Reversible information-energy conversions in a quantum hybrid system

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Maxwell’s demon paradox

The demon can decrease the entropy of the gas without paying work!

J.C. Maxwell - 1871
Actually, the demon stores the information about the particles in its memory...

L. Brillouin - 1949
...and work is required to erase the memory and thus to perform cycles!

R. Landauer - 1961
Landauer’s Erasure of 1 bit

Where Shannon’s entropy of the bit is:

\[
H = - P_1 \log_2 P_1 - P_0 \log_2 P_0
\]

(in bits)
Landauer’s principle

\[ W \geq W_0 = kT \ln 2 \]
Landauer’s Erasure of a bit

If the erasure is a reversible (very slow) transformation:

\[ W = kT \ln 2 \]
If the erasure is a *reversible* (very slow) transformation:

$$W = kT \ln 2$$

where $P_0 = 50\%$, $P_1 = 50\%$, or $P_0 = 100\%$. 

**Szilard’s engine**

$H_i = 1 \quad \rightarrow \quad H_f = 0$

**Work extracted** $W$
$W_0 = kT \ln 2$ is the elementary work corresponding to 1 bit of information.
If information becomes quantum…

**EPR state**

\[
\rho_{AB} = |\Psi\rangle\langle\Psi| \\
\Psi = (|0_A1_B\rangle + |1_A0_B\rangle)/\sqrt{2}
\]

Alice’s point of view

\[\text{Tr}_B \rho_{AB} = \mathbb{I}/2\]

Maximally mixed state, no work extraction possible

Global point of view

Pure state, \(H = 0\)

Can perform a Szilard engine and convert the information into work
If information becomes quantum…

EPR state

$$\rho_{AB} = |\Psi\rangle\langle\Psi|$$

The peculiarities of quantum information let their imprint on the work we can extract / have to pay

$$\text{Tr}_B \rho_{AB} = \text{Maximally mixed state, no work extraction possible}$$

$$\Rightarrow \text{Many theoretical results linking work to quantum correlations, discord, entanglement...}$$

L. del Rio et al., Nature 474, 61--63 (2011)
Oppenheim, Horodecki, PRL 89 (2002)
Zurek, PRA 67 (2003) …

Alice’s point of view

$$\text{Tr}_B \rho_A = 0$$

Global point of view

Zubrzilard engine

Information

Work

$$\Rightarrow \text{Information into work}$$
To check quantum info thermodynamics theorems we need:

- Reversible transformations
- State of the battery monitored
- Battery
- qubit
- Heat bath
- $W$
• I) Standard Landauer’s erasure protocol
  • Standard protocol
  • Optical protocol

• II) A battery enabling to monitor work exchanges

• III) Full cycle of energy-information conversions

• IV) Proposal for optical Carnot engine
Standard erasure protocol

\[ H_i = 1 \text{ bit} \]

Heat bath \( T \) at \( t = 0 \)
Standard erasure protocol

$H_i = 1 \text{ bit}$

Heat bath $T$

External Operator

$t = 0$
Standard erasure protocol

Work performed by the operator while raising one of the states

\[ W(t) = \int_0^t P(E) \, dE \]

Population of the state

\[ t = 0 \]

\[ H_i = 1 \text{ bit} \]

Heat bath \( T \)

External Operator

Heat bath \( T \)

Work performed by the operator while raising one of the states

\[ W(t) = \int_0^t P(E) \, dE \]
The qubit is at any time in equilibrium with the bath

\[ P(E) = e^{-E/kT}/Z \]
The qubit is at any time in equilibrium with the bath

\[ P(E) = \frac{e^{-E/kT}}{Z} \]
The qubit is in a known state and isolated from the bath
Szilard engine protocol

The empty state is raised with no work cost

\( W = 0 \)

Heat bath \( T \)

Battery

\( E \gg kT \)

\( t < 0 \)
The qubit is put in equilibrium with the bath

$t = 0$
The empty state is lowered very slowly. For an energy low enough it gets populated.

\[ 0 < t < t_f \]
Then lowering the occupied state enables to extract work

$$E \sim kT$$

$$\begin{array}{c}
E \\
\downarrow
\end{array} \begin{array}{c}
W
\end{array}$$

$$\begin{array}{c}
Battery
\end{array}$$

$$\begin{array}{c}
Q
\end{array}$$

$$\begin{array}{c}
Heat\ bath
\end{array}$$

$$0 < t < t_f$$
The elementary work $W_0$ is extracted from the bath and stored in the battery.
Direct implementation faces challenges:
- reaching reversiblility
- measuring work
Outline

• I) Standard Landauer’s erasure protocol
  • Standard protocol
  • Optical protocol

• II) A battery enabling to monitor work exchanges

• III) Full cycle of energy-information conversions

• IV) Proposal for optical Carnot engine
An optical version of the protocol

**Resonantly driven atom**
- $\gamma$ spontaneous emission rate
- $g$ classical Rabi frequency
- $\delta$ atom-laser detuning

**Totally mixed state**

**Non resonantly driven atom**
- $g$ atom-laser detuning

**Saturated regime**
- $g \gg \gamma$
An optical version of the protocol

Bringing the atom out of resonance, we erase the qubit encoded in its states

\[ \gamma \] spontaneous emission rate
\[ g \] classical Rabi frequency
\[ \delta \] atom-laser detuning
The laser mimicks a bath

After some $1/\gamma$, the population of the excited state is in the steady state:

$$\frac{1/2}{1 + (\delta / g)^2}$$

The Rabi frequency $g$ plays the role of the bath temperature
After some $1/\gamma$, the population of the excited state is in the steady state:

$$P_e(\delta) = \frac{1/2}{1 + (\delta / g)^2}$$

The effective thermalization time $1/\gamma$ is very short → Reaching reversibility is easier than with a thermal bath.
After some $1/\gamma$, the population of the excited state is in the steady state:

$$P_e(\delta) = \frac{1/2}{1 + (\delta/g)^2}$$

$\delta$ must vary on a time scale slow with respect to $1/\gamma$.
A colored bath

**Thermal bath**

\[ P_e(E) = \frac{e^{-E/kT}}{1 + e^{-E/kT}} \]

**Optical bath**

\[ P_e(\delta) = \frac{1/2}{1 + (\delta/g)^2} \]

- The right side of the plot is very similar.
- Behaviour is different for negative detuning.
A new value of the elementary work

Thermal bath

\[ P_e(E) = \frac{e^{-E/kT}}{1 + e^{-E/kT}} \]

Optical bath

\[ P_e(\delta) = \frac{1/2}{1 + (\delta / g)^2} \]

\[ W_0 = kT \ln 2 \quad \longleftrightarrow \quad W_L = \hbar \int_0^\infty P_e(\delta) d\delta = \hbar g \frac{\pi}{4} \]
\[ W_L = \hbar g \frac{\pi}{4} \]

\( \gamma \) spontaneous emission rate
\( g \) classical classical Rabi frequency
\( \delta \) atom-laser detuning

We need an external operator able to increase the detuning adiabatically and that we can monitor.
• I) Standard Landauer’s erasure protocol
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Specifications for the external operator device

This device must

1) Evolve very slowly with respect to the effective thermalization time $1/\gamma$ (adiabtic evolution)

2) Change its state in a measurable way when it performs work on the qubit
Specifications for the external operator device

This device must

1) Evolve very slowly with respect to the effective thermalization time $1/\gamma$ (adiabtic evolution)

2) Change its state in a measurable way when it performs work on the qubit

→ A solution: couple the atom to an oscillating nanowire
Hybrid optomechanical set up

**External operator:**
Oscillating nanowire.

**Bath:**
Laser resonant with the bare atomic frequency + vacuum

**Set up : nano ‘trumpets’**
I.Yeo et al., arXiv:1306.4209 (2013)
accepted in Nature Nano

**Qubit:**
Artificial atom (Quantum dot).
Strain-mediated coupling

Source: I.Yeo et al., arXiv:1306.4209 (2013), accepted in Nature Nano
Experimental characterization

Fluorescence spectroscopy of the embedded atom

Atomic frequency variation $\delta \omega(t)$ (µeV)

Source: I. Yeo et al., arXiv:1306.4209 (2013), accepted in Nature Nano
Using the coupling to change the detuning

External operator: Oscillating nanowire.

Nano ‘trumpets’

Bath: Laser resonant with the bare atomic frequency + vacuum

Qubit: Artificial atom.

Resonance with the laser

Mechanical oscillation phase (degree)

Detuning $\delta(t)$ ($\mu$eV)

Photon counts $n_c$

Source: I.Yeo et al., arXiv:1306.4209 (2013), accepted in Nature Nano
Using the coupling to change the detuning

Laser

Resonance with the laser

Detuning

Photon counts $n_c$

Phase (degree)

Detuning $\delta(t)$ ($\mu$eV)
Using the coupling to change the detuning

Adiabatic evolution possible: the mechanical frequency is much smaller than $\gamma$
Using the coupling to change the detuning

Laser

Resonance with the laser

Detuning \( \delta(t) \) (\( \mu eV \))

Phase (degree)

Photon counts \( n_c \)

How does the oscillator state change when it performs work?
Model: spin-boson hamiltonian

\[ H = \hbar \omega_0 (\sigma_z + 1/2) + \hbar g (\sigma_+ e^{-i\omega t} + \sigma_- e^{i\omega t}) \]

- **Atom**
- **Coupling with the laser**

**Optomechanical coupling**

\[ + \hbar g_m (b + b^\dagger) \sigma_z \]

**Mechanical mode**

\[ + \Omega b^\dagger b \]

**Coupling strength**

\[ g_m = \frac{\partial \omega_0}{\partial x} \bigg|_{x=0} \delta x_{ZPF} \]

**Zero point fluctuations**

**I. Yeo et al., arXiv:1306.4209 (2013), accepted in Nature Nano**

**A. Auffèves et al., arXiv:1305.4252 (2013)**
We consider a coherent state of the mechanical oscillator $\rightarrow$ Complex amplitude $\beta(t) = \langle b \rangle$

\[ x(t) = 2 \text{Re}(\beta) \delta x_{ZPF} \]
\[ \delta(t) = 2 g_m \text{Re}(\beta) \delta x_{ZPF} \]

**Mechanical energy:**
\[ E_{MO}(t) = \hbar \Omega |\beta(t)|^2 \]

Ex: free oscillations trajectory
Measuring the work of the work

H $\rightarrow$ Optical Bloch equations

\[ E_{MO}(t) = \hbar \Omega |\beta(t)|^2 \]

Mechanical energy

\[ E_{MO}(t) - E_{MO}(0) = \int_0^t dt \, \delta P_e(t) = w(t) \]

Measuring $\beta(t)$ gives access to the work performed on the qubit!

$\rightarrow$ Light deflexion techniques
At $t=0$, we kick the oscillator and let it evolve ...
I) Standard Landauer’s erasure protocol
   - Standard protocol
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Observing work exchanges

Resonance with the laser

\[ \beta(t=0) \]
Observing work exchanges

Resonance with the laser

Free oscillation when the atom is far from resonance:
- Atom erased: $W=0$
- Atom decoupled from the bath
Observing work exchanges

Resonance with the laser

Variation of $|\beta|$ when leaving or coming in resonance $\rightarrow$ exchange of work
Observing work exchanges

Resonance with the laser

Typically $W_L$ corresponds to:

$\Delta x = 0.4$ pm  
Amplitude: 1.2 pm  
Signal/Shot noise = 40  
Signal/Thermal noise = 0.3

$(g = 3$ GHz, $g_m = 30$ MHz,  
$\beta_0 = 10^2$, $\Omega/2\pi = 550$ kHz,  
$T = 100$ mK)

$\rightarrow$ Measurable with current deflexion techniques  
B. Sanii et al. PRL 104 (2010)

Variation of $|\beta|$ when leaving or coming in resonance $\rightarrow$ exchange of work
Observing work exchanges

Resonance with the laser

Typically $W_L$ corresponds to:

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(\(g = 3\) GHz, \(g_m = 30\) MHz, \(\beta_0 = 10^2\), \(\Omega/2\pi = 550\) kHz, \(T = 100\) mK)

NB: \(g_m > \Omega\) needed

Variation of $|\beta|$ when leaving or coming in resonance $\rightarrow$ exchange of work
First quarter of oscillation
Landauer’s erasure

- $W_L$ provided by the battery
- $W_L$ dissipated in the bath
- Qubit erased
Second quarter of oscillation
Szilard’s engine

- $W_L$ stored in the battery
- $W_L$ extracted from the bath
- Qubit in a mixed state
Third quarter of oscillation
Inverse Landauer’s erasure

- $W_L$ stored in the battery
- $W_L$ extracted from the bath
- Qubit erased

Consequence of colored bath
Last quarter of oscillation

Inverse Szilard’s engine

- \( W_L \) provided by the battery
- \( W_L \) dissipated in the bath
- Qubit in a mixed state

Consequence of colored bath
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Principle of the engine

Elementary work

\[ W_L = \hbar g \frac{\pi}{4} \]

Rabi frequency (temperature)

\[ W_{stored} = -\hbar g \frac{\pi}{4} + \hbar g \frac{\pi}{4} + \hbar g \frac{\pi}{4} - \hbar g \frac{\pi}{4} = 0 \]
Principle of the engine

Two different Rabi frequencies $g_2 > g_1$ = thermalization of the qubit with a hot source and a cold source.

$$W_{\text{stored}} = -\hbar g_1 \frac{\pi}{4} + \hbar g_2 \frac{\pi}{4} + \hbar g_2 \frac{\pi}{4} - \hbar g_1 \frac{\pi}{4} > 0$$
Carnot efficiency in finite time

\[ \eta = 1 - \frac{g_2}{g_1} \]

\[ \Leftrightarrow \eta_C = 1 - \frac{T_2}{T_1} \]

Carnot ideal efficiency reached with realistic parameters!
Carnot efficiency in finite time

Carnot ideal efficiency reached

\[ \eta = 1 - \frac{g_2}{g_1} \]

\[ \leftrightarrow \eta_C = 1 - \frac{T_2}{T_1} \]

\[ P = 10^{-17} \text{ W} \]

3 order of magnitudes over existing proposals of single qubit heat engines

O. Abah et al., PRL 109, 203006 (2012).
Conclusion

• A set up enabling reversible information energy conversion in a qubit

• Direct observation of work exchanges in a quantum battery

• Mechanical oscillations perform Carnot cycles at maximum efficiency


O. Arcizet et al., Nature Physics 7 (2011) 879
Now that the building blocks Landauer’s erasure & Szilard engine are ensured, we can go to the fully quantum regime

- Erasure cost of two entangled qubits
  
  L. del Rio et al., Nature 474, 61--63 (2011)

- Measurement of work distribution during conversions → in situ verification of quantum fluctuation theorems
  
  L. Mazzola et al., PRL 110 (2013)


O. Arcizet et al., Nature Physics 7 (2011) 879
Thank you for your attention

$$W_L = \hbar g \frac{\pi}{4}$$

More details in:
Cyril Elouard, Maxime Richard, Alexia Auffèves, arXiv:1309.5276
A double Carnot cycle

1st cycle: Landauer’s erasure + Szilard engine

Equivalent to (P,V)

Equivalent to (S,T)
A double Carnot cycle

2nd cycle: inverse Landauer’s erasure + inverse Szilard engine

Equivalent to (P,V)  
Equivalent to (S,T)
1) Experimental implementation

NV center in a nanowire under magnetic field gradient

O. Arcizet et al., Nature Physics 7 (2011) 879
2) Two entangled atoms $S$ and $Q$

Pure entangled state

\[ \rho_1 = |\Psi\rangle_{QS}\langle\Psi| \]
\[ \Psi_{QS} = (|01\rangle + |10\rangle)/\sqrt{2} \]

Szilard engine on $S$ and $Q$

\[ \rho_2 = |11\rangle\langle11| \]
\[ \rho_3 = \mathbb{I}/4 \]
\[ \rho_4 = |0\rangle_s\langle0| \otimes \mathbb{I}/2 \]

Landauer’s erasure on $S$

\[ \text{Cost } W_L \]

From $S$ point of view

\[ \text{Tr}_Q \rho_1 = \mathbb{I}/2 \]
\[ \text{Erasure of the qubit } S \]
\[ \text{Total work gain } W_L \]
\[ \text{Tr}_Q \rho_3 = |0\rangle_s\langle0| \]

2W_L extracted

From $S$ point of view

\[ \text{Total work gain } W_L \]
3) Direct measurement of work distribution

Can be extracted from quantum jump trajectories of the qubit during erasure

Verification *in situ* of Fluctuation theorems such as Jarzinsky equality

4) Color of the emitted photons

Color of the photon emitted during erasure contain information about the dissipated heat and then must quantify the irreversibility of the engine.

Alternative devices with monitored environment

Pekola et al., arXiv:1212.5808
B.Huard group set up
Hamiltonian describing the optomechanical device

\[ H = \hbar \omega_0 (\sigma_z + 1/2) + \hbar g (\sigma_+ e^{-i\omega t} + \sigma_- e^{i\omega t}) + \hbar g_m (b + b^\dagger) \sigma_z + \hbar \Omega b^\dagger b \]

- \( b \) phonon annihilation operator
- \( \sigma_+ = |e><g| \)
- \( \sigma_- = |g><e| \)

Diagram:
- Laser
  - \( \omega \)
  - \( g \)
  - Rabi freq.
- Atom
  - \( \omega_0 \)
  - Optomech. coupling
- Oscillator
  - \( \Omega \)
Energy balance during the erasure

\[ \Delta U = -\frac{\hbar \omega_0}{2} = \hbar \int d(P_e(\omega_0 + \delta)) \]

\[ \Delta U = \hbar \int_0^{\delta_f} P_e d\delta + \hbar \int_0^{\delta_f} \omega_0 \frac{dP_e}{d\delta} d\delta + \hbar \int_0^{\delta_f} \delta \frac{dP_e}{d\delta} d\delta \]

\[ W_L \quad -\frac{\hbar \omega_0}{2} \quad Q_L = -W_L \]

\[ \Delta E_{em} \]
Heat ≠ all the photon emitted

\[ \omega_0 + \delta \]

Constant detuning

\[ \omega_0 \]

\[ 0 \]

\begin{align*}
\Delta U &= 0 \\
W &= 0
\end{align*}

\[ Q = 0 \]

• But Rabi oscillations cause many photon emissions
Heat ≠ all the photon emitted

When detuning is varying, the change in emitted photons frequency cause an energy excess which is equivalent to the dissipated heat