Noise Resistant Manipulation of Protected Qubits

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Co-Workers

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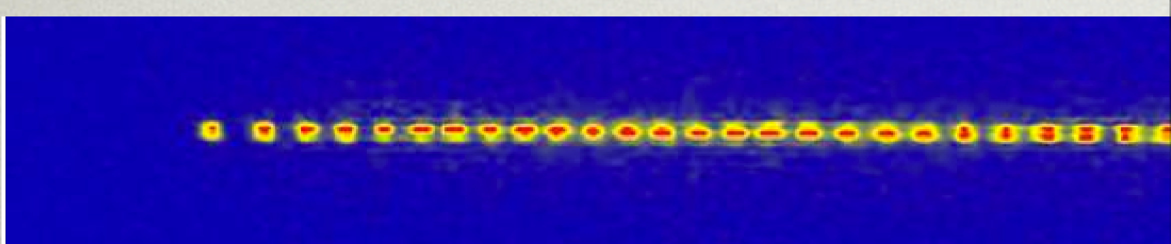
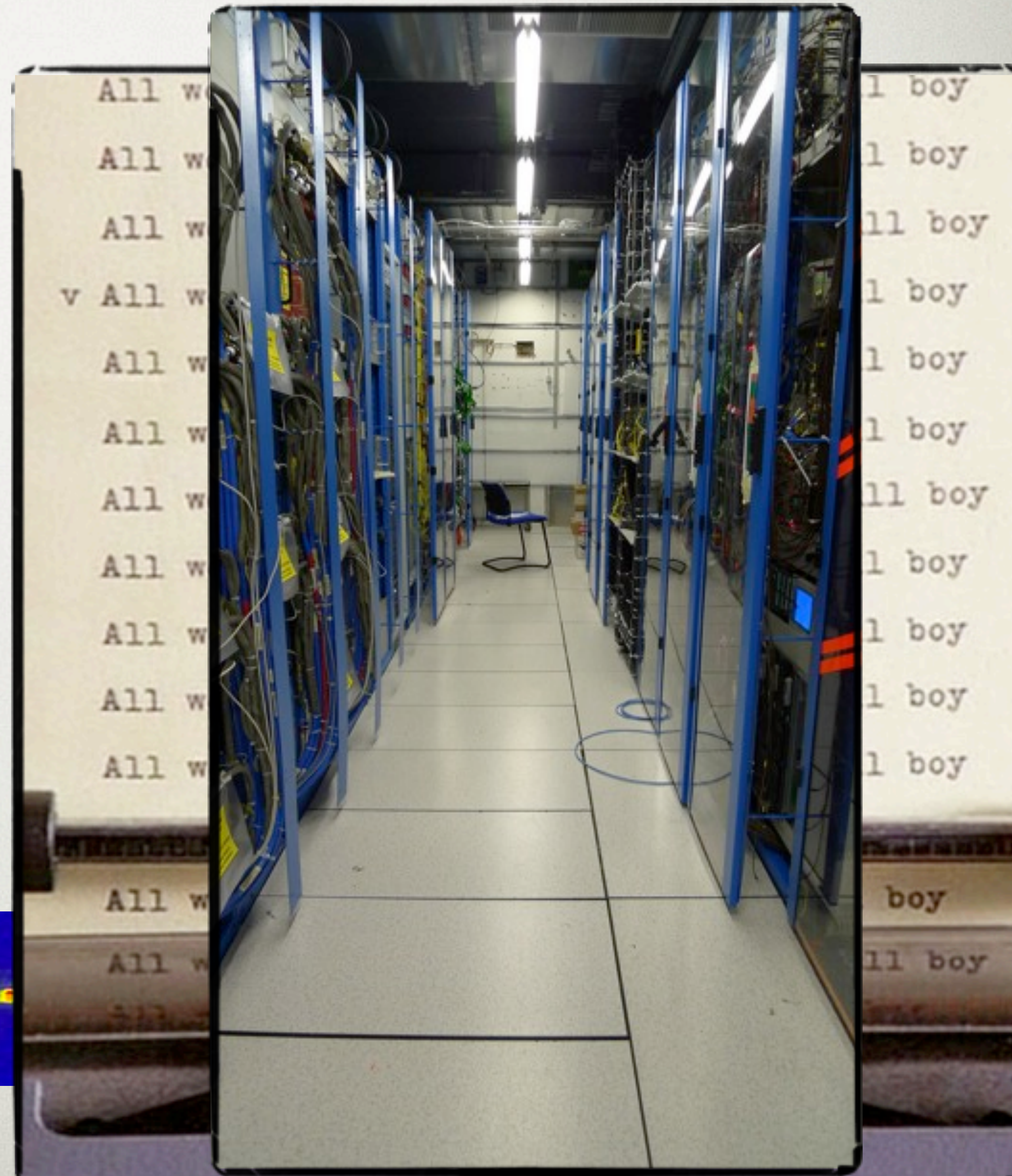
- Romain Dubessy - MPQ, now at LPL (UP13 & CNRS)

Context

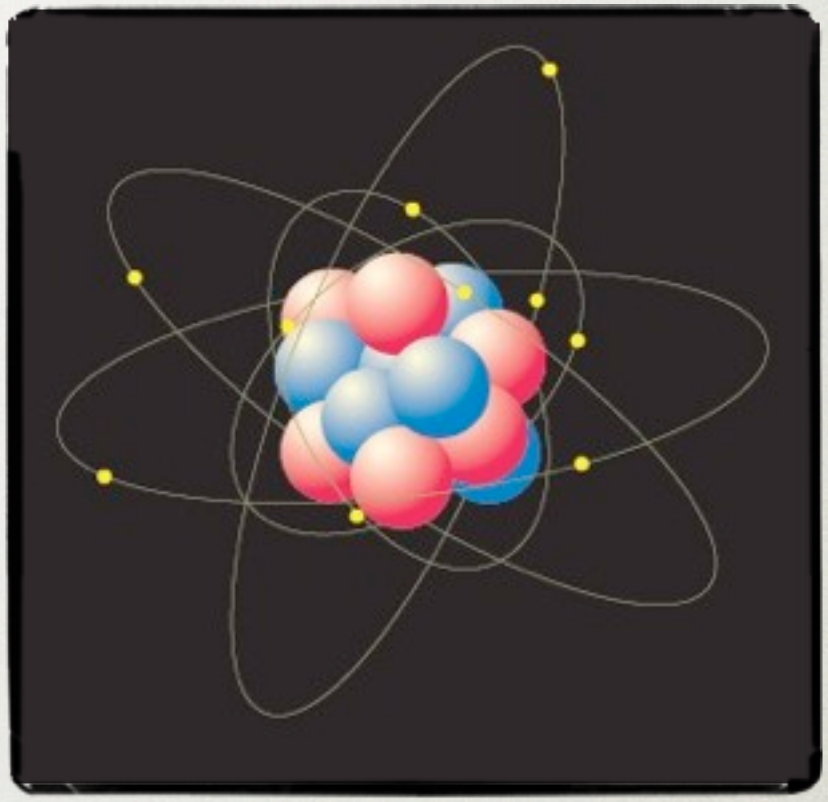
- Qubits are useful
- Qubits are hard to keep & manipulate
- Qubits are fun





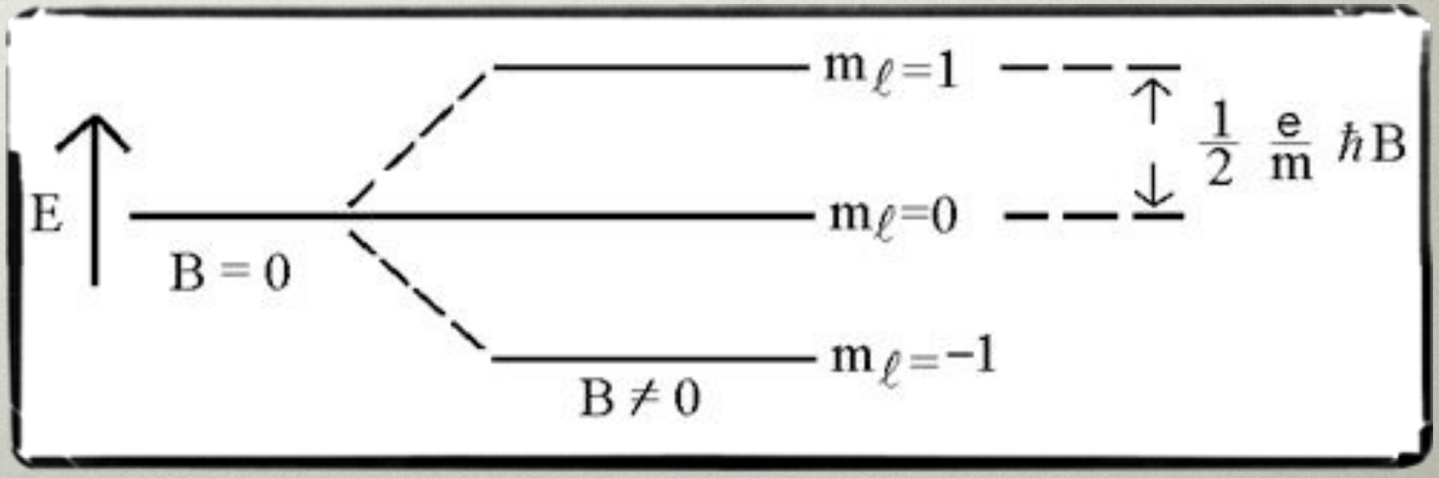
Interacting Spins

- In an atom, spins (neutrons, protons and electrons) are coupled to form a system with discrete energy levels



- The sensitivity of *e.g.* electronic energy level to external perturbation varies

- One can design an artificial atom which is less sensitive to external perturbations



Other Approaches

- CS approach: quantum error correction

PHYSICAL REVIEW A **73**, 012340 (2006)

Operator quantum error-correcting subsystems for self-correcting quantum memories

Dave Bacon

Department of Computer Science & Engineering, University of Washington, Seattle, Washington 98195, USA

(Received 30 June 2005; published 30 January 2006)

- Condensed matter approach: strongly correlated fermions

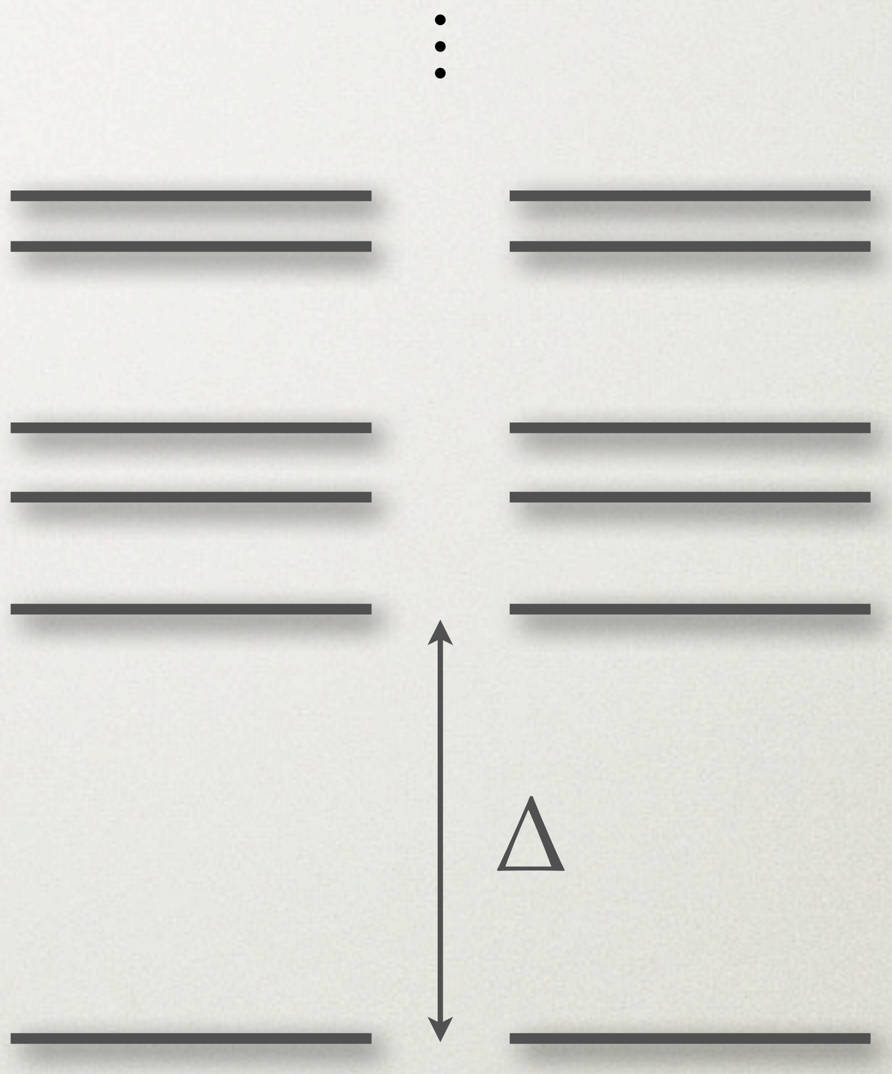
PHYSICAL REVIEW B **71**, 024505 (2005)

Protected qubits and Chern-Simons theories in Josephson junction arrays

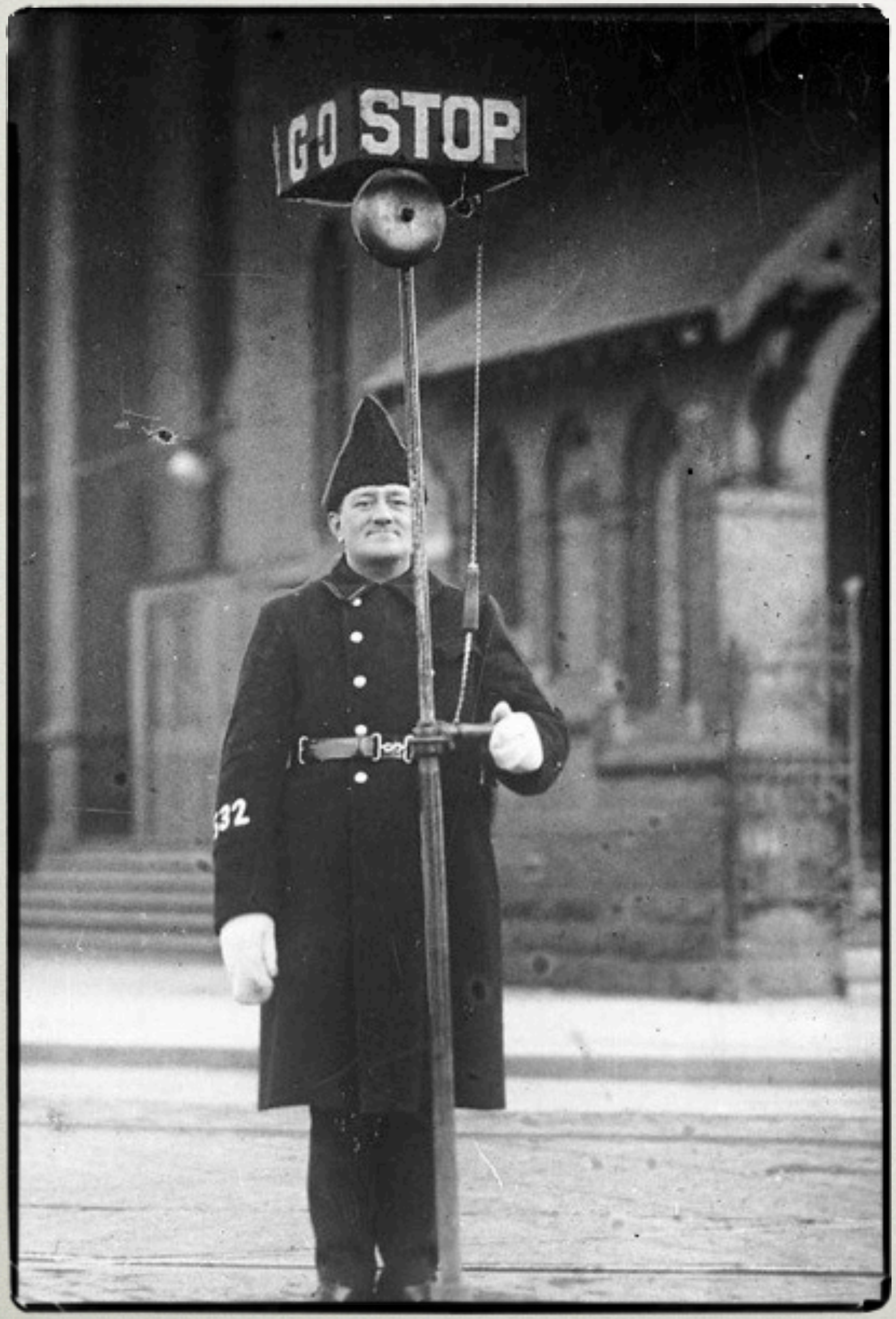
B. Douçot,[1,*] M. V. Feigel'man,[2] L. B. Ioffe,[3,*] and A. S. Ioselevich[2]

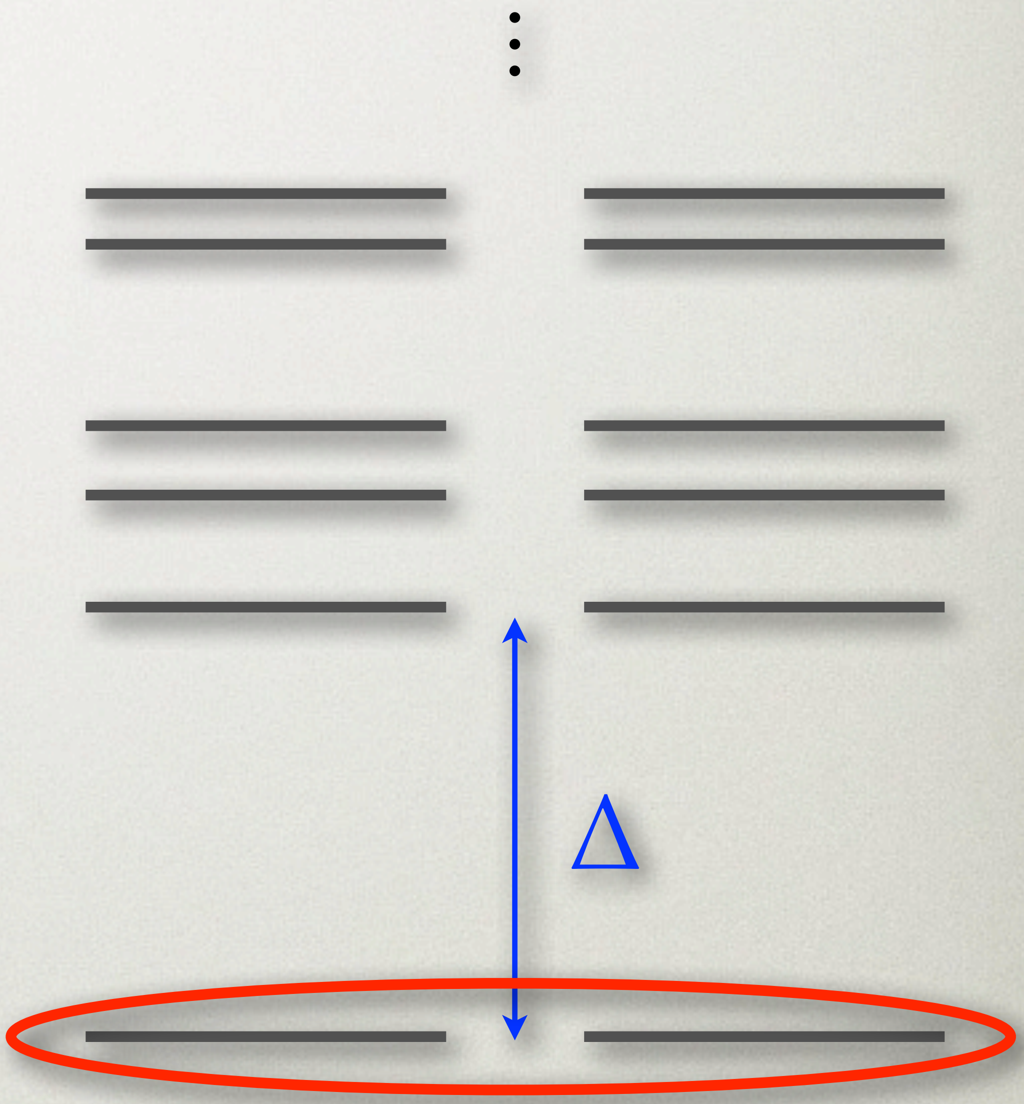
Energetic Error Correction

- Encode a logical qubit in N^2 spins 1/2.
- The qubit is encoded in the **degenerate** ground state of a well-designed Hamiltonian.
- An error costs energy.



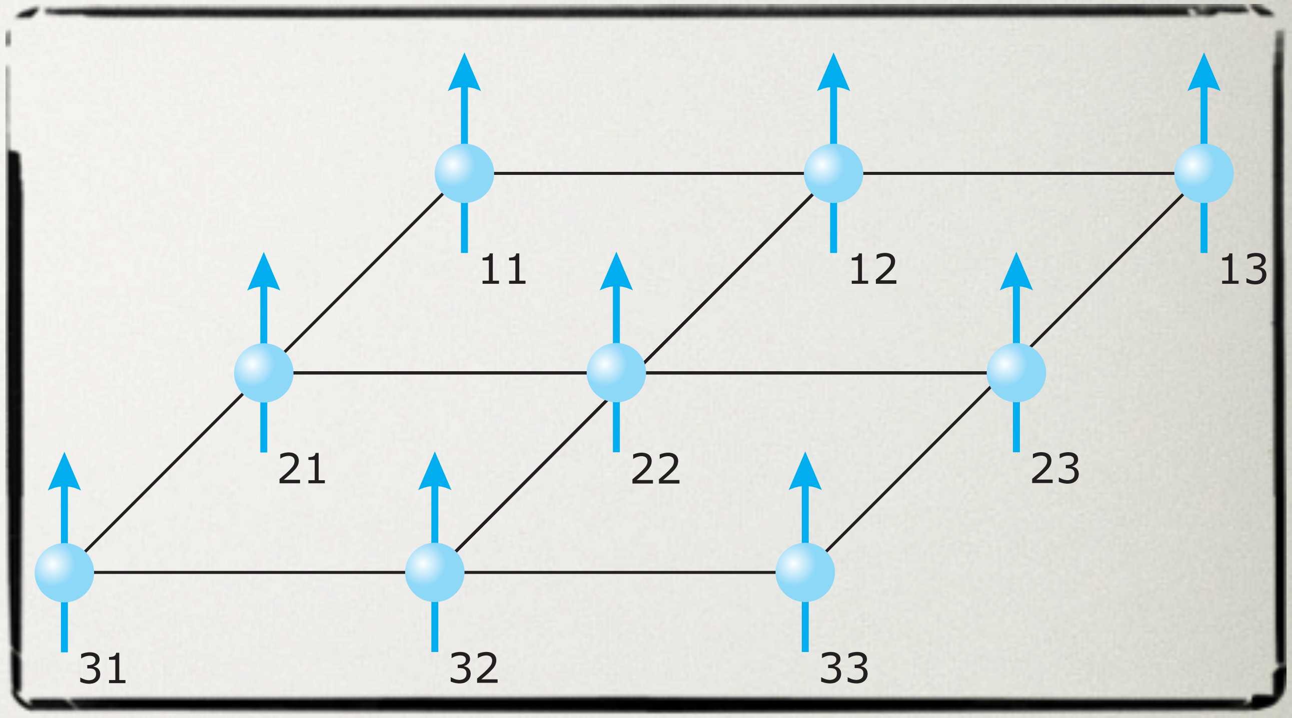
Using Symmetries

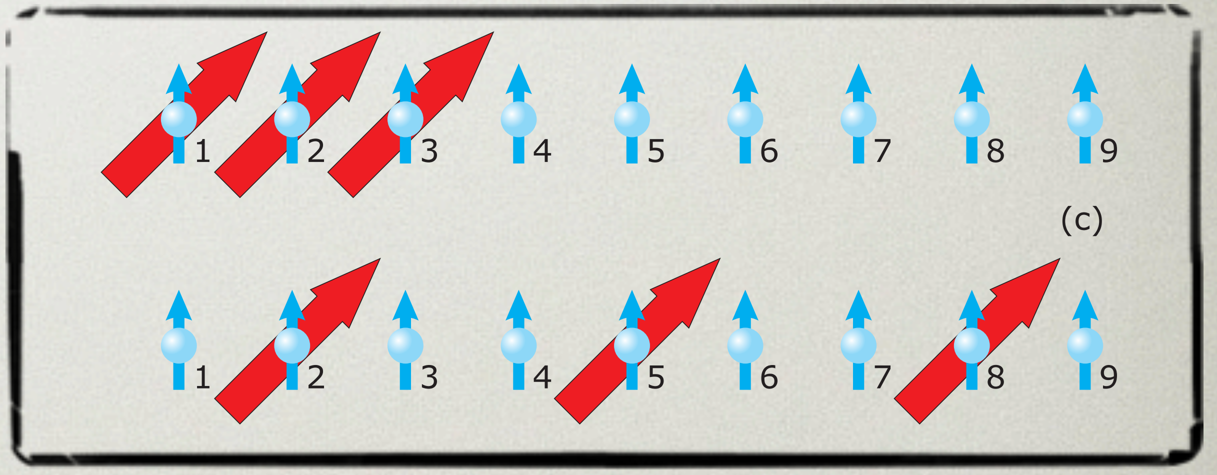




Details: System

- System: square array of NxN spins 1/2
- Implementation:
  - Trapped ions
  - SC qubits

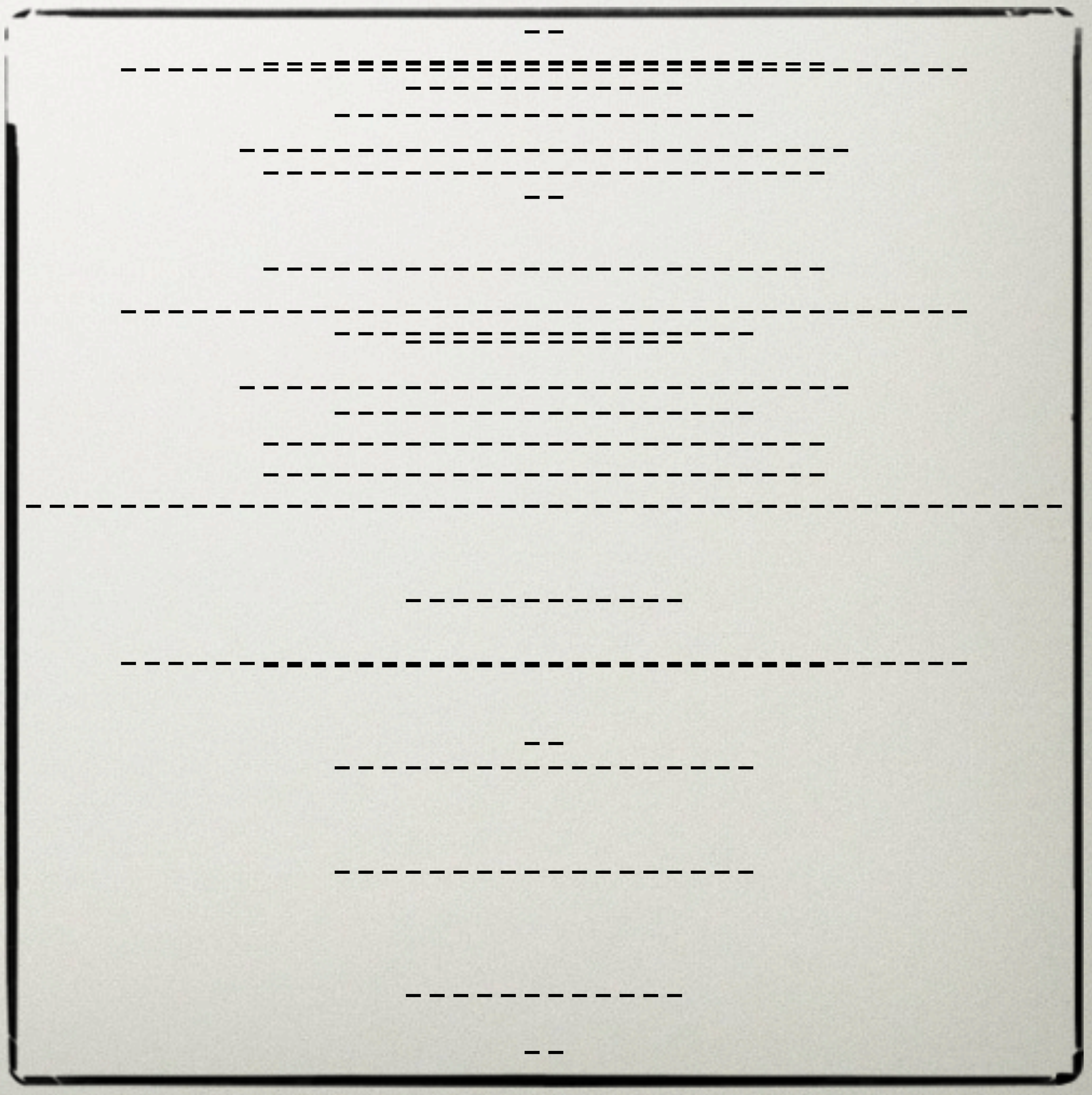




Details: Hamiltonian

$$\hat{H} = -\hbar\chi_x \sum_{\text{rows}} \left(\sum_{\text{col.}} \sigma^x_{i,j} \right)^2 - \hbar\chi_y \sum_{\text{col.}} \left(\sum_{\text{rows}} \sigma^y_{i,j} \right)^2$$

- Analogous to the compass model
- «Large» gap
- Doubly degenerate ground state



Details: Symmetries

- 2N symmetries
- Ensure degeneracy (no exact control of the Hamiltonian required)
- The symmetries act on the qubit ... but are hard to implement

$$\boxed{\begin{aligned}\hat{P}_i &= \prod_j \hat{\sigma}^y_{i,j} \\ \hat{Q}_j &= \prod_i \hat{\sigma}^x_{i,j}\end{aligned}}$$

$$\hat{P}_i|0,1\rangle = \pm|0,1\rangle \Rightarrow \hat{P}_i \equiv \hat{\Sigma}^z$$
$$\hat{Q}_j|0,1\rangle = |1,0\rangle \Rightarrow \hat{Q}_j \equiv \hat{\Sigma}^x$$

Potential Problem

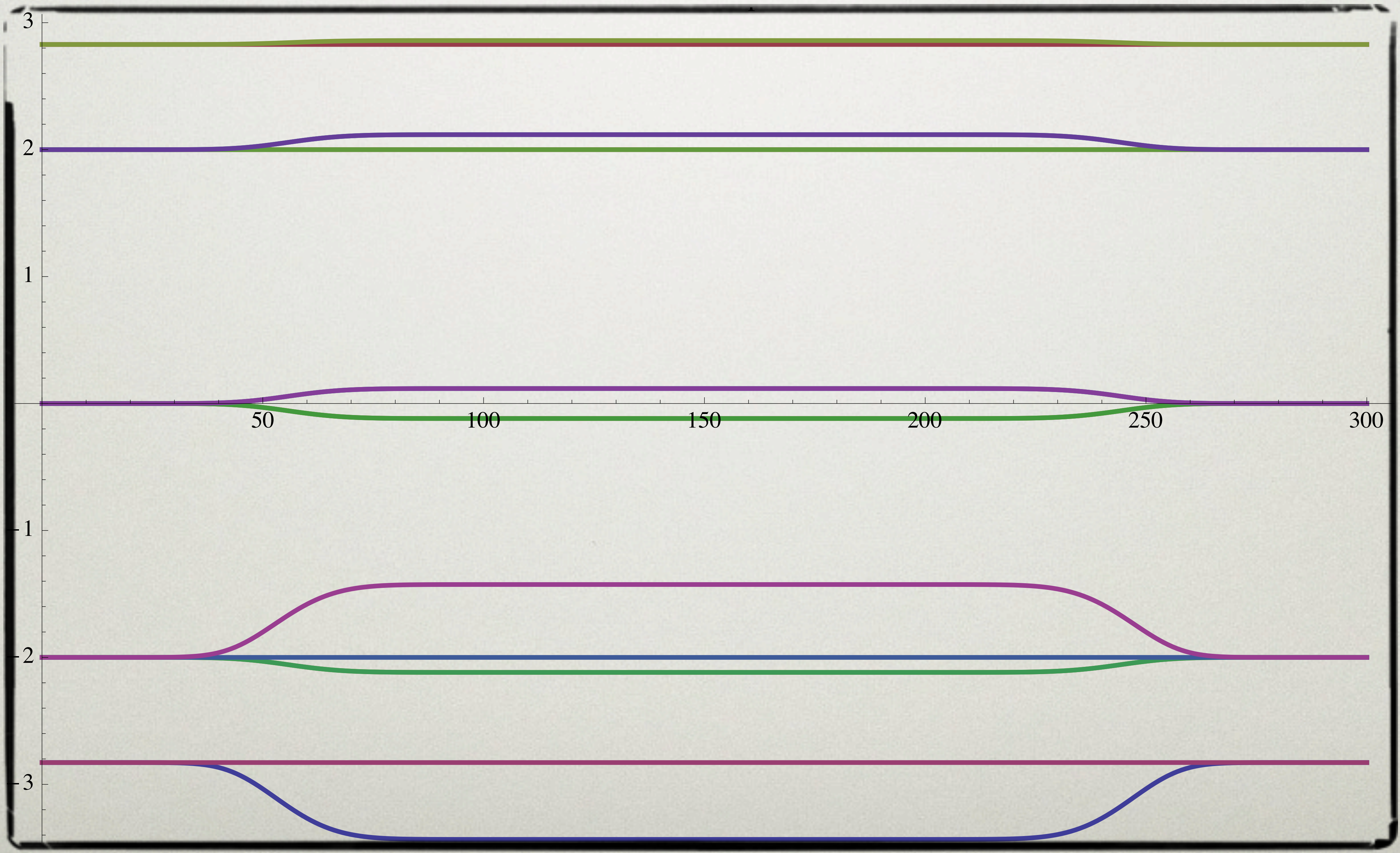
- The protection fails in the thermodynamic limit and for finite temperature $(\Delta \to 0 \leq k_B T)$
- AFAIK, this limit has not been reached experimentally for qubits.

Manipulation: Principle & Problems

- Method: use a time-varying Hamiltonian
- Symmetries prevent manipulation !
- Solution: break some symmetries
- … but what about error correction ?

Constraints

- Single qubit rotation: implement two orthogonal rotations with arbitrary angle
- No level-crossing
- Slow (adiabatic)



Operation

$$\boxed{\alpha|0\rangle + \beta|1\rangle \xrightarrow{\hat{U}} \alpha'|0\rangle + \beta'|1\rangle}$$

$$\hat{U} = \exp\left(-\frac{i}{\hbar}\int \hat{H}(t)dt\right) = \alpha_1 \hat{\mathbb{I}}_2 + \sum_{i=x,y,z} \alpha_i \hat{\Sigma}^i$$

As long as the modification remains adiabatic, the state

remains within the protected subspace

Operation & Symmetries

- For an operation to remain within the logical qubit space, it must either preserve or change **all** eigenvalues **simultaneously**.
- *i.e.,* it must either commute or anti-commute with all the symmetries.

Perturbation Theory

- The properties of U can be deduced from the properties of H
- A given term of the expansion H_M may either change all or preserve all eigenvalues

	Change all p_i	Keep all p_i
Change all q_j	?	?
Keep all q_j	?	?

Using Symmetries (1)

- Hypothesis: no p_i change
- H_M commutes with all the P_i
- H_M commutes with $\sum_z$
- H_M is of the form $I+a_z \sum_z$

- Hypothesis: all q_j change
- H_M anticommutes with all the Q_j
- H_M anticommutes with $\sum_x$
- H_M is of the form $a_y\sum_y+a_z\sum_z$

Symmetries & M$^{\text{th}}$ Order Terms

	Keep all q_j (commutes with Σ_x)	Change all q_j (anticommutes with Σ_x)
Keep all p_i (commutes with Σ_z)	$\alpha_1 \mathbb{I} + \alpha_z \Sigma_z$ $\alpha_1 \mathbb{I} + \alpha_x \Sigma_x$ $\Rightarrow \alpha_1 \mathbb{I}$	$\alpha_1 \mathbb{I} + \alpha_z \Sigma_z$ $\alpha_y \Sigma_y + \alpha_z \Sigma_z$ $\Rightarrow \alpha_z \Sigma_z$
Change all p_i (anticommutes with Σ_z)	$\alpha_x \Sigma_x + \alpha_y \Sigma_y$ $\alpha_1 \mathbb{I} + \alpha_x \Sigma_x$ $\Rightarrow \alpha_x \Sigma_x$	$\alpha_x \Sigma_x + \alpha_y \Sigma_y$ $\alpha_y \Sigma_y + \alpha_z \Sigma_z$ $\Rightarrow \alpha_y \Sigma_y$

- Only based on symmetries
- Valid for any operator and order

Case of $\mathcal{S}_y = \sum_{i,j} \sigma^y_{i,j}$

- At any order S_y conserves all the P_i
- The first line of the above table applies
- $H_M \propto \sigma^y_{i_1,j_1} \sigma^y_{i_2,j_2} \ldots \sigma^y_{i_M,j_M}$
- For $M<N$, at least one q_j is untouched
- The first square of the table applies
- The system is unchanged
- Protection !
- For $M=N$, all q_j change sign
- **Σ_z operation**

Case of $\mathcal{S}_x = \sum_{i,j} \sigma^x_{i,j}$

- At any order S_x conserves all the Q_j
- The first column of the above table applies
- $H_M \propto \sigma^x_{i_1,j_1}\sigma^x_{i_2,j_2}\ldots\sigma^x_{i_M,j_M}$
- For $M<N$, at least one p_i is untouched

- The first square of the table applies
- The system is unchanged
- Protection !
- For $M=N$, all p_i change sign
- **Σ_x operation**

Strength of the manipulation

- The dominant term is the N^{th} order term.
- It depends on
  - the strength of the manipulation
  - the gap
  - the manipulation time
- Rotation angle $\propto \left(\frac{g^u_{max}}{\Delta}\right)^N \frac{\Delta t}{\hbar}$

Noise Model

- Noises are considered local
- $$H_N = \sum_{i,j} f_{i,j} \sigma_{i,j}^{u_{i,j}}$$
- Only the dominant term is considered relevant
- $H_N = f \sum_{i,j} \sigma_{i,j}^{u}$ where $f = \mathrm{Max}_{i,j}\left(f_{i,j}\right)$
- Although this model may seem simplistic it is very general

Manipulation and Noise are Parallel

- The contribution of the noise adds directly on the manipulation
- $\propto \left(\frac{g^u_{max} + f}{\Delta}\right)^N \frac{\Delta t}{\hbar}$
- Change of the rotation angle
- Noise acts like it would in standard qubit manipulation

Manipulation and Noise are Orthogonal

- Noise and manipulation add
- The Mth order term of the expansion reads

$$H_M \propto \left(\sigma^u_{i_1,j_1}\sigma^u_{i_2,j_2}\ldots\sigma^u_{i_m,j_m}\right)\left(\sigma^v_{i_1,j_1}\sigma^v_{i_2,j_2}\ldots\sigma^v_{i_n,j_n}\right)$$

with $m+n = M$

- Example : $u=y$ and $v=x$
- The manipulation acts on q_j and noise on p_i

Effect of the Noise

- $H_M \propto \left(\sigma^y_{i_1,j_1}\sigma^y_{i_2,j_2}\ldots\sigma^y_{i_m,j_m}\right)\left(\sigma^x_{i_1,j_1}\sigma^x_{i_2,j_2}\ldots\sigma^x_{i_n,j_n}\right)$
- The dominant term corresponds to $m+n=N$
- The smallest order for manipulation is
  - no q_j change sign $\Rightarrow m=0$
  - all q_j change sign $\Rightarrow m=N$
- The smallest (non zero) order for noise is
  - no p_i change sign $\Rightarrow n=2$ (two changes in the same row)
  - all p_i change sign $\Rightarrow n=N$

Effect of the Noise

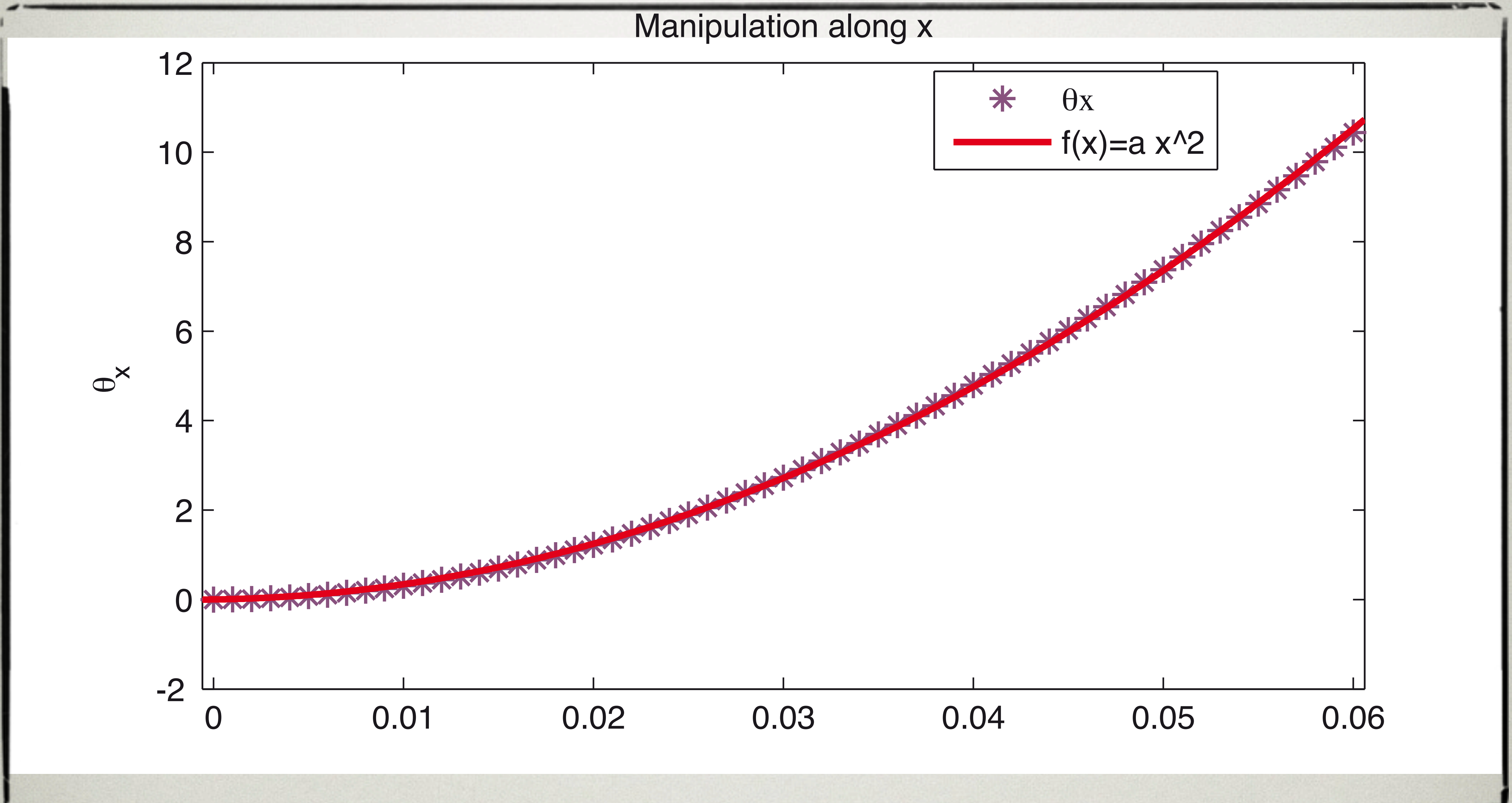
Noise \ Manipulation	Keep all q_j ($m=0$)	Change all q_j ($m=N$)
Keep all p_i ($n=2$)		$\left(\frac{f}{\Delta}\right)^2 \left(\frac{g}{\Delta}\right)^N / \left(\frac{g}{\Delta}\right)^N \Rightarrow \left(\frac{f}{g}\right)^2$
Change all p_i ($n=N$)	$\left(\frac{f}{\Delta}\right)^N / \left(\frac{g}{\Delta}\right)^N \Rightarrow \left(\frac{f}{g}\right)^N$	$\left(\frac{f}{\Delta}\right)^N \left(\frac{g}{\Delta}\right)^N / \left(\frac{g}{\Delta}\right)^N \Rightarrow \left(\frac{f}{\Delta}\right)^N$

- Noise is **always** reduced at least quadratically

Numerical Simulations

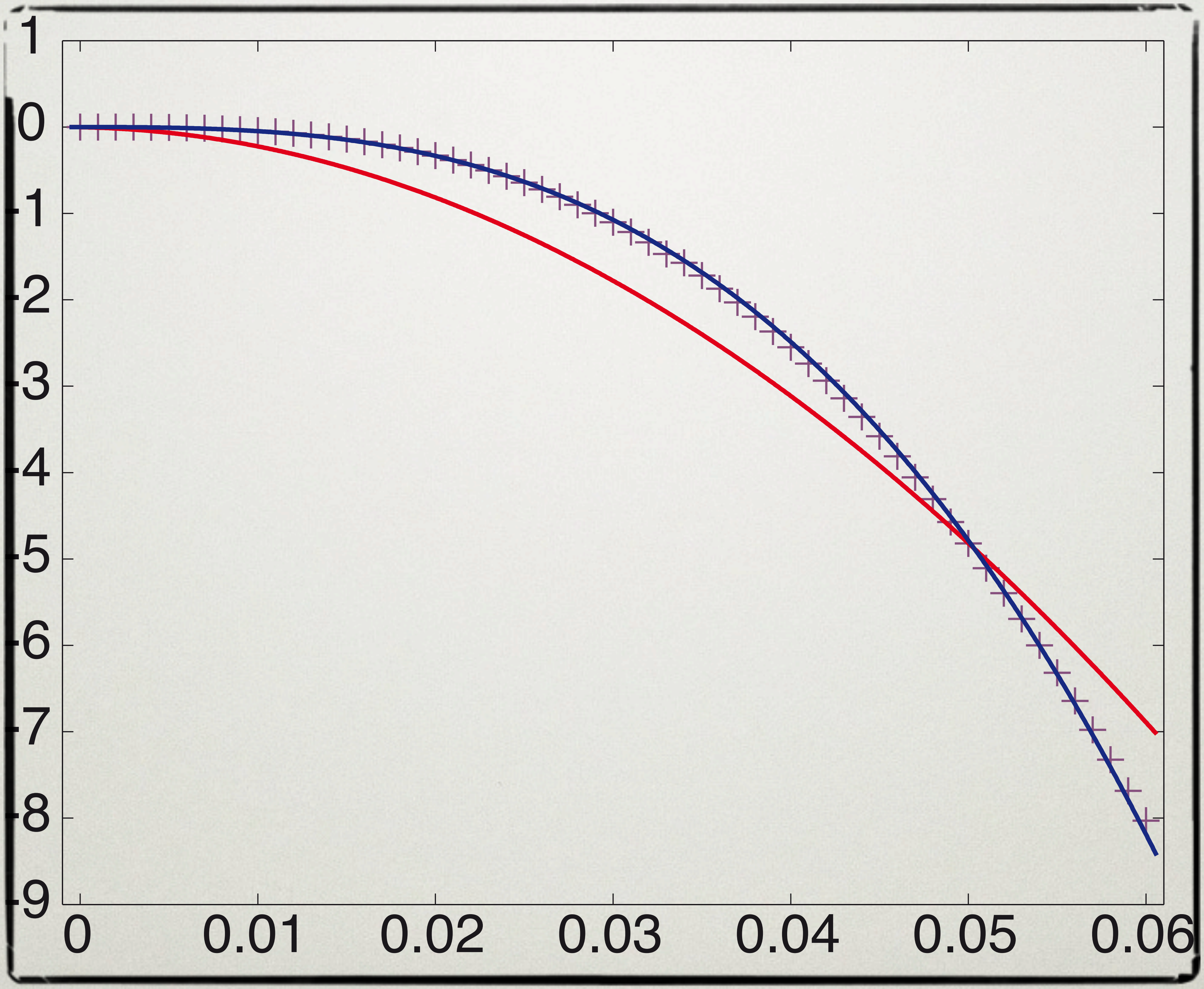
- Matlab
- 2x2 or 3x3 systems

Manipulation 2x2



- Quadratic variation of the rotation angle

Manipulation : 3x3



- Cubic variation of the rotation angle

Conclusion

- Qubit protection against **arbitrary** noise
- Efficient & noise-resistant preparation
- Efficient & noise-resistant manipulation
- References
  - P. Milman *et al*., Phys. Rev. Lett. **99** 020503 (2007)
  - T. Coudreau *et al,* Phys. Rev. Lett. **107** 030502 (2011)

Perspectives

- Further precise experimental conditions
- Two qubits manipulation
- Information propagation on spin chains

Announcement: CS-Physics Graduate School

- Who: graduate students in CS & physics ?
- Why: strengthen the links between both fields
- When: fall 2012 / spring 2013
- Where: ?