycle of energy-information conversions
- IV) Proposal for optical Carnot engine

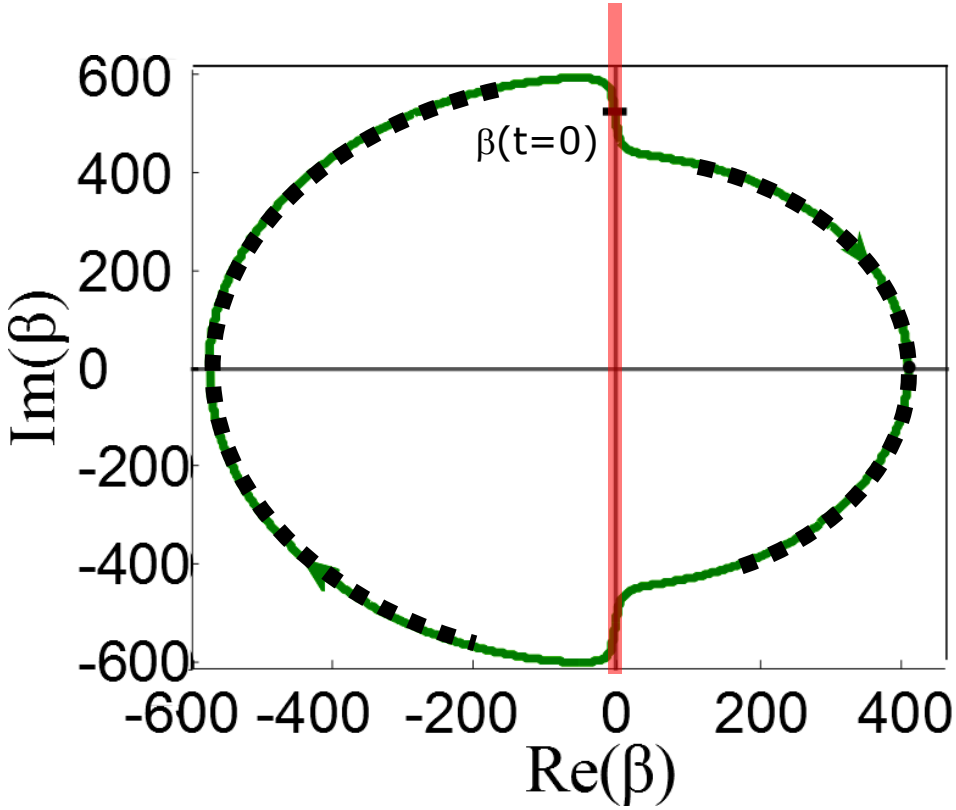
Observing work exchanges

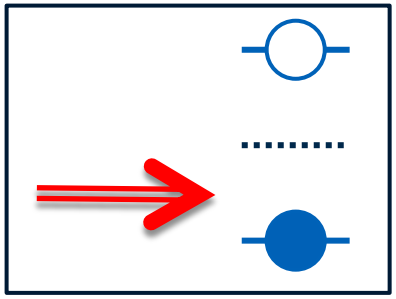
Resonance with the laser



Observing work exchanges

Resonance with the laser

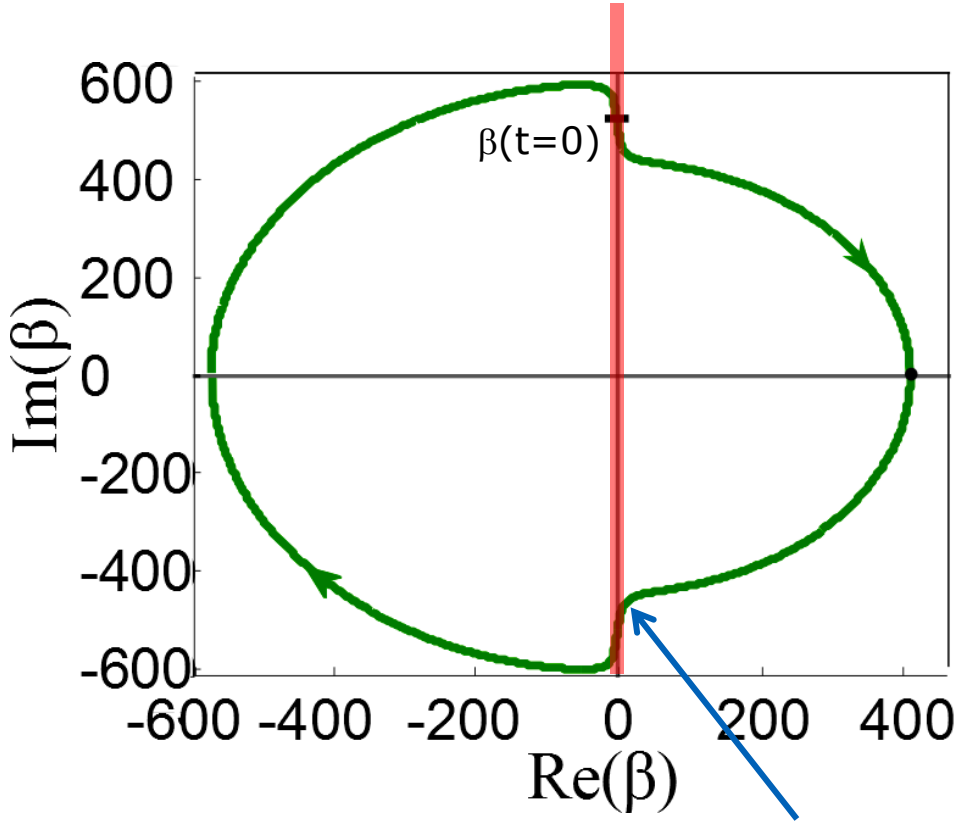




Free oscillation when the atom is far from resonance:
- Atom erased: W=0
- Atom decoupled from the bath

Observing work exchanges

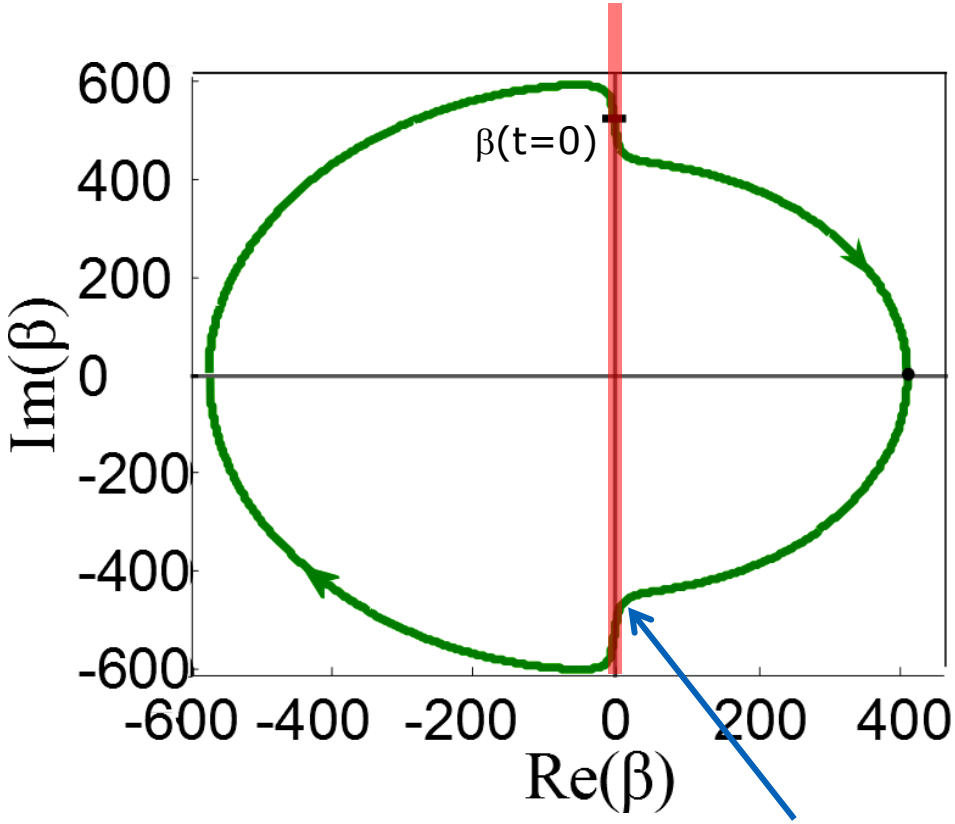
Resonance with the laser



Variation of |β| when leaving or coming in resonance → exchange of work

Observing work exchanges

Resonance with the laser



Typically W_L corresponds to:

Δx = 0.4 pm
Amplitude: 1.2 pm
Signal/Shot noise = 40
Signal/Thermal noise = 0.3

(g = 3 GHz, g_m = 30 MHz, $\beta_0 = 10^2$, $\Omega/2\pi$ = 550 kHz, T = 100 mK)

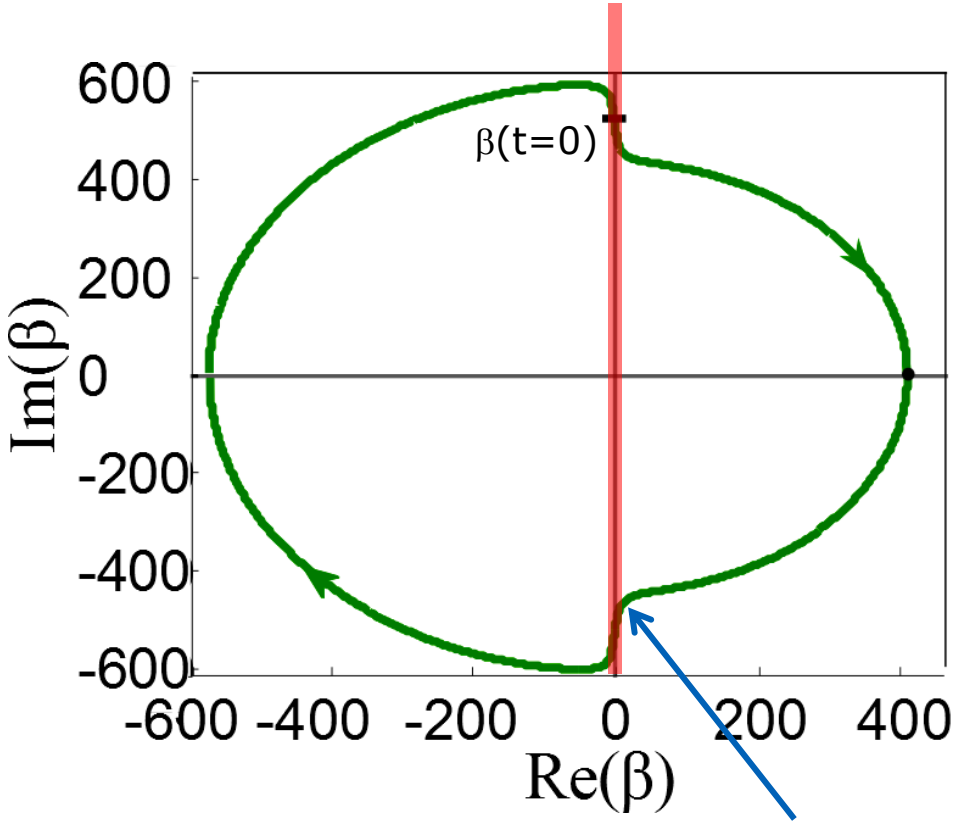
→ Measurable with current deflexion techniques

B. Sanii et al. PRL 104 (2010)

Variation of $|\beta|$ when leaving or coming in resonance → exchange of work

Observing work exchanges

Resonance with the laser



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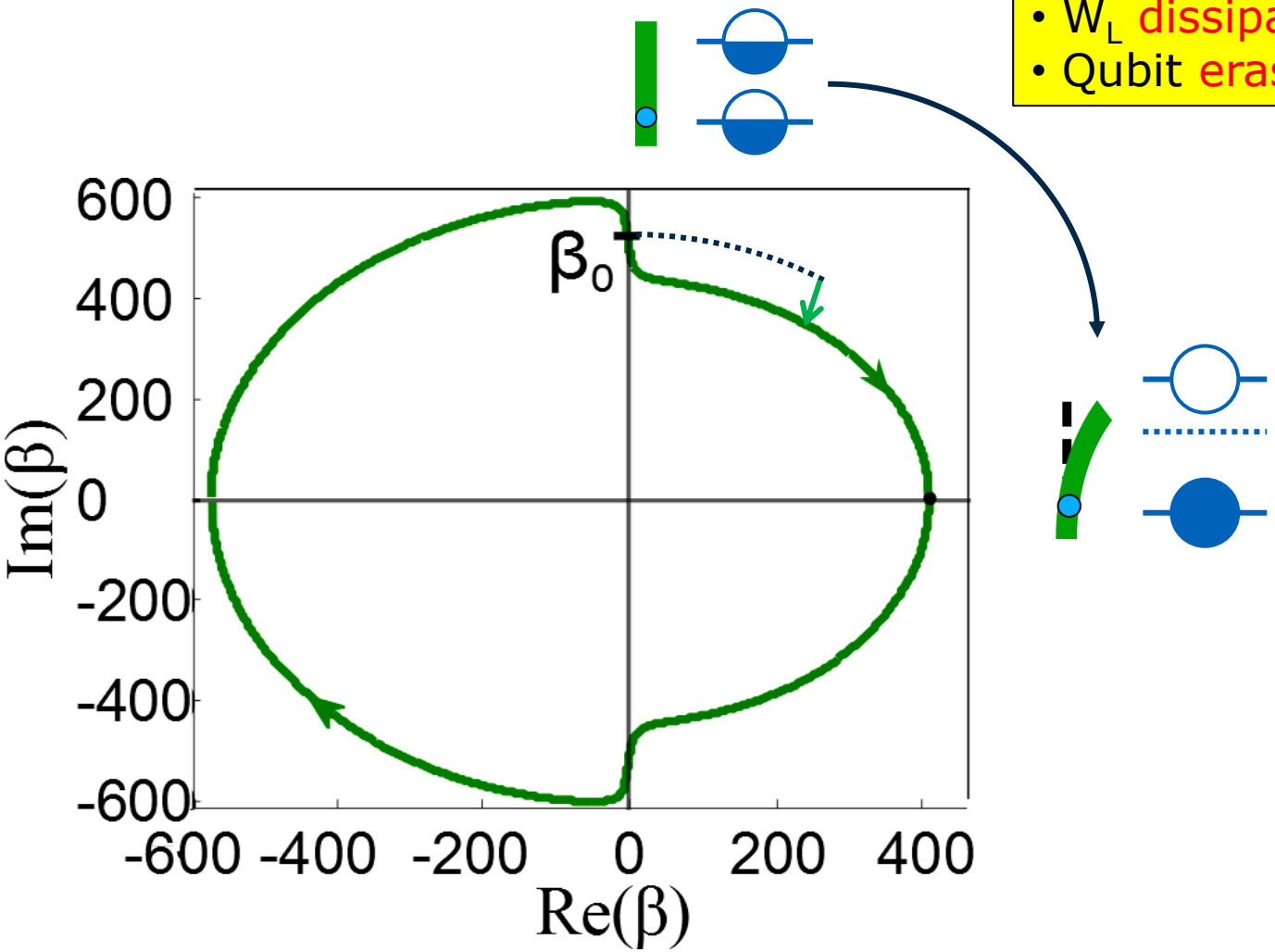
(g = 3 GHz, g_m = 30 MHz, $\beta_0 = 10^2$, $\Omega/2\pi$ = 550 kHz, T = 100 mK)

NB: $g_m > \Omega$ needed

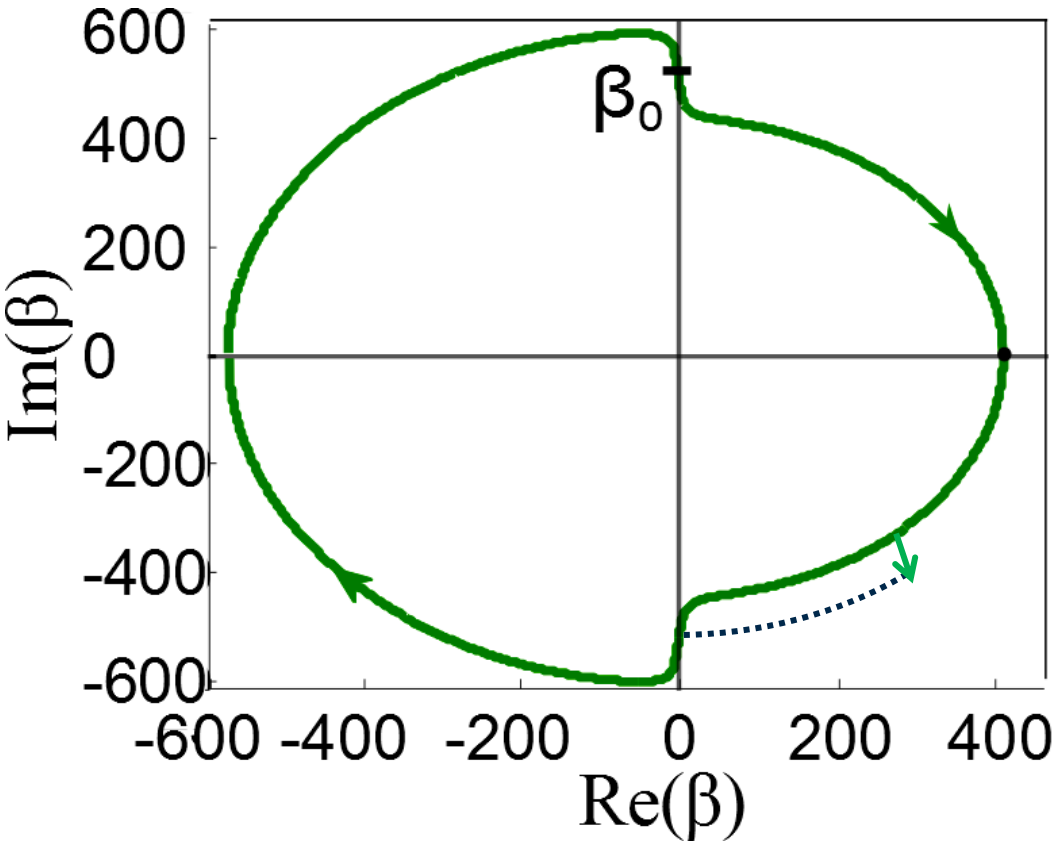
Variation of $|\beta|$ when leaving or coming in resonance → exchange of work

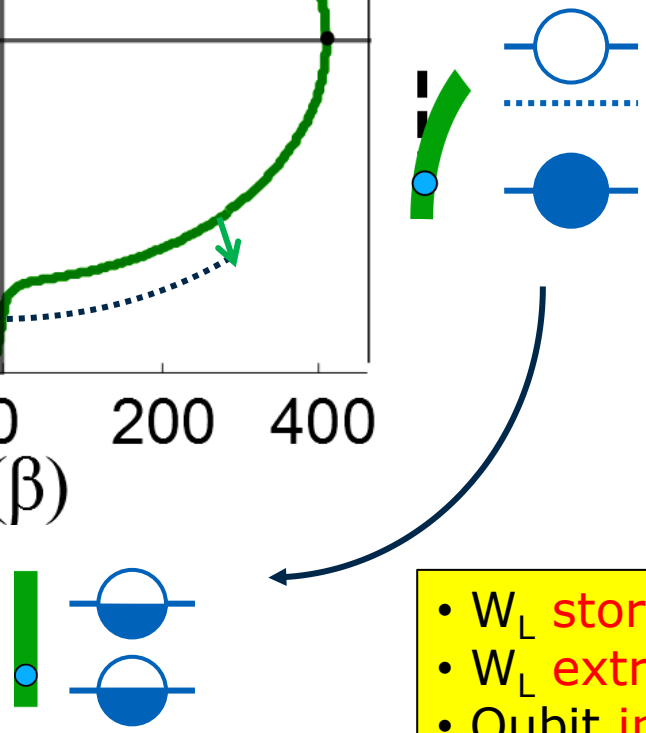
First quarter of oscillation
Landauer's erasure

- W_L provided by the battery
- W_L dissipated in the bath
- Qubit erased



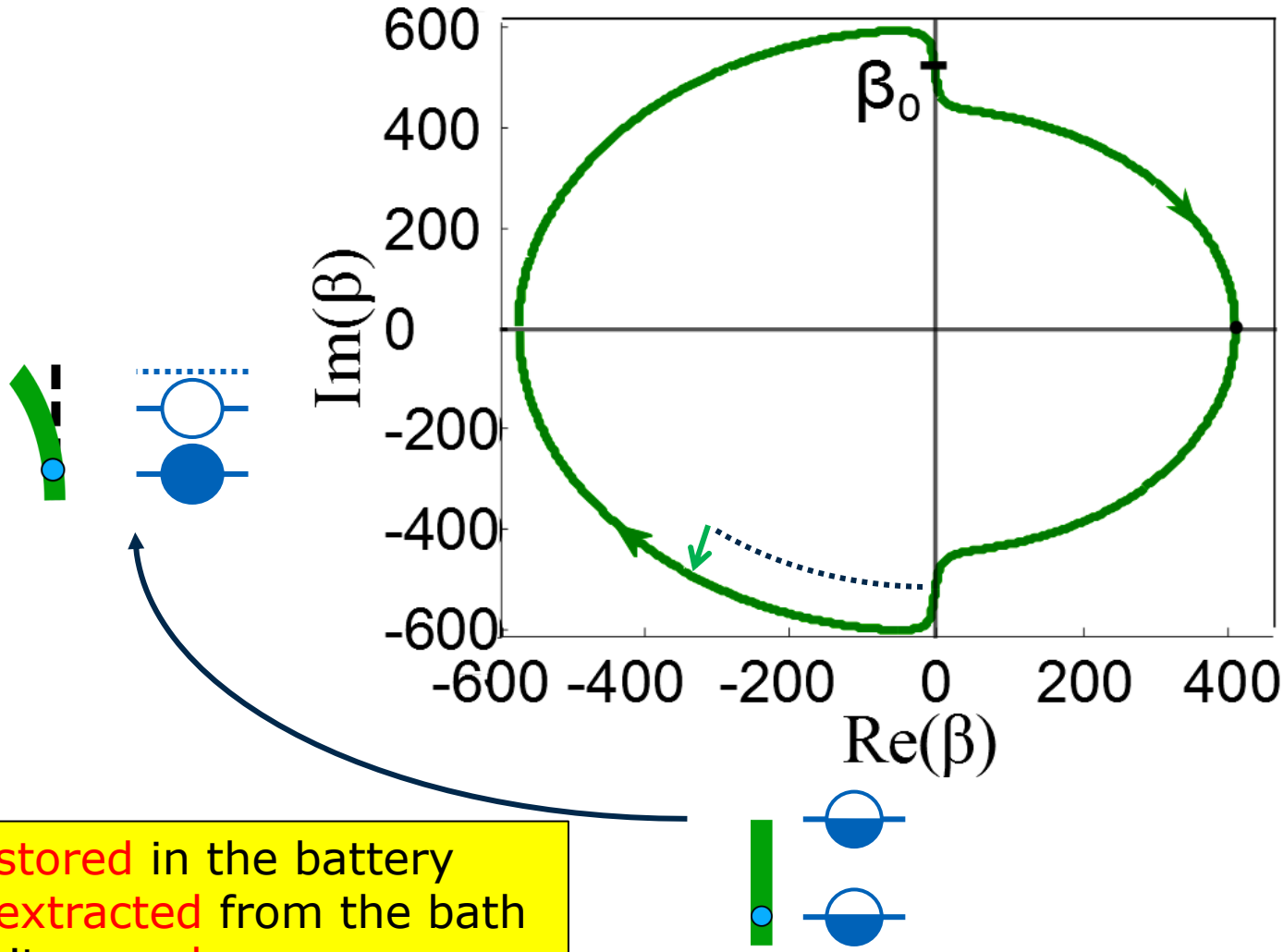
Second quarter of oscillation
Szilard's engine





- W_L stored in the battery
- W_L extracted from the bath
- Qubit in a mixed state

Third quarter of oscillation
Inverse Landauer's erasure



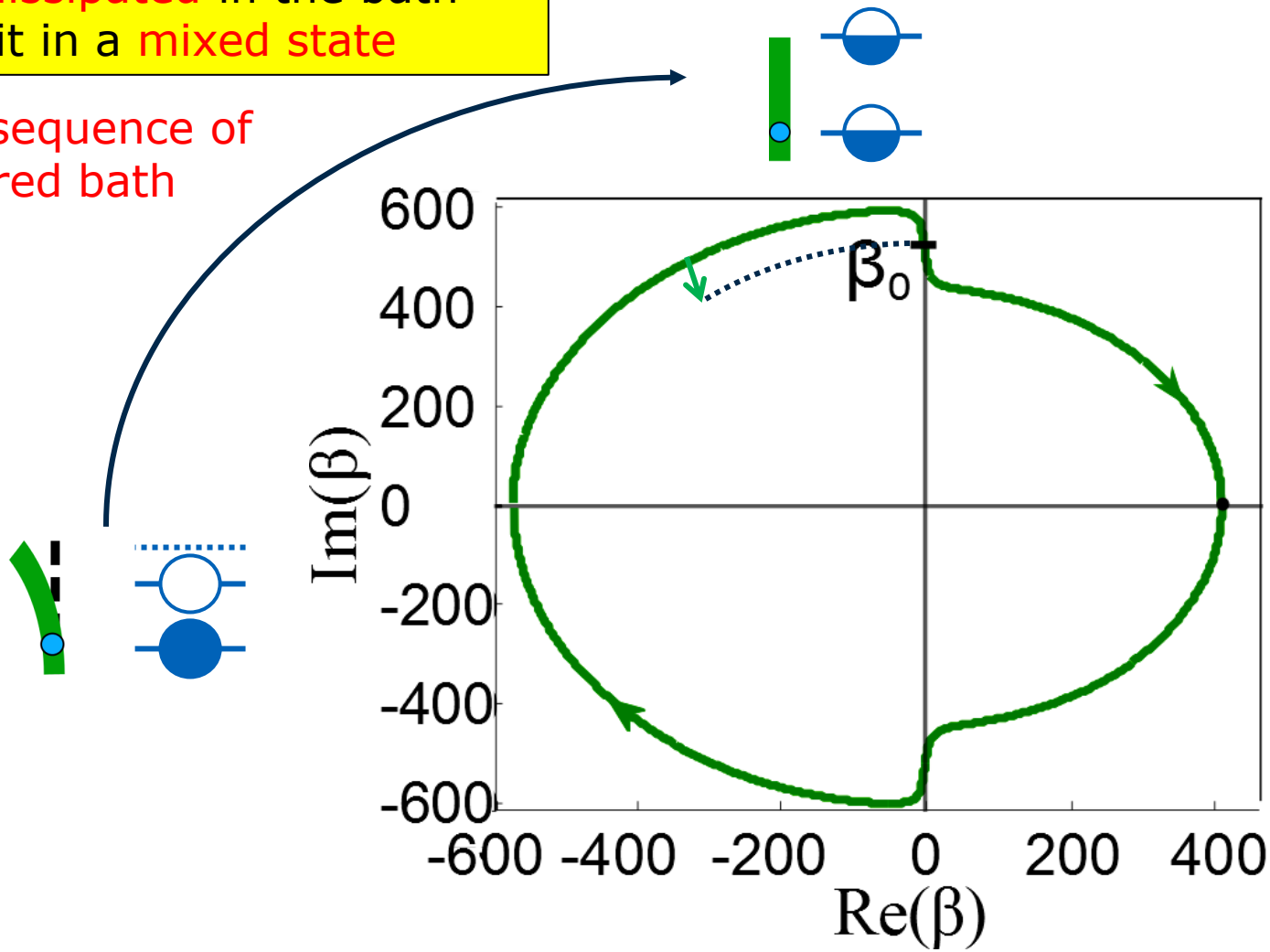
- W_L stored in the battery
- W_L extracted from the bath
- Qubit erased

Consequence of colored bath

Last quarter of oscillation
Inverse Szilard's engine

- W_L provided by the battery
- W_L dissipated in the bath
- Qubit in a mixed state

Consequence of colored bath



Outline

- I) Standard Landauer's erasure protocol
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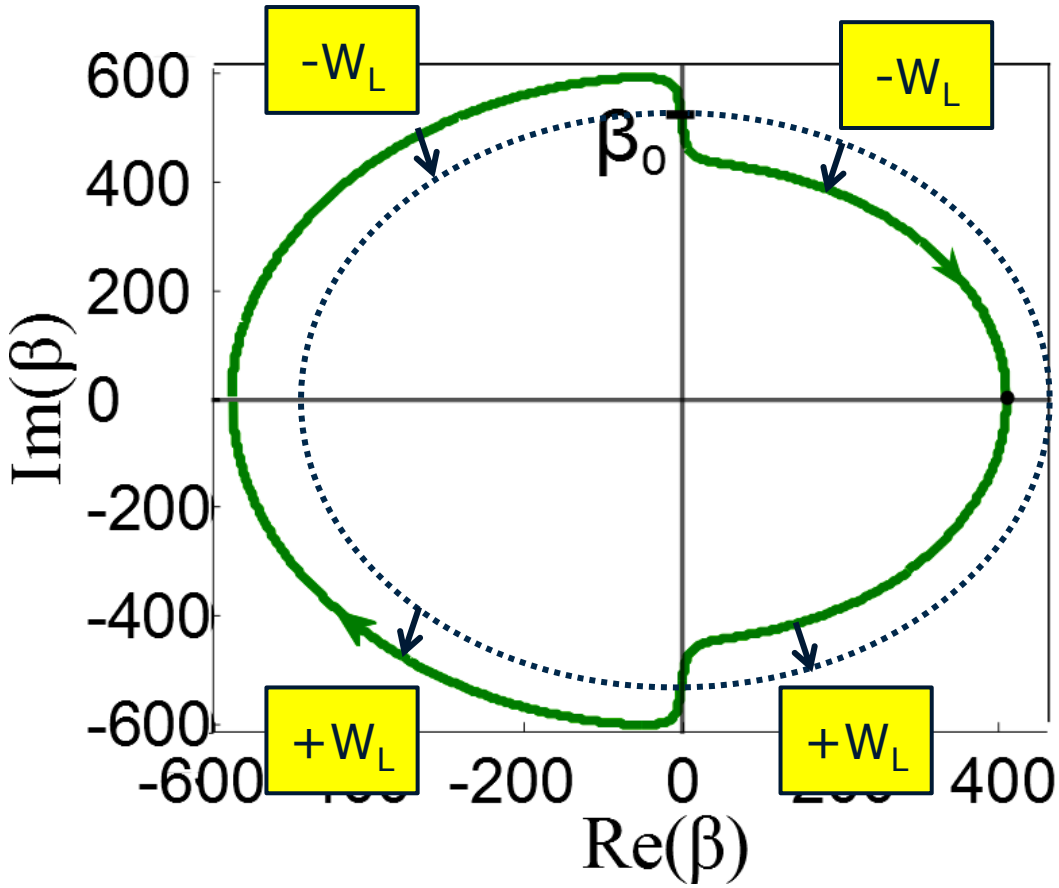
- IV) Proposal for optical Carnot engine

Principle of the engine



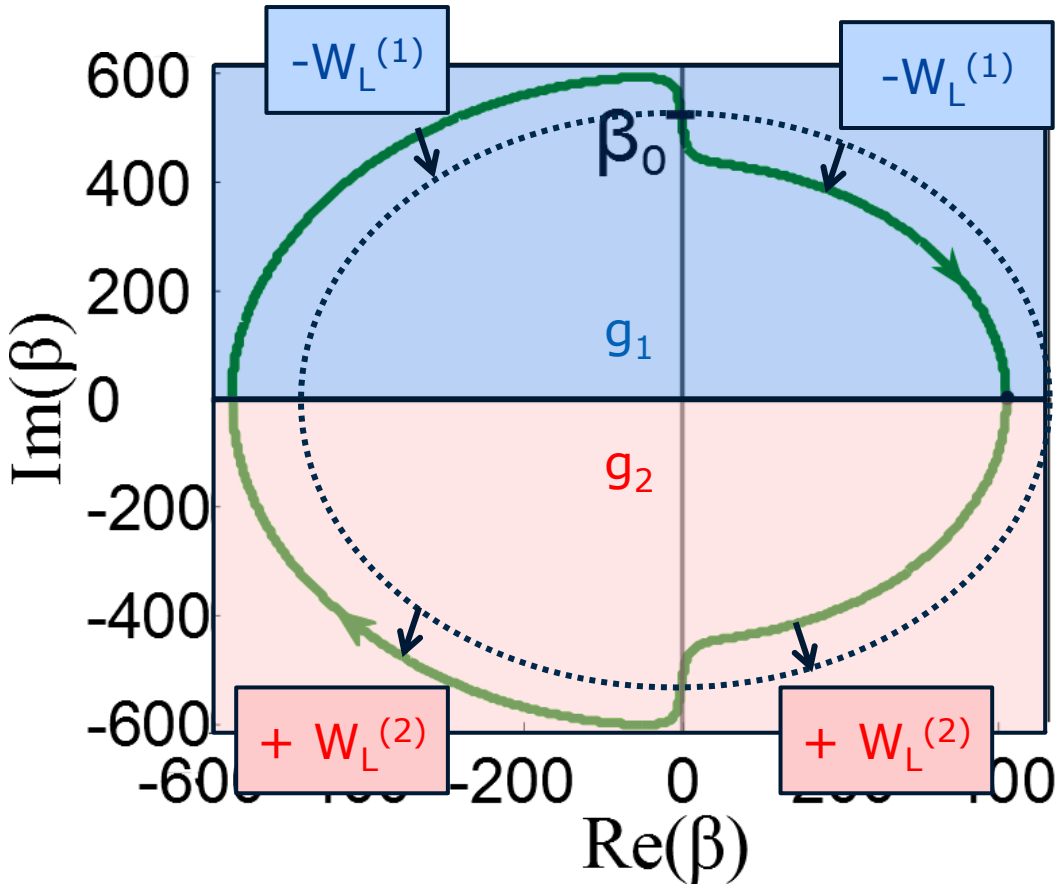
Elementary work

$$W_L = \hbar g \frac{\pi}{4}$$

Rabi frequency (temperature)

$$W_{stored} = -\hbar g \frac{\pi}{4} + \hbar g \frac{\pi}{4} + \hbar g \frac{\pi}{4} - \hbar g \frac{\pi}{4} = 0$$

Principle of the engine



Two different Rabi frequencies

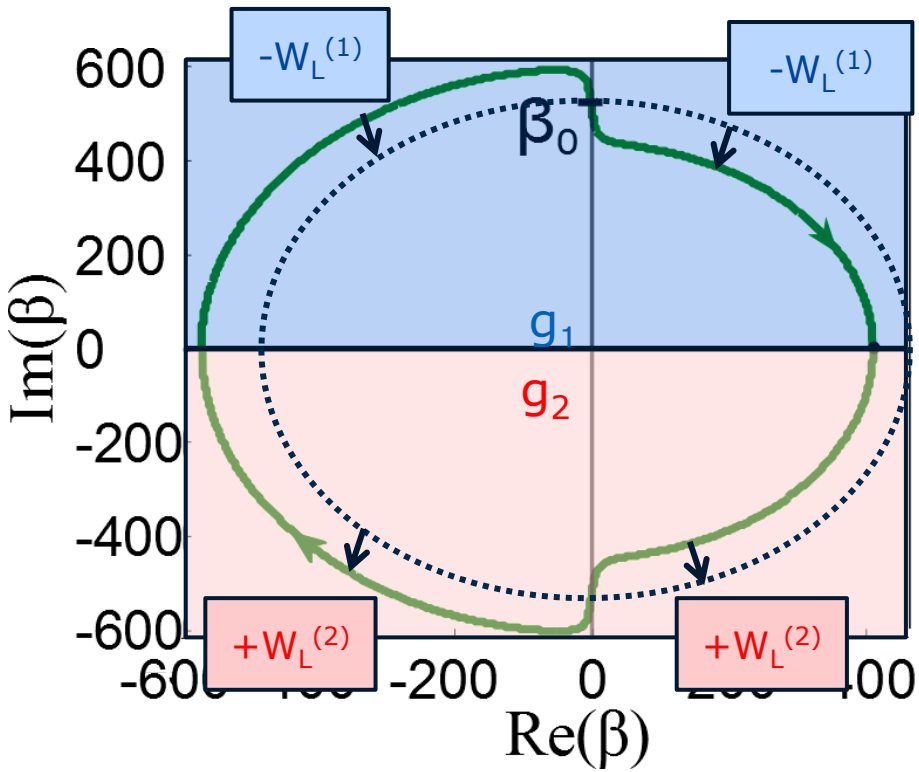
$g_2 > g_1$

=

thermalization of the qubit with a hot source and a cold source

$$W_{stored} = -\hbar g_1 \frac{\pi}{4} + \hbar g_2 \frac{\pi}{4} + \hbar g_2 \frac{\pi}{4} - \hbar g_1 \frac{\pi}{4} > 0$$

Carnot efficicieny in finite time

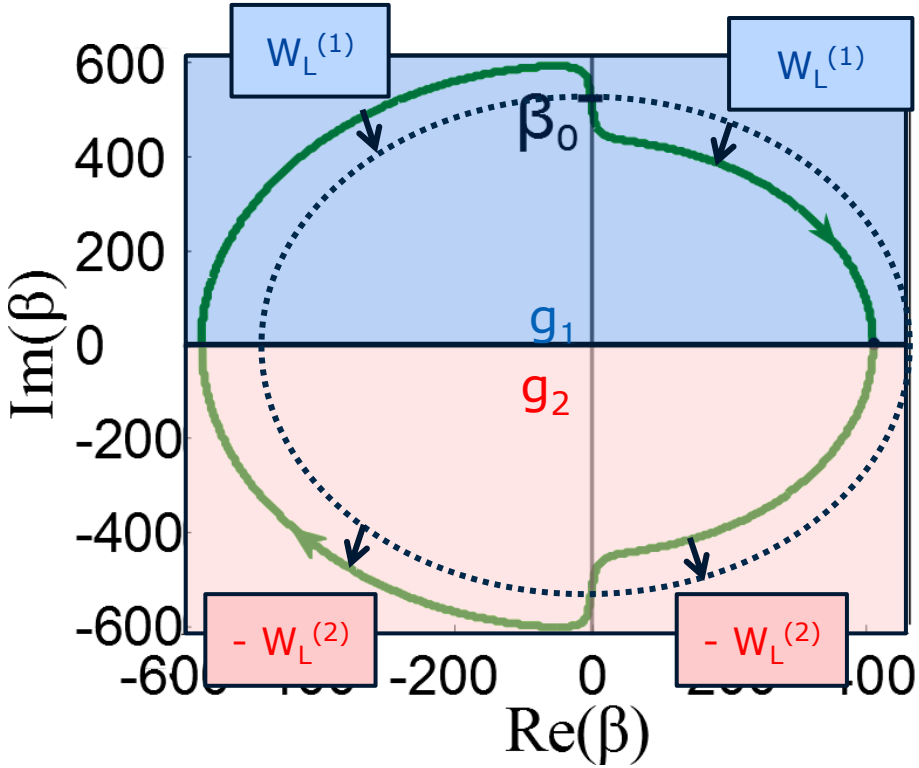


$$\eta = 1 - g_2 / g_1$$

$$\Leftrightarrow \quad \eta_C = 1 - T_2 / T_1$$

Carnot ideal efficiency reached with realistic parameters!

Carnot efficicieny in finite time



Carnot ideal efficiency reached

$$\eta = 1 - g_2 / g_1$$

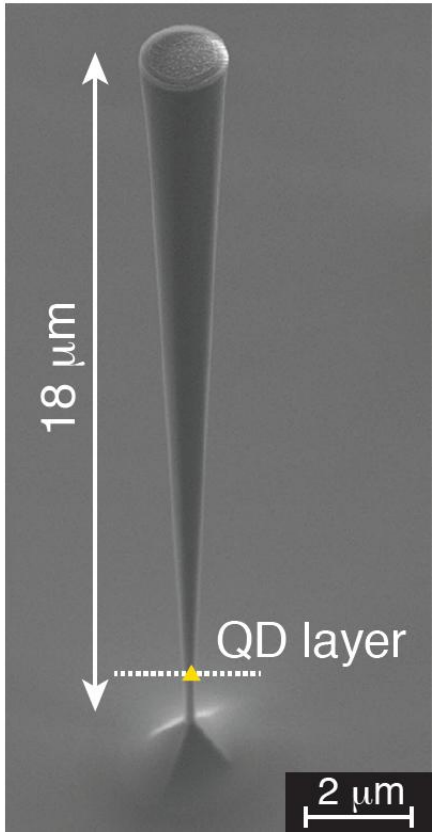
$$\Leftrightarrow \quad \eta_C = 1 - T_2 / T_1$$

$\mathcal{P} = 10^{-17}$ W

3 order of magnitudes over existing proposals of single qubit heat engines

O. Abah et al., PRL 109, 203006 (2012).

Conclusion

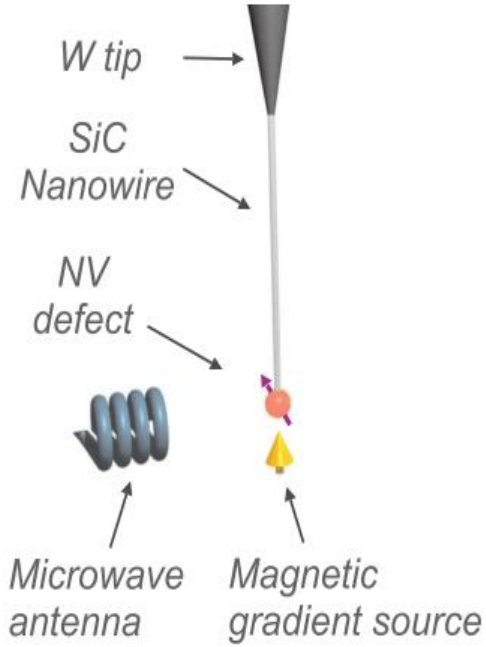


I.Yeo et al.,
arXiv:1306.4209 (2013)
accepted in Nature Nano

- A set up enabling reversible information energy conversion in a qubit

- Direct observation of work exchanges in a quantum battery

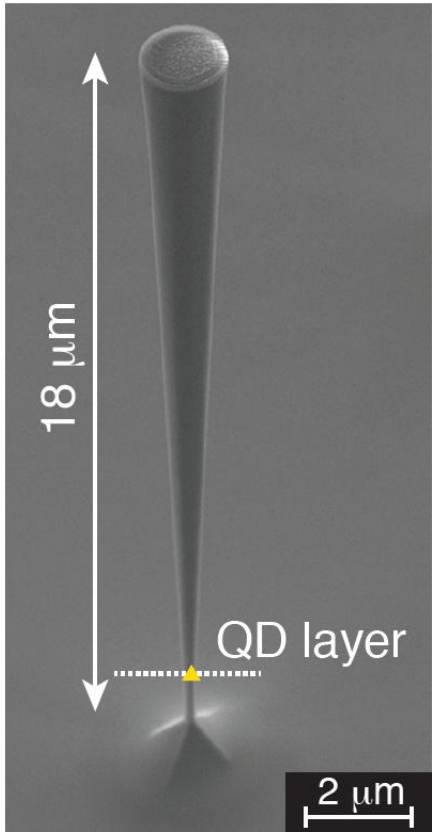
- Mechanical oscillations perform Carnot cycles at maximum efficiency



O. Arcizet et al.,
Nature Physics 7 (2011) 879

Perspectives

Now that the building blocks Landauer's erasure & Szilard engine are ensured, we can go to the fully quantum regime



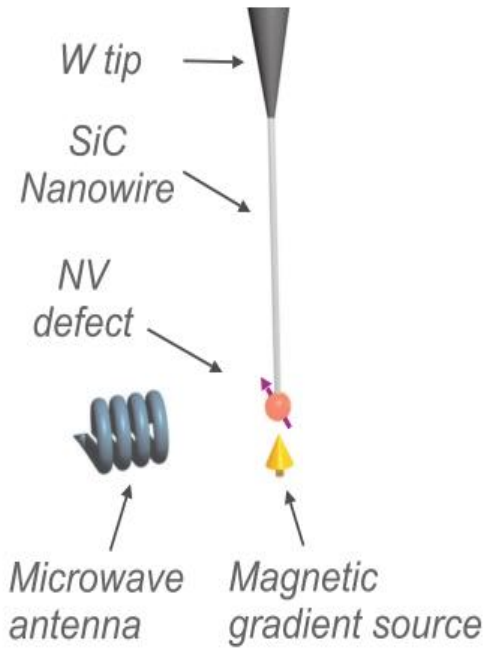
I.Yeo et al.,
arXiv:1306.4209 (2013)
accepted in Nature Nano

- Erasure cost of two entangled qubits

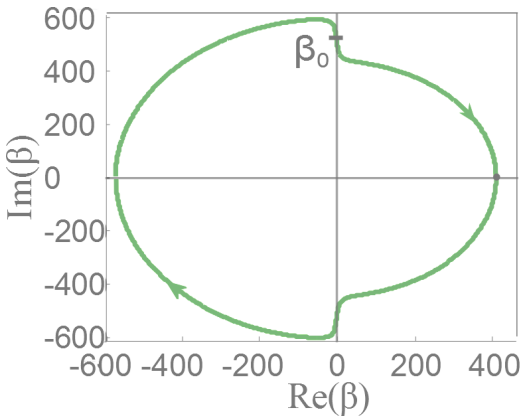
  L. del Rio et al., Nature 474, 61--63 (2011)

- Measurement of work distribution during conversions → in situ verification of quantum fluctuation theorems

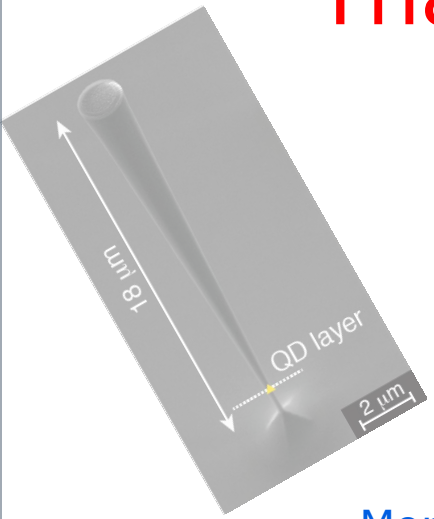
  L. Mazzola et al., PRL 110 (2013)

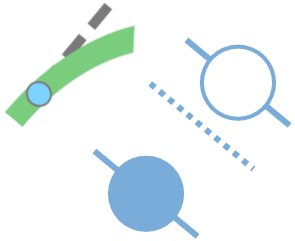


O. Arcizet et al.,
Nature Physics 7 (2011) 879



Thank you for your attention



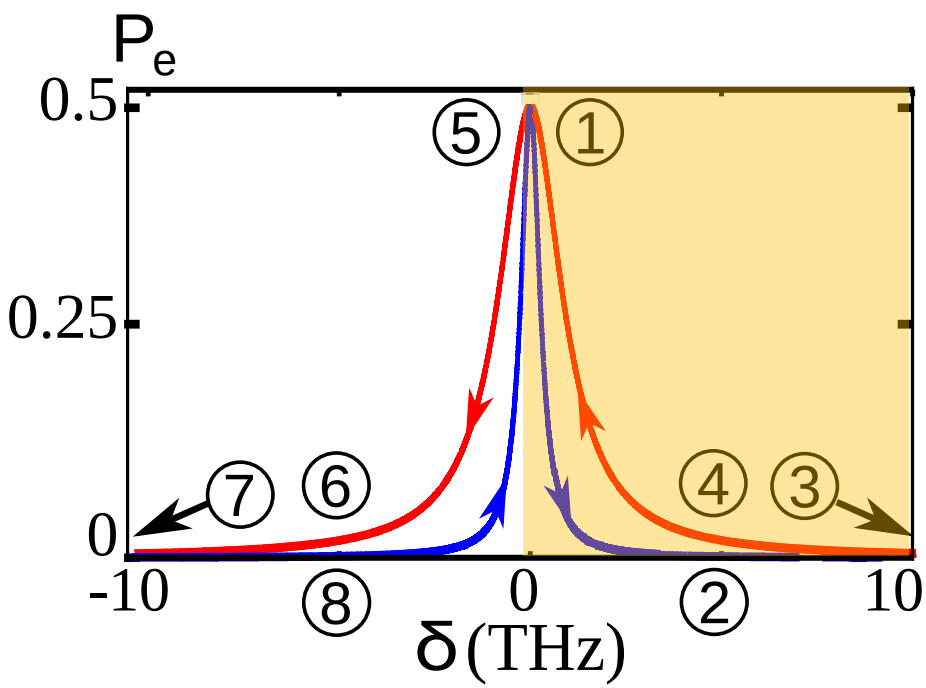


$$W_L = \hbar g \frac{\pi}{4}$$

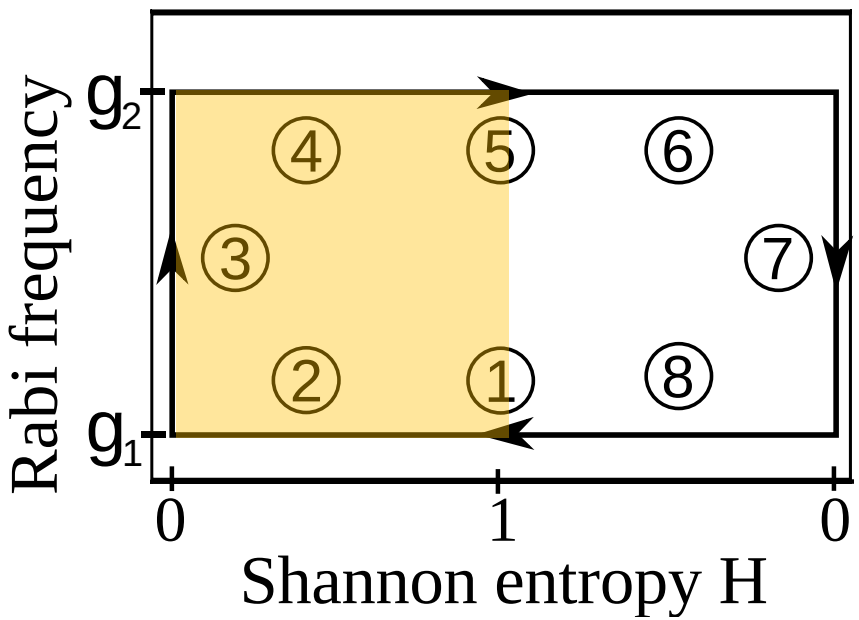
More details in:
Cyril Elouard, Maxime Richard, Alexia Auffèves, arXiv:1309.5276

A double Carnot cycle

1st cycle: Landauer's erasure + Szilard engine



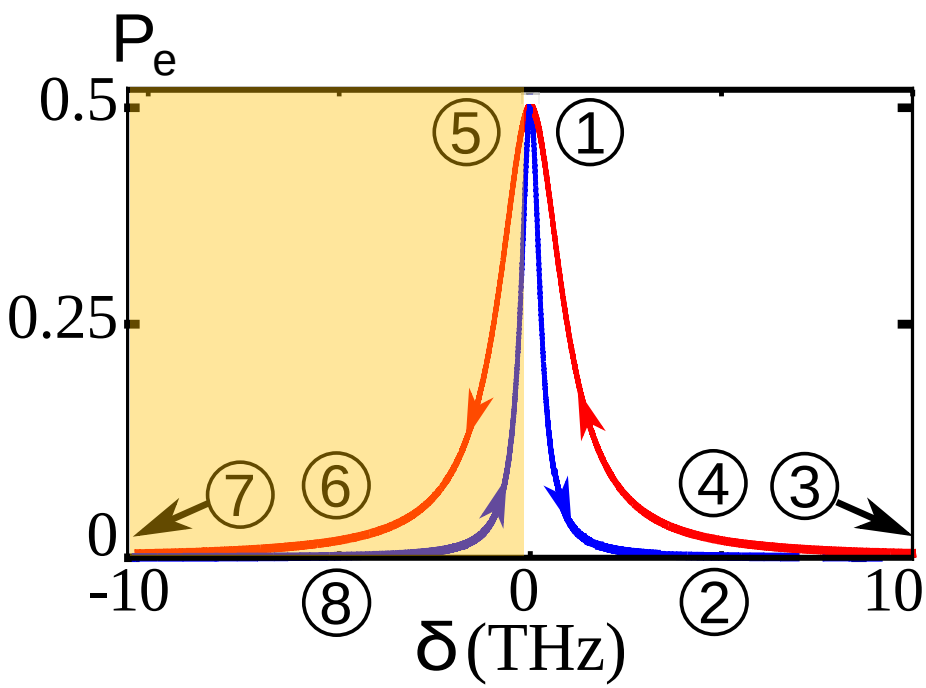
Equivalent to (P,V)



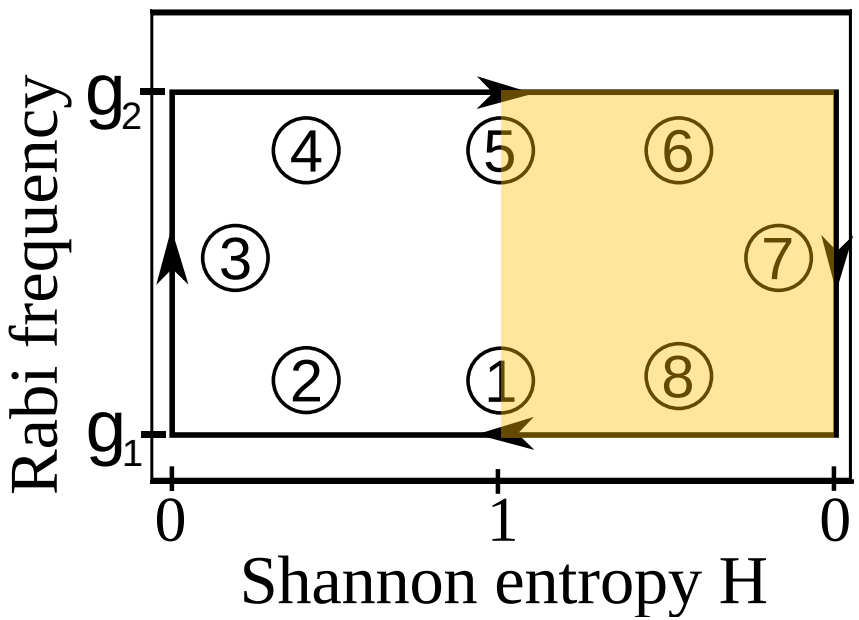
Equivalent to (S,T)

A double Carnot cycle

2nd cycle: *inverse* Landauer's erasure + *inverse* Szilard engine



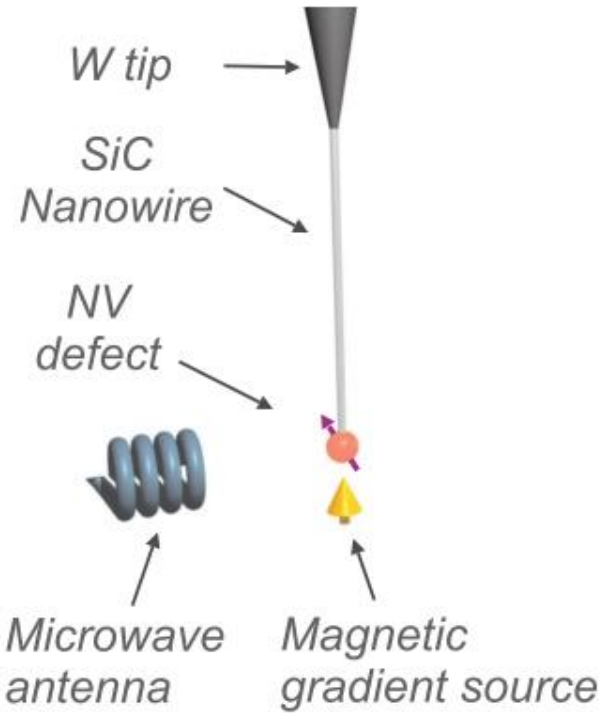
Equivalent to (P,V)



Equivalent to (S,T)

1)Experimental implementation

NV center in a nanowire under magnetic field gradient



O. Arcizet et al., Nature Physics 7 (2011) 879

2) Two entangled atoms S and Q

Total work gain W_L

Pure entangled state

$$\rho_1 = |\Psi\rangle_{QS}\langle\Psi| \qquad \Psi_{QS} = (|01\rangle + |10\rangle)/\sqrt{2}$$

$\xrightarrow{U}$ $\rho_2 = |11\rangle\langle 11|$

Szilard engine on S and Q
$2W_L$ extracted

$\longrightarrow$ $\rho_3 = \mathbb{I}/4$

Landauer's erasure on **S**
cost W_L

$\longrightarrow$ $\rho_4 = |0\rangle_S\langle 0| \otimes \mathbb{I}/2$

From S point of view

$$\mathrm{Tr}_Q\, \rho_1 = \mathbb{I}/2 \xrightarrow{\text{Erasure of the qubit S}} \mathrm{Tr}_Q\, \rho_3 = |0\rangle\langle 0|$$

Perspectives

3) Direct measurement of work distribution

Can be extracted from quantum jump trajectories of the qubit during erasure

→ Verification *in situ* of Fluctuation theorems such as Jarzinsky equality

4) Color of the emitted photons

Color of the photon emitted during erasure contain information about the dissipated heat and then must quantifiy the irreversibility of the engine.

→ Alternative devices with monitored environment

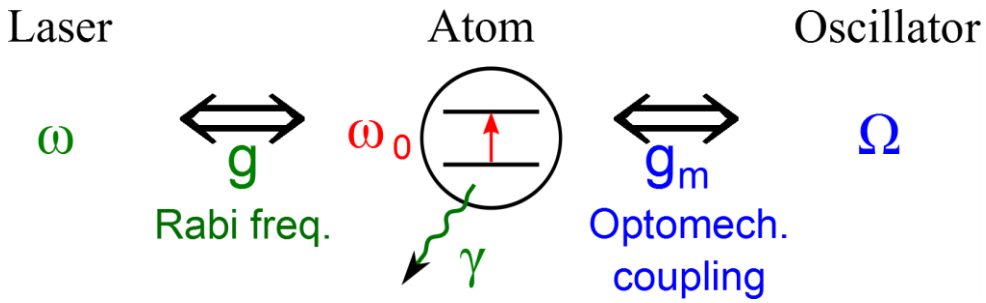
Pekola et al., arXiv:1212.5808

B.Huard group set up

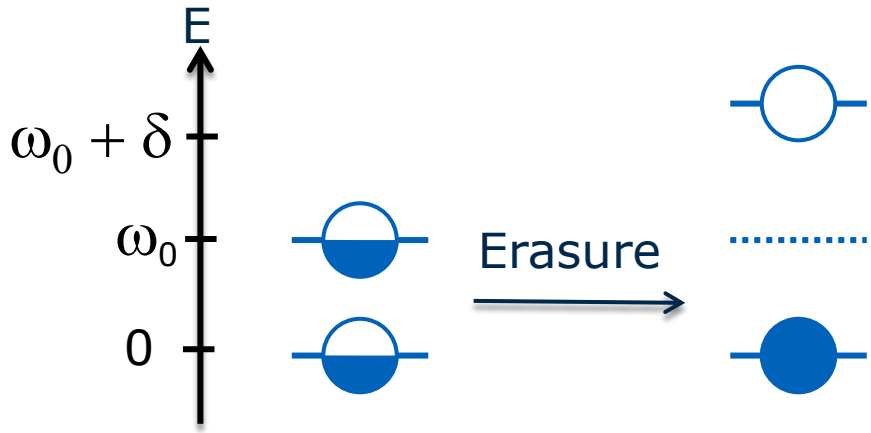
Hamiltonian describing the optomechanical device

$$H = \underbrace{\hbar\omega_0(\sigma_z + 1/2)}_{atom} + \underbrace{\hbar g(\sigma_+ e^{-i\omega t} + \sigma_- e^{i\omega t})}_{light\ coupling} + \underbrace{\hbar g_m(b + b^\dagger)\sigma_z}_{Mech.\ coupling} + \underbrace{\hbar\Omega b^\dagger b}_{phonon\ mode}$$

- b phonon annihilation operator
- σ_+ = |e><g|
- σ_- = |g><e|



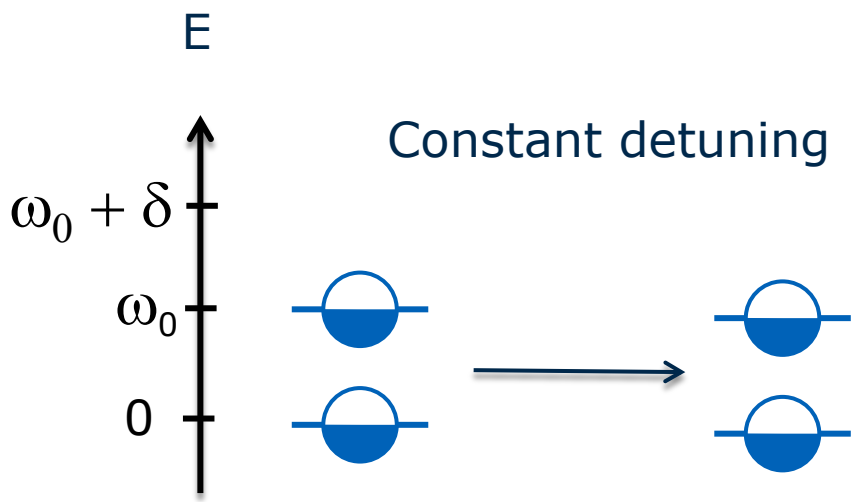
Energy balance during the erasure



$$\Delta U = -\frac{\hbar\omega_0}{2} = \hbar \int d(P_e(\omega_0 + \delta))$$

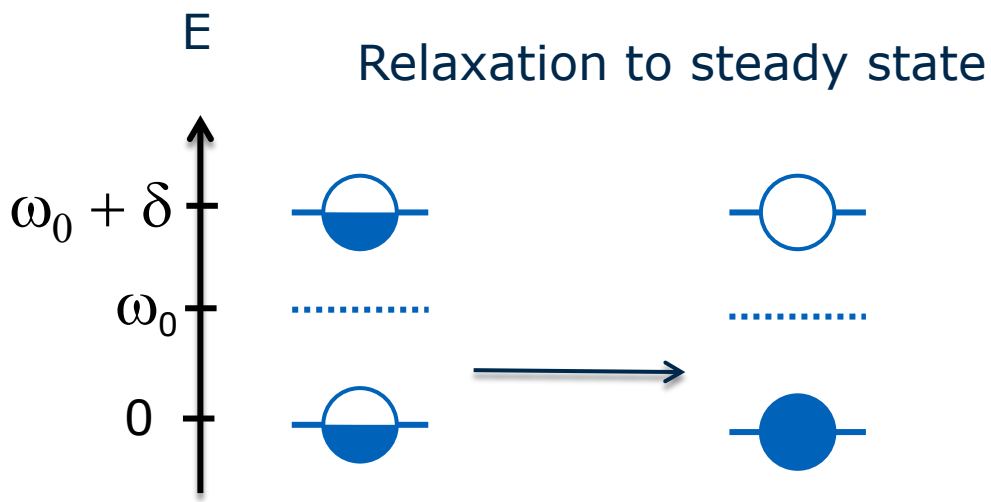
$$= \underbrace{\hbar \int_0^{\delta_f} P_e d\delta}_{W_L} + \overbrace{\underbrace{\hbar \int_0^{\delta_f} \omega_0 \frac{dP_e}{d\delta} d\delta}_{-\hbar\omega_0/2} + \underbrace{\hbar \int_0^{\delta_f} \delta \frac{dP_e}{d\delta} d\delta}_{Q_L = -W_L}}^{\Delta E_{em}}$$

Heat ≠ all the photon emitted



- $\Delta U = 0$, $W = 0$ } $Q = 0$
- But Rabi oscillations cause many photon emissions

Heat ≠ all the photon emitted



When detuning is varying, the change in emitted photons frequency cause an energy excess which is equivalent to the dissipated heat